Announcements

- No class next Tues, makeup Mon 10/19, 9:30, HEC 18
- Project proposals due next Fri 10/23; recommend meeting
- MT details provided next week

Capacity of Fading channels

- Recap Optimal Rate/Power Adaptation with TX/RX CSI
- Channel Inversion with Fixed Rate

Capacity of Freq. Selective Fading Channels

Linear Digital Modulation Review

Performance of Linear Modulation in AWGN
Review of Last Lecture

- **Channel Capacity**
  - Maximum data rate that can be transmitted over a channel with arbitrarily small error

- **Capacity of AWGN Channel: Blog₂[1+γ] bps**
  - $γ = \frac{P_r}{(N_0B)}$ is the receiver SNR

- **Capacity of Flat-Fading Channels**
  - Nothing known: capacity typically zero
  - Fading Statistics Known (few results)
  - Fading Known at RX (average capacity)

\[
C = \int_0^\infty B \log_2 \left(1 + \gamma \right) p(\gamma) d\gamma \leq B \log_2 (1 + \bar{\gamma})
\]
Review of Last Lecture (ctd)

- Capacity in Flat-Fading: $\gamma$ known at TX/RX

$$C = \max_{P(\gamma): E[P(\gamma)] = \bar{P}} \int_0^\infty B \log_2 \left( 1 + \frac{\gamma P(\gamma)}{\bar{P}} \right) p(\gamma) d\gamma$$

- Optimal Rate and Power Adaptation

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \text{else} \end{cases}$$

$$\frac{C}{B} = \int_{\gamma_0}^\infty \log_2 \left( \frac{\gamma}{\gamma_0} \right) p(\gamma) d\gamma.$$  

- The instantaneous power/rate only depend on $p(\gamma)$ through $\gamma_0$
Channel Inversion

- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
    - Close to optimal
Capacity in Flat-Fading

Rayleigh

Log-Normal
Frequency Selective Fading Channels

- For time-invariant channels, capacity achieved by water-filling in frequency
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
  - Each subband has width $B_c$ (like MCM/OFDM).
  - Independent fading in each subband
  - Capacity is the sum of subband capacities

\[
P = \frac{1}{|H(f)|^2}
\]
Review of Linear Digital Modulation

- Signal over $i$th symbol period:

$$s(t) = s_{i1}g(t)\cos(2\pi f_c t + \phi_0) - s_{i2}g(t)\sin(2\pi f_c t + \phi_0)$$

- Pulse shape $g(t)$ typically Nyquist
- Signal constellation defined by $(s_{i1}, s_{i2})$ pairs
- Can be differentially encoded
- $M$ values for $(s_{i1}, s_{i2}) \Rightarrow \log_2 M$ bits per symbol

- $P_s$ depends on
  - Minimum distance $d_{min}$ (depends on $\gamma_s$)
  - # of nearest neighbors $\alpha_M$
  - Approximate expression:
    - Standard/alternate Q function
    $$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$
Main Points

- Channel inversion practical, but should truncate or get a large capacity loss.

- Capacity of wideband channel obtained by breaking up channel into subbands.
  - Similar to multicarrier modulation.

- Linear modulation dominant in high-rate wireless systems due to its spectral efficiency.

- \[ P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right) \]
  - Alternate Q function useful in diversity analysis.