Phase Retrieval and Computer-generated Holography

EE367/CS448I: Computational Imaging
stanford.edu/class/ee367
Lecture 13

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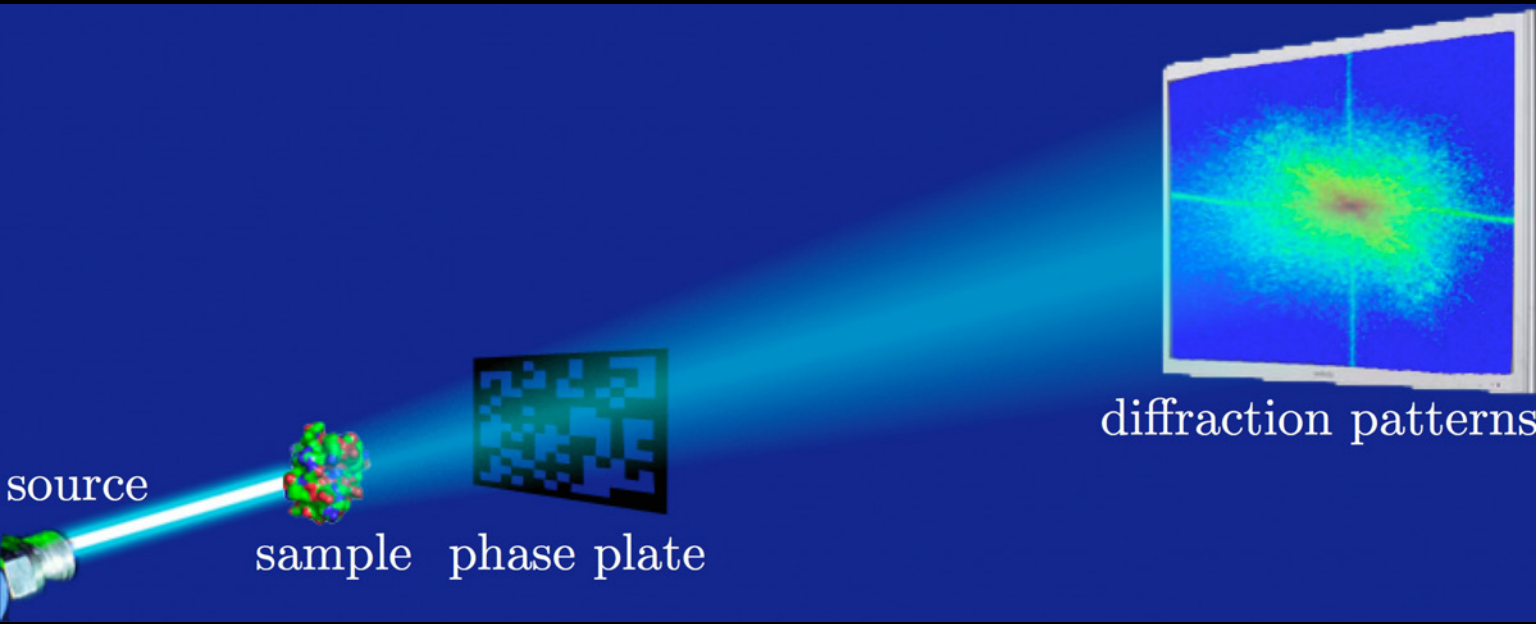
Brief Recap of (some) Wave Optics

- Free-space wave propagation is often modeled by the Fourier transform of the field, i.e., for large distances or with lenses

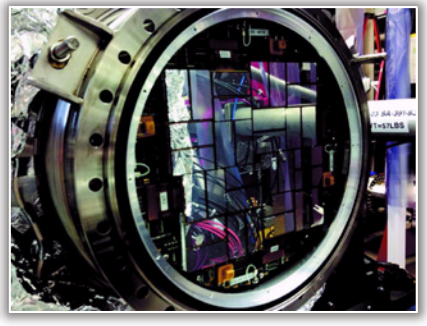
- Cannot measure complex field, only intensity (i.e., amplitude squared)

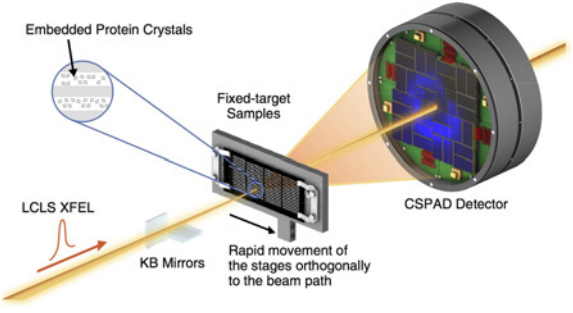
Phase Retrieval



Phase Retrieval @ SLAC

x-ray crystallography or single-particle imaging (SPI)





Applications of Phase Retrieval

- Crystallography
- Transmission electron microscopy
- Astronomy
- Coherent diffractive imaging
- Fourier ptychography
- Lensless imaging
- Computer-generated holography
- …

Phase Retrieval

- Objective function: $\text{minimize}_x\, J(x) = \frac{1}{2}\| |Fx| - b \|_2^2$
- F is discrete Fourier transform matrix, b are amplitude-only measurements, x is complex-valued unknown vector

- Subgradient of objective: $F^H\left((|Fx| - b) \circ \frac{Fx}{|Fx|}\right) \in \nabla J(x)$
- Absolute value not differentiable, but can work with subgradients

Phase Retrieval

- (Sub)gradient descent:

$$
\begin{aligned}
x^{(k+1)} &= x^{(k)} - \alpha \nabla J(x) \\
&= x^{(k)} - \alpha F^H F x^{(k)} + \alpha F^H \left(b \circ \frac{F x^{(k)}}{\left| F x^{(k)} \right|} \right) \\
&\overset{\alpha=1}{\approx} F^H \left(b \circ \frac{F x^{(k)}}{\left| F x^{(k)} \right|} \right)
\end{aligned}
$$

- Interesting: $b \circ \frac{a e^{i\phi}}{|a e^{i\phi}|} = b e^{i\phi}$

Phase Retrieval

- Generalized Gerchberg-Saxton or Error Reduction (ER) algorithm [GS 1972, Fienup 1982]:

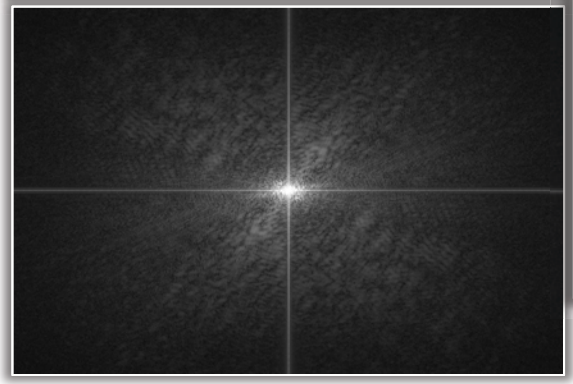
$$x^{(k+1)} = \Pi_{\mathcal{C}} \left(F^H \left(b \circ \frac{F x^{(k)}}{\left| F x^{(k)} \right|} \right) \right)$$

- Projection on feasible set $\Pi_{\mathcal{C}}$ enforces additional constraints, such as nonnegativity of x or limited support, via projected (sub)gradient descent
- Approach is 40 years old, but should be a great starting point

