EE359 – Lecture 5 Outline

- **Announcements:**
  - HW posted, due Friday 5pm
  - Background on random processes in Appendix B
  - Afternoon blues cure: Chocolate!

- **Review of Last Lecture: Narrowband Fading**
- **Auto and Cross Correlation of In-Phase and Quadrature Signal Components**
- **Correlation and PSD in uniform scattering**
- **Signal Envelope Distributions**
- **Wideband Channels and their Characterization**
Review of Last Lecture

- Model Parameters from Measurements
- Random Multipath Model
- Channel Impulse Response

\[ c(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j\phi_n(t)}\delta(\tau - \tau_n(t)) \]

- Many multipath components, Amplitudes change slowly, Phases change rapidly
- For delay spread \( \max |\tau_n(t) - \tau_m(t)| \ll 1/B, u(t) \approx u(t-\tau) \).

- Received signal given by

\[ r(t) = R\left\{ u(t)e^{j2\pi f_c t} \sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)} \right\} \]

- No signal distortion in time
- Multipath yields complex scale factor in brackets
Review Continued:
Narrowband Model

- For \( u(t) = e^{j\phi_0} \), \( r(t) = r_I(t)\cos(2\pi f_c t + \phi_0) - r_Q(t)\sin(2\pi f_c t + \phi_0) \)
- In phase and quadrature signal components:
  \[
  r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\cos \phi_n(t), \quad r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\sin \phi_n(t)
  \]
  \[
  \phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n}(t) - \phi_0
  \]
- For \( N(t) \) large, \( r_I(t) \) and \( r_Q(t) \) jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If \( \phi_n(t) \) uniform, the in-phase/quad components are mean zero (shown), indep., and stationary (today).
Auto and Cross Correlation

\[ r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)} \cos(2\pi f_c t), \quad r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)} \sin(2\pi f_c t), \quad \phi_n \sim U[0, 2\pi] \]

- Recall that \( \theta_n \) is the multipath arrival angle
- Autocorrelation of inphase/quad signal is
  \[ A_{r_I}(\tau) = A_{r_Q}(\tau) = \text{PE}_{\theta_n} [\cos 2\pi f_{D_n} \tau], \quad f_{D_n} = v \cos \theta_n / \lambda \]
- Cross Correlation of inphase/quad signal is
  \[ A_{r_I,r_Q}(\tau) = \text{PE}_{\theta_n} [\sin 2\pi f_{D_n} \tau] = -A_{r_I,r_Q}(\tau) \]
- Autocorrelation of received signal is
  \[ A_r(\tau) = A_{r_I}(\tau) \cos(2\pi f_c \tau) - A_{r_I,r_Q}(\tau) \sin(2\pi f_c \tau) \]
Uniform AOAs

- Under uniform scattering, in phase and quad comps have no cross correlation and autocorrelation is

\[ A_{r_l}(\tau) = A_{r_Q}(\tau) = PJ_0(2\pi f_D \tau) \]

*Decorrelates over roughly half a wavelength*

- The PSD of received signal is

\[ S_r(f) = .25[S_{r_l}(f - f_c) + S_{r_l}(f + f_c)] \]

\[ S_{r_l}(f) = F[PJ_0(2\pi f_D \tau)] \]

*Used to generate simulation values*
Signal Envelope Distribution

- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models “worse than Rayleigh”
  - Lends itself better to closed form BER expressions
Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
- Requires statistical characterization of $c(\tau, t)$
  - Assume CLT, stationarity and uncorrelated scattering
  - Leads to simplification of its autocorrelation function

\[ \Delta \tau < \frac{1}{B_u} \]

Narrowband

\[ \Delta \tau > \frac{1}{B_u} \]

Wideband

\[ \Delta \tau_1 \quad \Delta \tau_2 \]
Main Points

- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath

- Uniform scattering makes autocorrelation of inphase and quad comps of RX signal follow Bessel function
  - Signal components decorrelate over half wavelength
  - The PSD has a bowel shape centered at carrier frequency

- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common

- Wideband channels have resolvable multipath
  - Will statistically characterize $c(\tau,t)$ for WSSUS model