Wideband Fading, Doppler and Delay Spread

Lecture Outline

- Nakagami Fading Distribution
- Wideband Channel Models
- Scattering Function
- Multipath Intensity Profile, Delay Spread, and Coherence Bandwidth
- Doppler Power Spectrum, Doppler Spread, and Coherence Time

1. Nakagami Fading Distribution:
   - Experimental results support a Nakagami distribution for some environments.
   - Similar to Rician, but can model “worse than Rayleigh.”
   - Model generally leads to closed-form expressions in BER and diversity analysis
   - Envelope distribution given by: 
     \[ p_Z(z) = \frac{2^m z^{2m-1}}{\Gamma(m) P_r} \exp \left[ -\frac{m z^2}{P_r} \right], \quad m \geq .5. \]

2. Wideband Channel Models
   - In wideband multipath channels the individual multipath components can be resolved by the receiver. True if \( T_m > 1/B. \)
   - If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

3. Channel Scattering Function:
   - Typically time-varying channel impulse response \( c(\tau, t) \) is unknown, so its wideband model must be characterized statistically.
   - Since under our random model with a large number of scatterers, \( c(\tau, t) \) is Gaussian. We assume it is WSS, so we only need to characterize its mean and correlation, which is independent of time. Similar to narrowband model, for \( \phi_n \) uniformly distributed, \( c(\tau, t) \) has mean zero.
   - Autocorrelation of \( c(\tau, t) \) is \( A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t) \delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t) \) since we assume channel response associated with different scatterers is uncorrelated.
   - Statistical scattering function defined as \( S(\tau, \rho) = F_\Delta [A_c(\tau, \Delta t)]. \) This function measures the average channel gain as a function of both delay \( \tau \) and Doppler \( \rho \).
   - \( S(\tau, \rho) \) easy to measure empirically and is used to get average delay spread \( T_M \), rms delay spread \( \sigma_\tau \), and Doppler spread \( B_d \) for empirical channel measurements.
4. Multipath Intensity Profile and Delay Spread

- Multipath intensity profile (delay power spectrum) defined as \( A_c(\tau; \Delta t = 0) = A_c(\tau) \), i.e. the autocorrelation relative to delay \( \tau \) at a fixed time.
- The average delay \( T_m \) and rms delay spread \( \sigma_\tau \) are defined relative to \( A_c(\tau) \). These parameters approximate the maximum delay of nontrivial multipath components.

5. Coherence Bandwidth

- The coherence bandwidth is defined relative to the Fourier transform of \( A_c(\tau) \), given by \( A_C(\Delta f) = \mathcal{F}[A_c(\tau)] \). Note that \( A_C(\Delta f) = A_C(\Delta f, \Delta t = 0) \).
- Since \( A_C(\Delta f) \) is the autocorrelation of a Gaussian process, multipath components separated by \( \Delta f_0 \) are independent if \( A_C(\Delta f_0) \approx 0 \).
- By the Fourier transform relationship, the bandwidth over which \( A_C(\Delta f) \) is nonzero is roughly \( B_c \approx 1/T_m \) or \( B_c \approx 1/\sigma_\tau \) (can also add constants to these denominators).
- \( B_c \) defines the coherence bandwidth of the channel, i.e. the bandwidth over which fading is correlated.
- A signal experiences frequency selective fading or ISI if its bandwidth exceeds the coherence bandwidth of the channel.

6. Doppler Power Spectrum, Doppler Spread, and Coherence Time:

- Doppler power spectrum defined as the Fourier transform of \( A_c(\tau = 0; \Delta t) = A_c(\Delta t) \), which is the autocorrelation of \( c(t, \tau) \) for a fixed delay over time \( \Delta t \).
- Specifically, the doppler power spectrum is \( S_c(\rho) = \mathcal{F}[A_C(\Delta f = 0, \Delta t)] \), which measures channel intensity as a function of Doppler frequency.
- The maximum value of \( \rho \) for which \(|S_c(\rho)| > 0\) is called the channel Doppler spread, which is denoted by \( B_d \).
- By the Fourier transform relationship, \( A_c(\Delta t) \approx 0 \) for \( \Delta t > 1/B_d \). Thus, the channel becomes uncorrelated over a time of \( 1/B_d \) seconds.
- We define the channel coherence time as \( T_c = 1/B_d \). If the coherence time greatly exceeds a bit time, the signal experiences error bursts.

Main Points

- Nakagami distribution obtained based on experiments. Exhibits “worse than Rayleigh” behavior.
- Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.
- Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.
- Multipath delay spread defines the maximum delay of significant multipath components. Its inverse is the channel coherence bandwidth. Signals separated in frequency by the coherence bandwidth have independent fading.
- Doppler spread defines the channel’s maximum nonzero Doppler. Its inverse is the channel coherence time. Signals separated in time by the coherence time have independent fading.