Phase Retrieval - Example

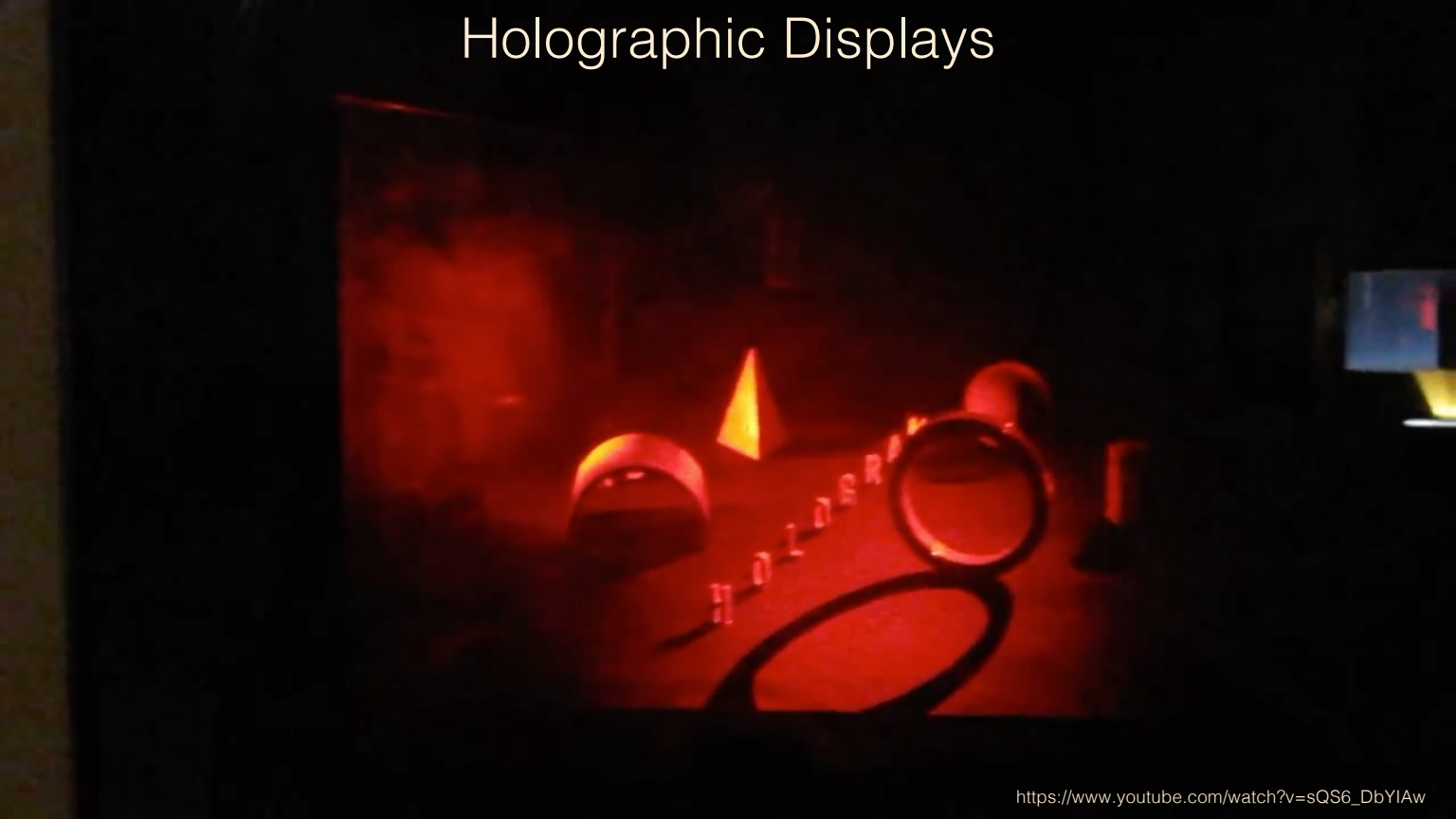
Ground Truth Image

Simulated measurements



Reconstructed Image

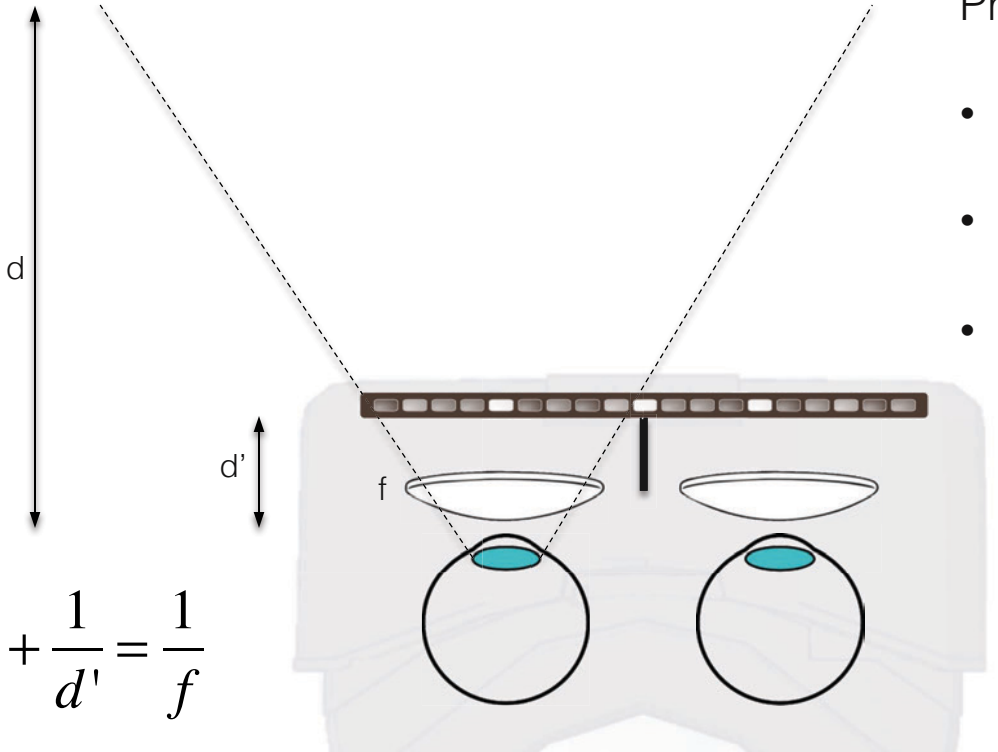
Holographic Displays



https://www.youtube.com/watch?v=sQS6_DbYIAw

Sir Charles Wheatstone, 1838

Virtual Image



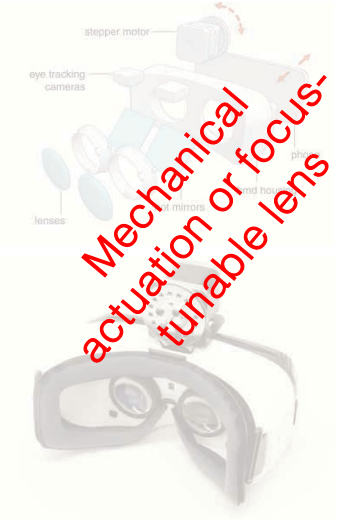
$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f}$$

