Lecture 6 - EE 359: Wireless Communications - Autumn 2016

**Wideband Fading. Doppler and Delay Spread. Introduction to Channel Capacity.**

Lecture Outline

- Wideband Channel Models
- Scattering Function
- Multipath Intensity Profile, Delay Spread, and Coherence Bandwidth
- Doppler Power Spectrum, Doppler Spread, and Coherence Time
- Introduction to Channel Capacity

1. **Wideband Channel Models**
   - In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
   - If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

2. **Channel Scattering Function:**
   - Typically time-varying channel impulse response $c(\tau, t)$ is unknown, so its wideband model must be characterized statistically.
   - Since under our random model with a large number of scatterers, $c(\tau, t)$ is Gaussian. We assume it is WSS, so we only need to characterize its mean and correlation, which is independent of time. Similar to narrowband model, for $\phi_n$ uniformly distributed, $c(\tau, t)$ has mean zero.
   - Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t)\delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.
   - Statistical scattering function defined as $S(\tau, \rho) = F_{\Delta \tau}[A_c(\tau, \Delta t)]$. This function measures the average channel gain as a function of both delay $\tau$ and Doppler $\rho$.
   - $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread $T_M$, rms delay spread $\sigma_\tau$, and Doppler spread $B_d$ for empirical channel measurements.

3. **Multipath Intensity Profile and Delay Spread**
   - Multipath intensity profile (delay power spectrum) defined as $A_c(\tau; \Delta t = 0) \triangleq A_c(\tau)$, i.e. the autocorrelation relative to delay $\tau$ at a fixed time.
   - The average delay $T_m$ and rms delay spread $\sigma_\tau$ are defined relative to $A_c(\tau)$. These parameters approximate the maximum delay of nontrivial multipath components.

4. **Coherence Bandwidth**
   - The coherence bandwidth is defined relative to the Fourier transform of $A_c(\tau)$, given by $A_C(\Delta f) = F[A_c(\tau)]$. Note that $A_C(\Delta f) = A_C(\Delta f, \Delta t = 0)$. 

Since $A_C(\Delta f)$ is the autocorrelation of a Gaussian process, multipath components separated by $\Delta f_0$ are independent if $A_C(\Delta f_0) \approx 0$.

- By the Fourier transform relationship, the bandwidth over which $A_C(\Delta f)$ is nonzero is roughly $B_c \approx 1/T_m$ or $B_c \approx 1/\sigma_\tau$ (can also add constants to these denominators).
- $B_c$ defines the coherence bandwidth of the channel, i.e. the bandwidth over which fading is correlated.
- A signal experiences frequency selective fading or ISI if its bandwidth exceeds the coherence bandwidth of the channel.

5. Doppler Power Spectrum, Doppler Spread, and Coherence Time:

- Doppler power spectrum is defined with respect to $A_c(\Delta f; \Delta t) = \mathcal{F}_\tau[A_c(\tau, \Delta t)]$.
- Specifically, the Doppler power spectrum is $S_c(\rho) = \mathcal{F}_\Delta t[A_c(\Delta f = 0, \Delta t) \triangleq A_c(\Delta t)]$, which measures channel intensity as a function of Doppler frequency.
- The maximum value of $\rho$ for which $|S_c(\rho)| > 0$ is called the channel Doppler spread, which is denoted by $B_d$.
- By the Fourier transform relationship, $A_c(\Delta t) \approx 0$ for $\Delta t > 1/B_d$. Thus, the channel becomes uncorrelated over a time of $1/B_d$ seconds.
- We define the channel coherence time as $T_c = 1/B_d$. If the coherence time greatly exceeds a bit time, the signal experiences error bursts.

6. Introduction to Channel Capacity

- A channel’s fundamental capacity limit was initiated in Shannon’s pioneering 1948 paper.
- Shannon defined channel capacity mathematically as the maximum mutual information of a channel, which depends on the joint distribution of the channel’s inputs and outputs.
- The significance of this definition comes from Shannon’s coding theorem and converse, which show that capacity is the maximum error-free data rate a channel can support.
- Capacity is a channel characteristic - not dependent on transmission or reception techniques or limitations. In particular, it imposes no constraints on complexity or delay.
- In AWGN, $C = B \log_2(1 + \gamma)$ bps, where $B$ is the signal bandwidth and $\gamma = S/N$ is the received signal-to-noise power ratio.

Main Points

- Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.
- Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.
- Multipath delay spread defines the maximum delay of significant multipath components. Its inverse is the channel coherence bandwidth. Signals separated in frequency by the coherence bandwidth have independent fading.
- Doppler spread defines the channel’s maximum nonzero Doppler. Its inverse is the channel coherence time. Signals separated in time by the coherence time have independent fading.
- Channel capacity defines the maximum error-free data rate of the channel. Has a simple formula for AWGN channels that depends only on the channel bandwidth and SNR.