due May 24, Friday 23:59pm

EE364b Homework 7

7.3 *ADMM for smart grid device coordination.* We consider an electrical grid consisting of N devices that exchange electricity over T time periods. Device i has energy profile $p^i \in \mathbf{R}^T$, with p_t^i denoting the energy consumed by device i in time period t, for $t = 1, \ldots, T$, $i = 1, \ldots, N$. (When $p_t^i < 0$, device i is producing energy in time period t.) Each device has a convex objective function $f_i : \mathbf{R}^T \to \mathbf{R}$, which we also use to encode constraints, by setting $f_i(p^i) = \infty$ for profiles that violate the constraints of device i. In each time period the energy flow has to balance, which means

$$\sum_{i=1}^{N} p_t^i = 0, \quad t = 1, \ldots, T.$$

The optimal profile coordination problem is to minimize the total cost, $\sum_{i=1}^{N} f_i(p^i)$, subject to the balance constraint, with variables p^i, $i = 1, \ldots, N$.

In this problem you will use ADMM to solve the optimal profile coordination problem in a distributed way, with each device optimizing its own profile, and exchanging messages to coordinate all of the profiles.

From this point on, we consider a specific (and small) problem instance. There are three devices: a generator, a fixed load, and a battery, with cost functions described below.

- *Generator.* The generator has upper and lower generator limits: $P^{\min} \leq -p_t \leq P^{\max}$, for $t = 1, \ldots, T$. (Note the minus sign, since a generator's profile is typically negative, using our convention.) The objective is

  $$f_{\text{gen}}(p) = \sum_{t=1}^{T} (\alpha(-p_t)^2 + \beta(-p_t)),$$

  where $\alpha, \beta > 0$ are given constants.
- *Fixed load.* The fixed load has zero objective function and the constraint that its power profile must equal a given consumption profile $d \in \mathbf{R}^T$.
- *Battery.* The battery has zero objective function, and charge/discharge limits given by C and D, respectively: $-D \leq p_t \leq C$, for $t = 1, \ldots, T$. The battery is initially uncharged (*i.e.*, $q_1 = 0$), so its charge level in period t is $q_t = \sum_{\tau=1}^{t-1} p_\tau$ (we neglect losses for this problem). The charge level must be nonnegative, and cannot exceed the battery capacity: $0 \leq q_t \leq Q$, $t = 1, \ldots, T+1$. The charge level is extended to time $T+1$ to allow the battery to charge/discharge in time T, subject to the operational constraints.

(a) Use CVX to solve the problem with data given in `admm_smart_grid_data.m`. Plot the (optimal) power profile for the generator and battery, as well as the battery charge level.

(b) Implement ADMM for this problem (you may use CVX to solve each device's local optimization in each ADMM iteration). Experiment with a few values of the parameter ρ to see its effect on the convergence rate of the algorithm. Plot the norm of the energy balance residual, versus iteration. Plot the power profiles of the generator and battery, as well as the energy balance residual, for several values of iteration (say, after one iteration, after 10 iterations, and after 50). Check the results against the solution found by CVX.