Problems:

- fixed focal plane
- no focus cues ☹
- vergence-accommodation conflict (nausea)

Computational Near-eye Displays with Focus Cues

Gaze-contingent Varifocal Displays

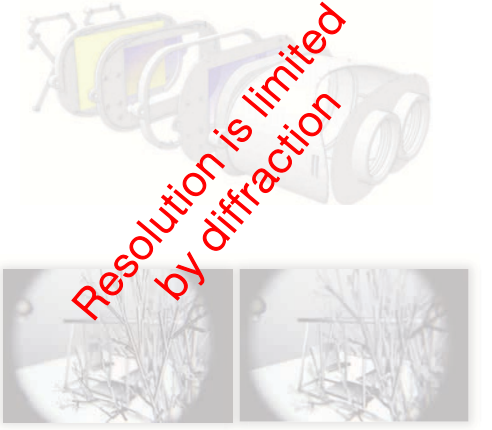


Shiwa et al. 1996; Liu et al. 2008;
Konrad et al. 2016; Padmanaban et al. 2016, 2017; …

Multiplane Displays

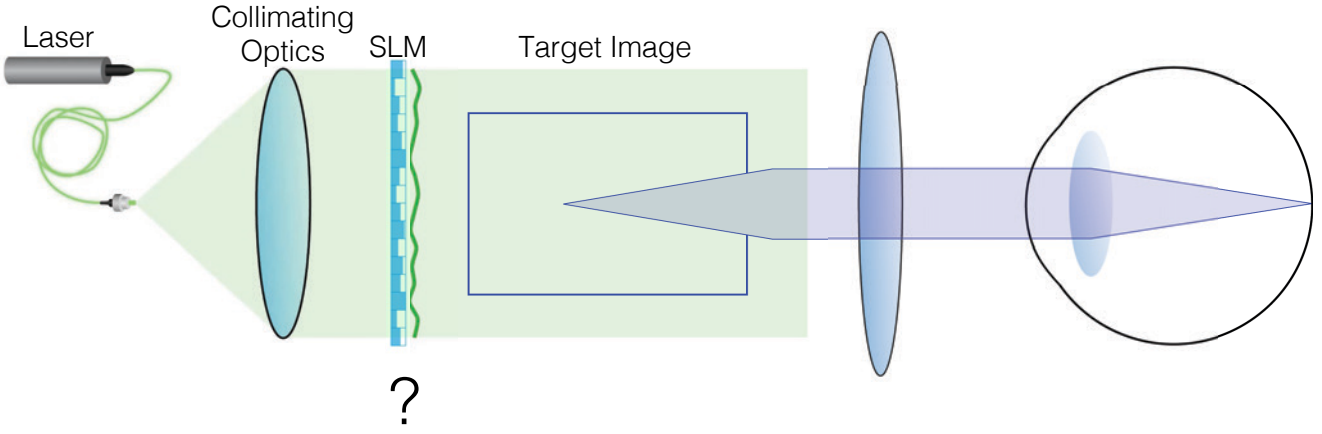
Rolland et al. 2000; Akeley et al. 2004
Liu et al. 2008; Love et al. 2009; …

Near-eye Light Field Displays



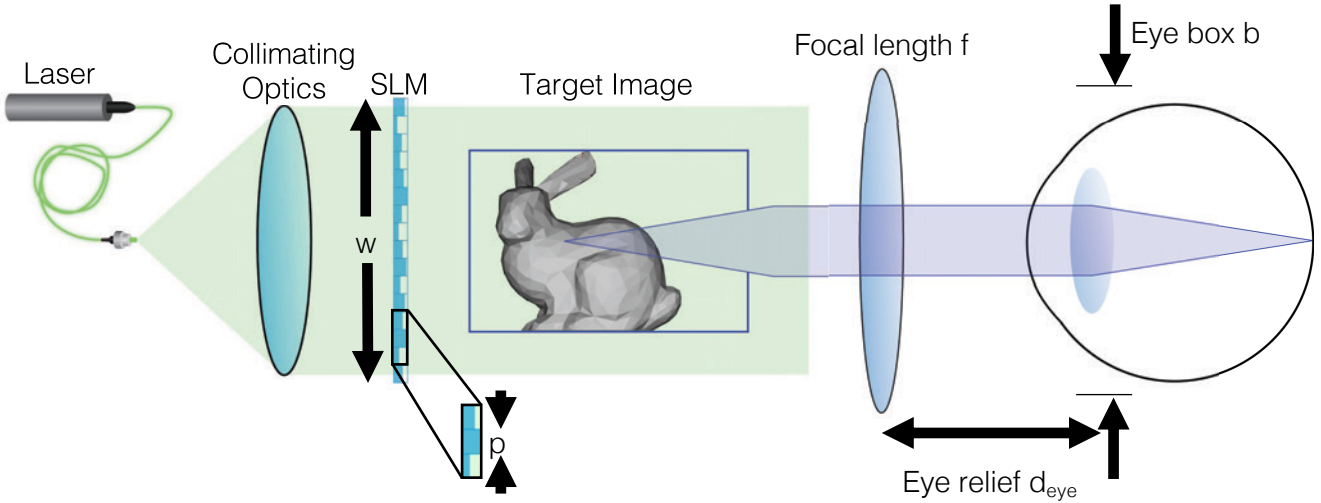
Lanman and Luebke 2013;
Hua and Javidi 2014; Huang et al. 2015

Holographic Near-eye Displays



[Maimone et al. 2017; Padmanaban et al. 2019; Peng et al. 2020; Shi et al. 2021…]

Holographic Near-eye Displays

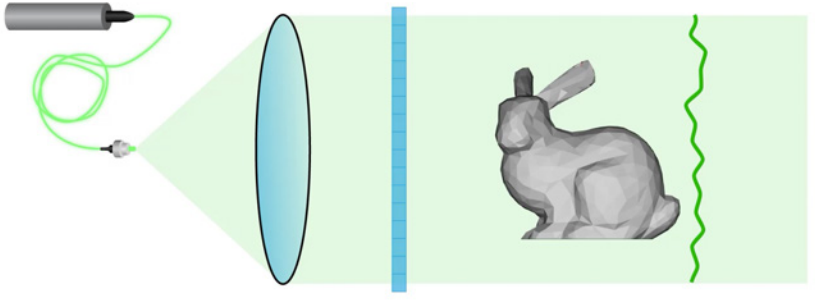


Field of view $FOV \approx 2\tan^{-1}\left(\frac{w}{2f}\right)$

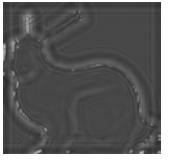
Eye box $b \approx d_{eye}\frac{\lambda}{p}$

[Padmanaban et al. 2019]

Computer-generated Holography: Direct Methods



SLM phase

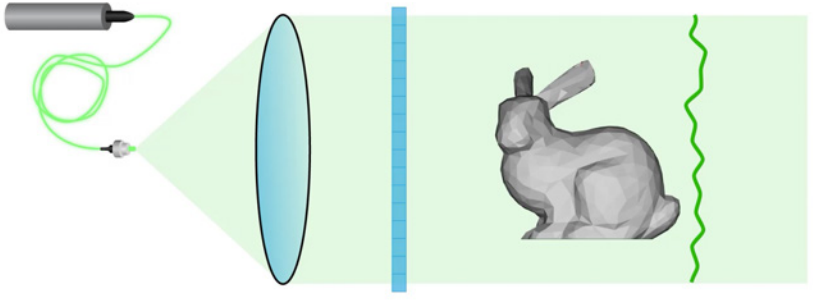


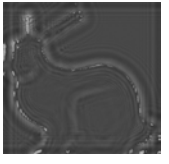
propagate

backward

Target Image

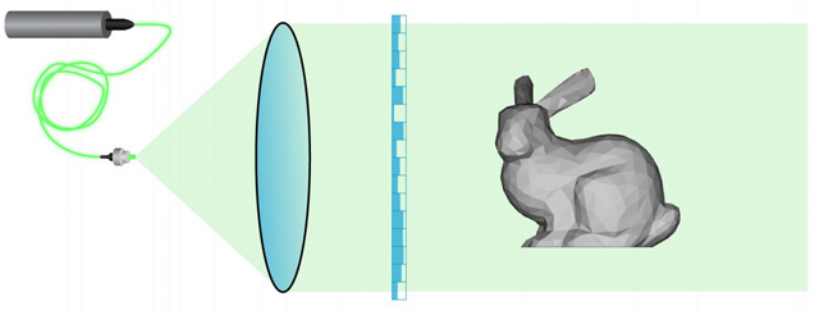
Computer-generated Holography: Direct Methods



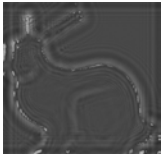


1. Only target intensity is provided, need to “make up” some target phase (e.g., 0)
2. Free-space propagation from target to SLM plane
3. Propagated field at SLM plane is complex, but SLM can only address phase → need phase encoding

Computer-generated Holography: Direct Methods



SLM phase



propagate backward

Target Image

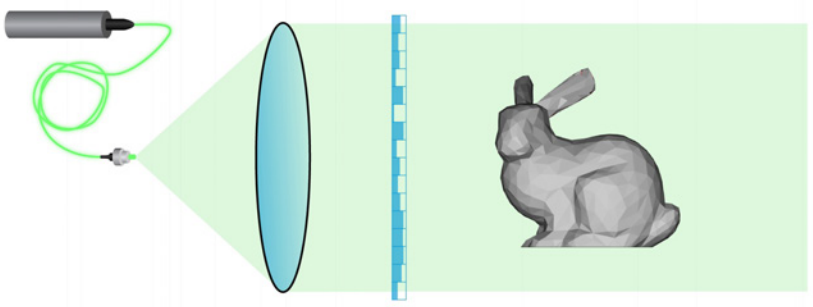
Free-space propagation:

$$u_{slm}(x,y) = \mathcal{F}^{-1}\left\{\mathcal{F}\left\{a(x,y)\,e^{i\phi(x,y)}\right\}\mathcal{H}(f_x,f_y,z)\right\}$$

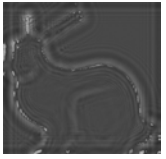
$$\mathcal{H}(f_x,f_y) = \begin{cases} e^{-i\frac{2\pi}{\lambda}\sqrt{1-(\lambda f_x)^2-(\lambda f_y)^2}\,z} & \text{if } \sqrt{f_x^2+f_y^2} < \frac{1}{\lambda} \\ 0 & \text{otherwise} \end{cases}$$

[Goodman, Fourier Optics]

Computer-generated Holography: Direct Methods



SLM phase



propagate backward

Target Image

Double phase-amplitude coding:

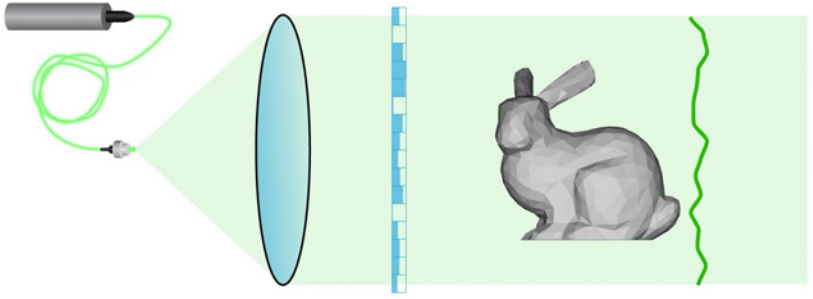
$$u_{slm}(x,y) = a(x,y)\, e^{i\phi(x,y)} = 0.5\left(e^{i\phi_1(x,y)} + e^{i\phi_2(x,y)}\right)$$

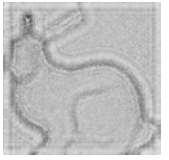
$$\phi_1(x,y) = \phi(x,y) - \cos^{-1}(a(x,y))$$

$$\phi_2(x,y) = \phi(x,y) + \cos^{-1}(a(x,y))$$

[Hsueh and Sawchuk 1978; Maimone et al. 2017]

Computer-generated Holography: Iterative Methods





propagate backward

propagate forward

SLM phase

Target Image

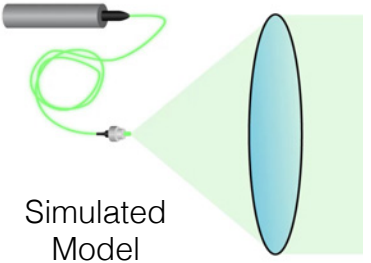
$$\underset{\phi}{\text{minimize}}\ \mathcal{L}\left(|\widehat{f}(\phi)|, a_{\text{target}}\right)$$

$\widehat{f}$ Free-space propagation model

f Unknown physical propagation, $\widehat{f} \neq f$

Iterations: $\phi^{(k)} \leftarrow \phi^{(k-1)} - \alpha \left(\frac{\partial \mathcal{L}}{\partial \phi}\right)^T \mathcal{L}\left(\left|\widehat{f}\left(\phi^{(k-1)}\right)\right|, a_{\text{target}}\right)$

Camera-in-the-loop (CITL) Optimization

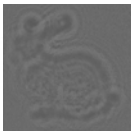


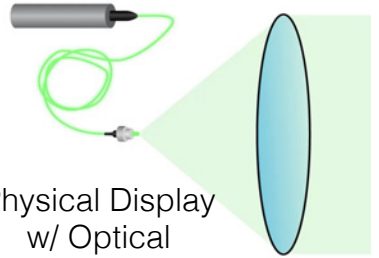
Simulated Model

Simulate SLM

phase ϕ

Display on SLM



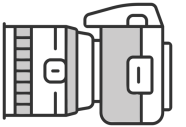


Physical Display w/ Optical Aberrations

$\widehat{f}$

Simulated amplitude

$$\mathcal{L}\left(\text{Target}, \right)$$

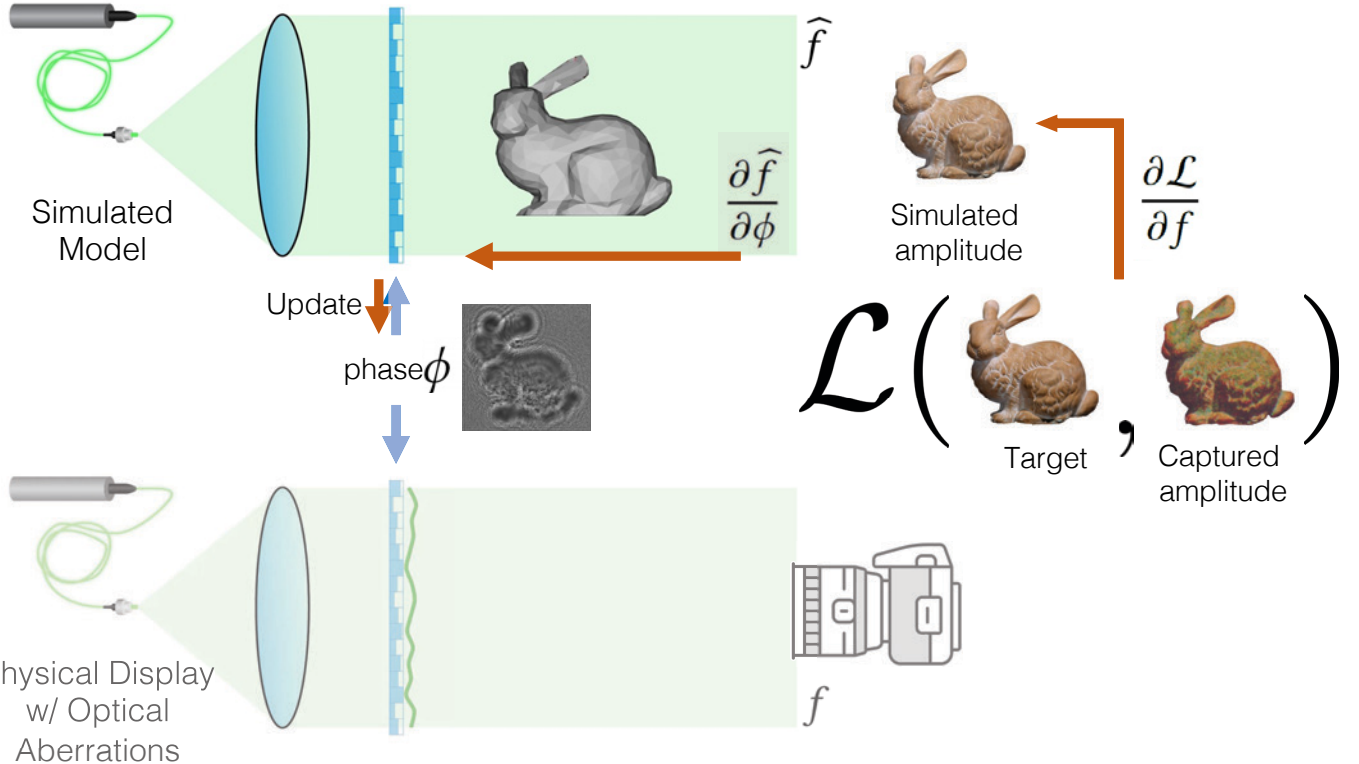


f

Captured amplitude

[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

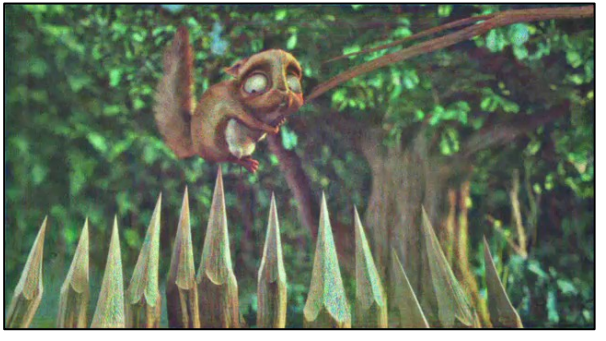
Camera-in-the-loop (CITL) Optimization

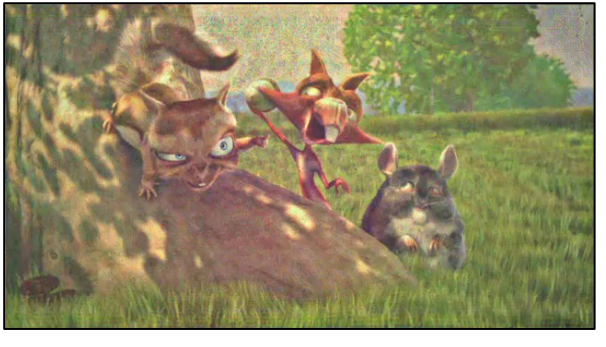


[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

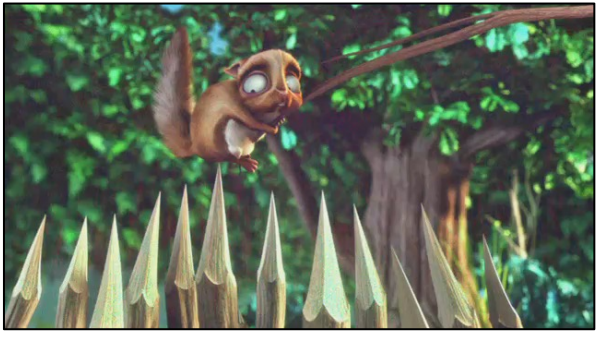
Camera-in-the-loop (CITL) Hologram Optimization

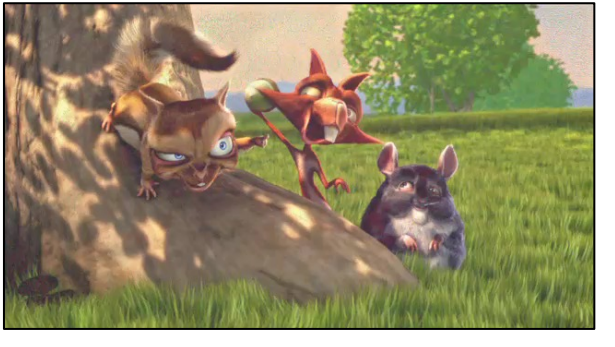
SGD with the ASM Model





Proposed CITL CGH

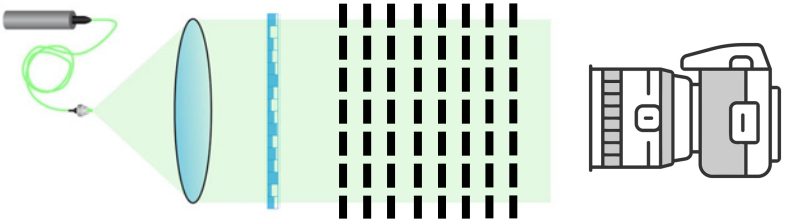


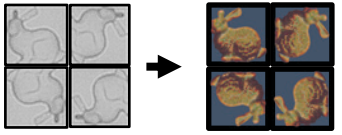


[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

Neural Holography

Physical Optics

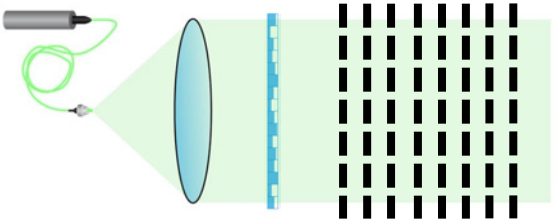


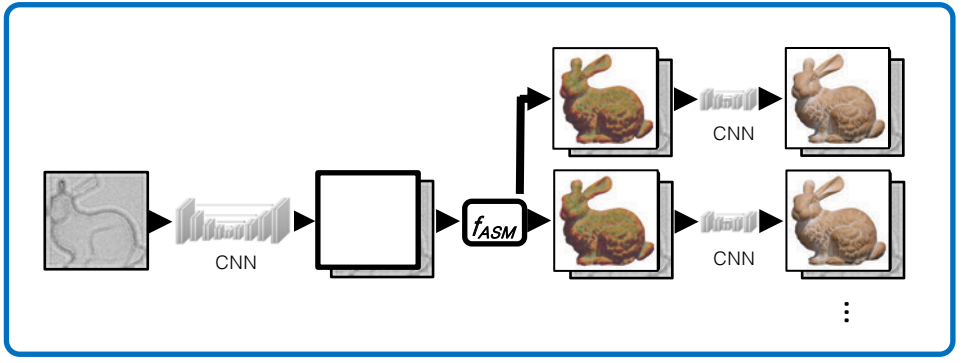


[Peng et al., SIGGRAPH Asia 2020; Choi et al., SIGGRAPH ASIA 2021; SIGGRAPH 2022; …]

Neural Holography

Physical Optics



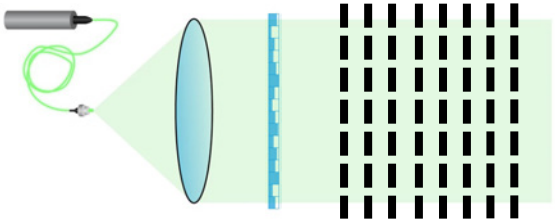


Camera-calibrated Wave Propagation Model

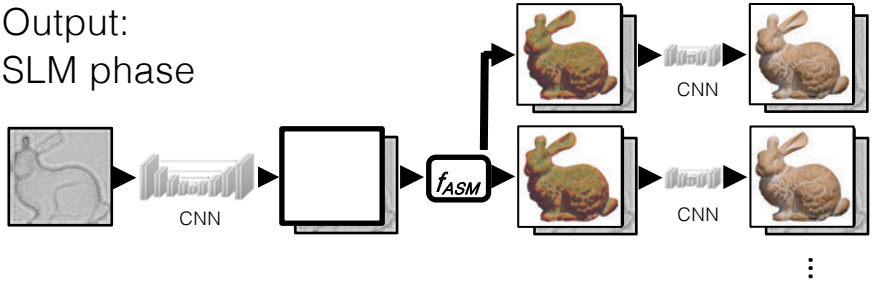
[Peng et al., SIGGRAPH Asia 2020; Choi et al., SIGGRAPH ASIA 2021; SIGGRAPH 2022; …]

Neural Holography

Physical Optics



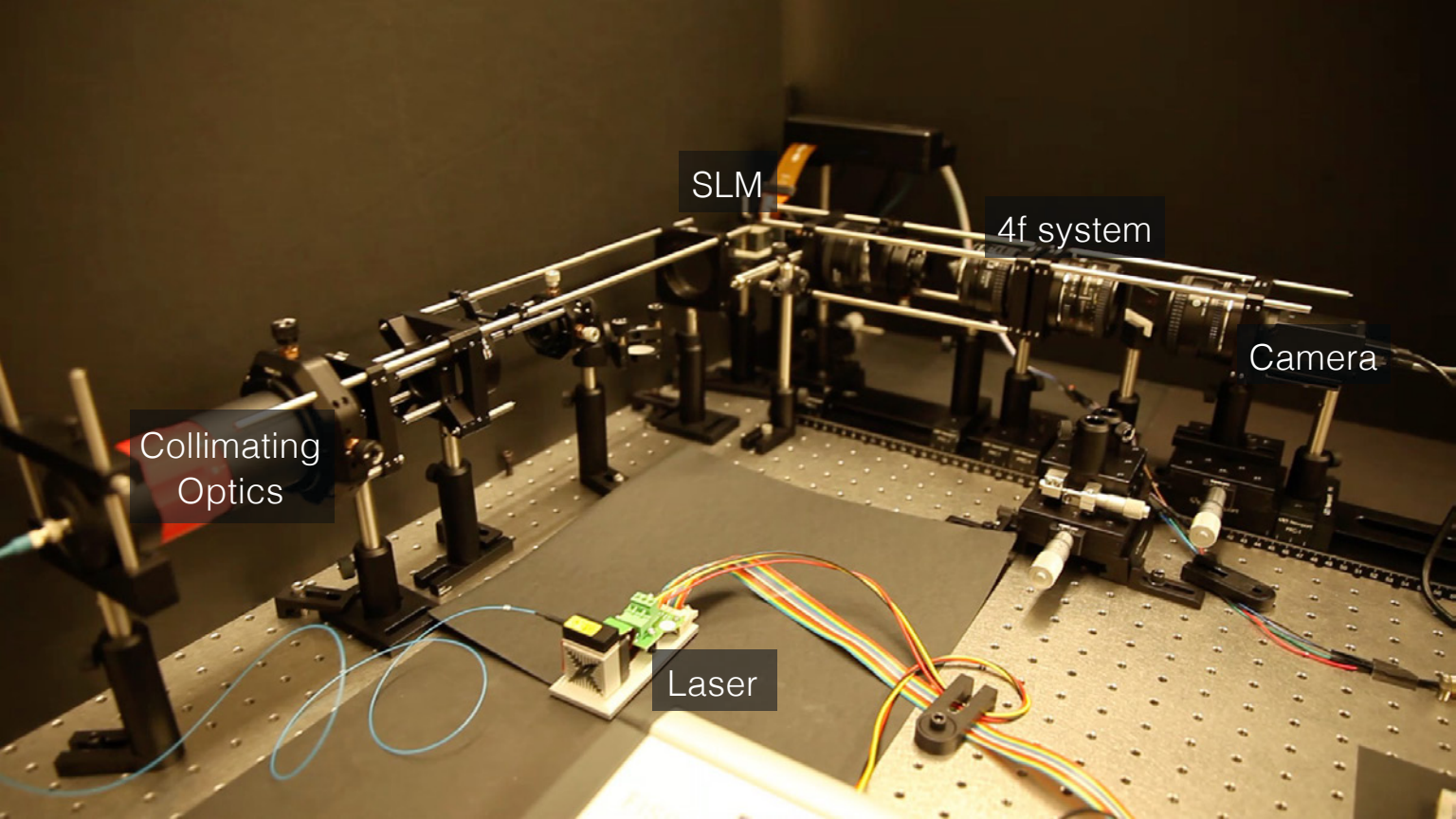
Output:
SLM phase

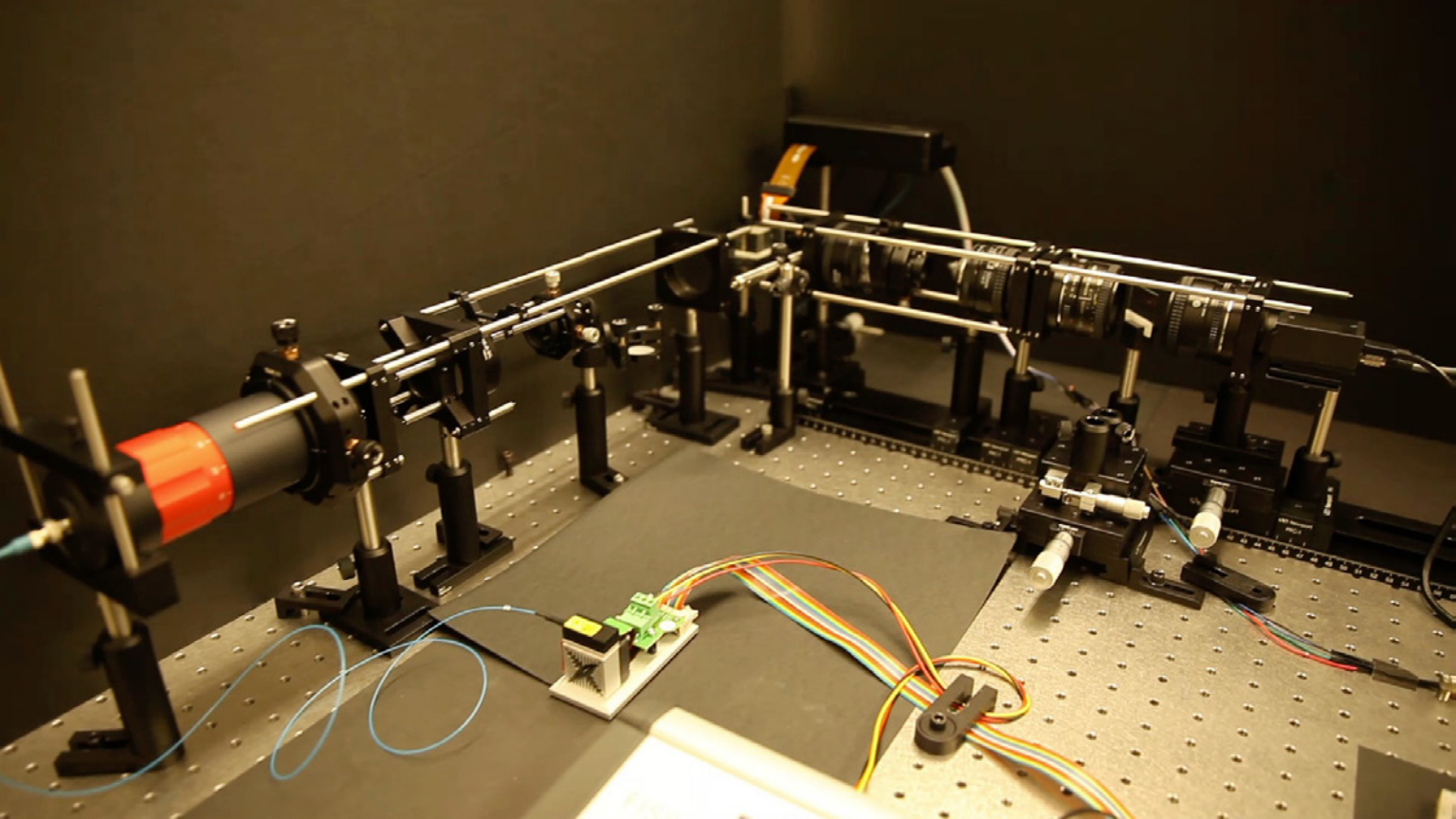


Camera-calibrated Wave Propagation Model

Input:
2D, 2.5D RGBD, 3D focal stack, 4D light field, …

[Peng et al., SIGGRAPH Asia 2020; Choi et al., SIGGRAPH ASIA 2021; SIGGRAPH 2022; …]


SLM
4f system
Camera
Collimating
Optics
Laser

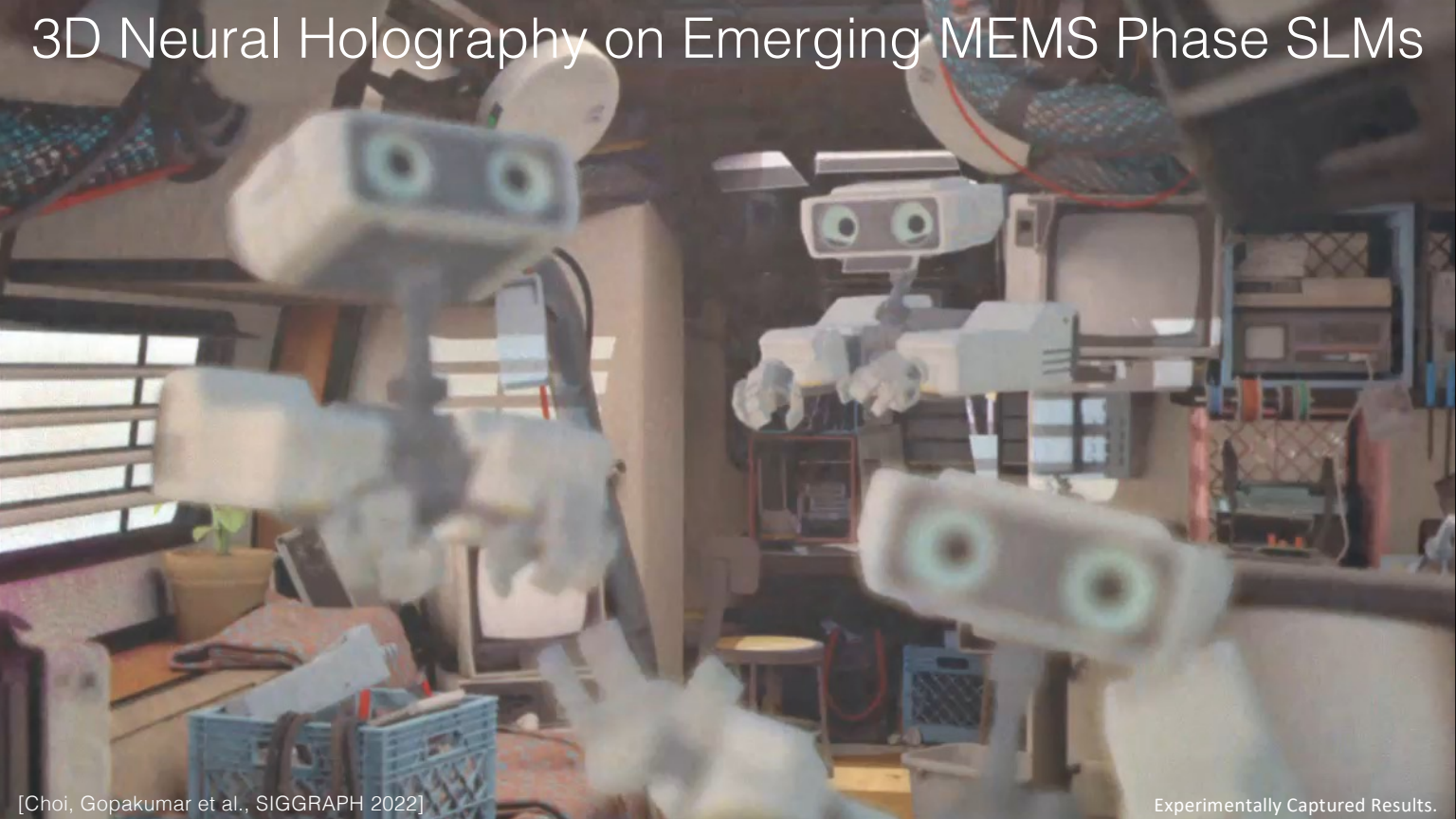


Gerchberg–Saxton

Neural Holography (CITL) 2020 Results

[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

3D Neural Holography on Emerging MEMS Phase SLMs



[Choi, Gopakumar et al., SIGGRAPH 2022]

Experimentally Captured Results.

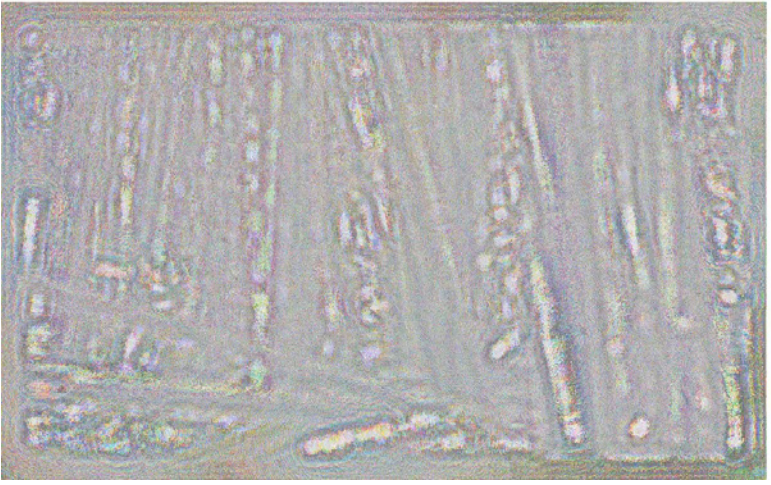
3D Neural Holography on Emerging MEMS Phase SLMs

[Choi, Gopakumar et al., SIGGRAPH 2022]

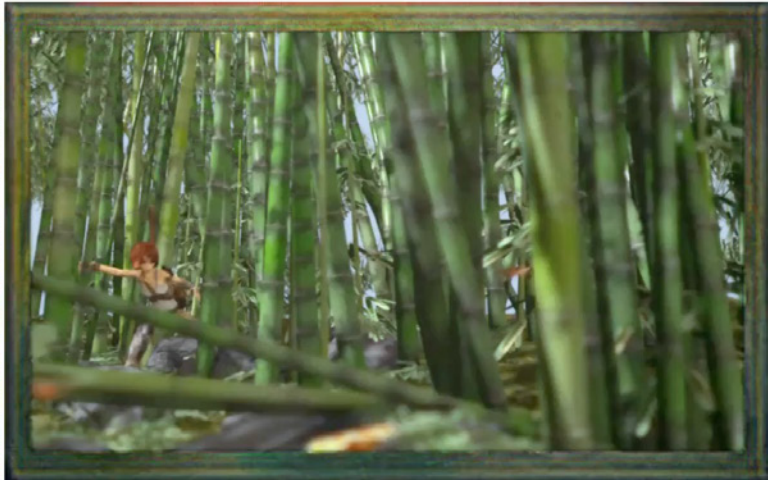
Experimentally Captured Results.

3D Neural Holography on Emerging MEMS Phase SLMs

Displayed patterns on phase SLM



Holograms captured with our prototype

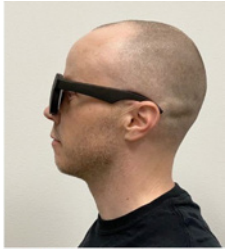


[Choi, Gopakumar et al., SIGGRAPH 2022]

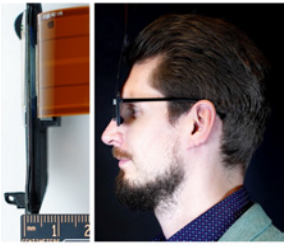
Experimentally Captured Results.

Additional Benefits of Holographic Near-eye Displays

Thin VR Display Form Factors



Maimone et al., SIGGRAPH 2020



Kim et al., SIGGRAPH 2022

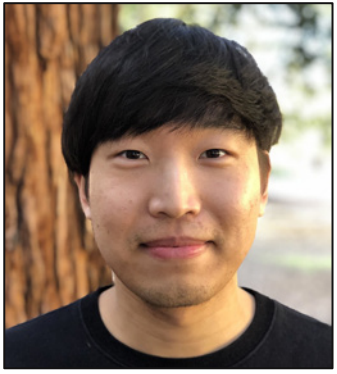
Other:

- Light-efficient AR Displays
- Prescription correction (including astigmatism and higher-orders)
- Correcting optical aberrations

…

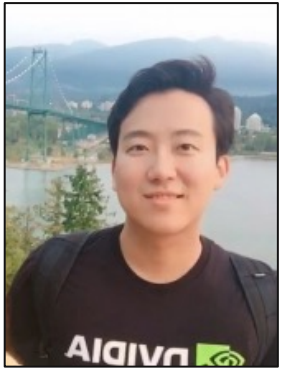
Acknowledgements

Evan Peng

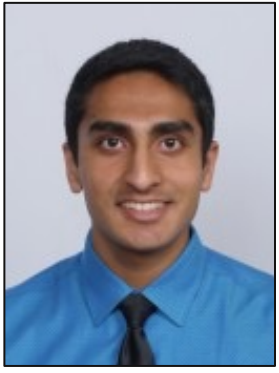


Suyeon Choi

Nitish Padmanaban



Jonghyun Kim



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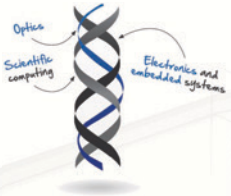

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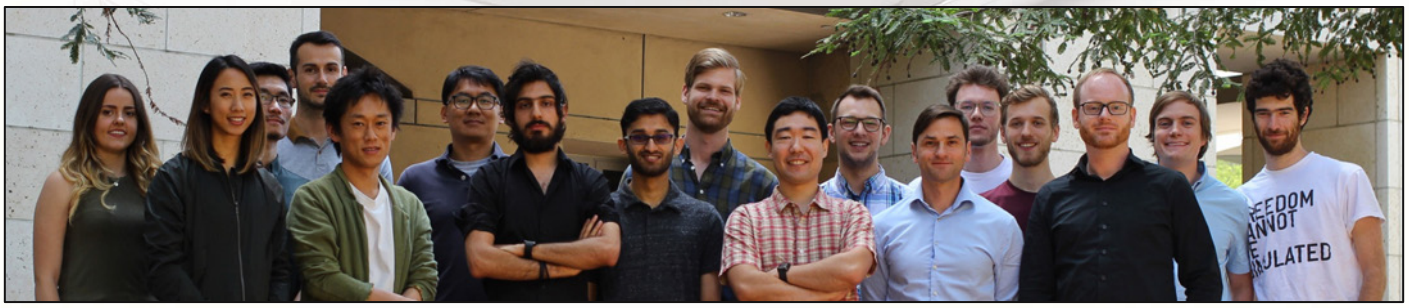

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NSF DARPA intel THE OKAWA FOUNDATION for Information and Telecommunications Ford

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