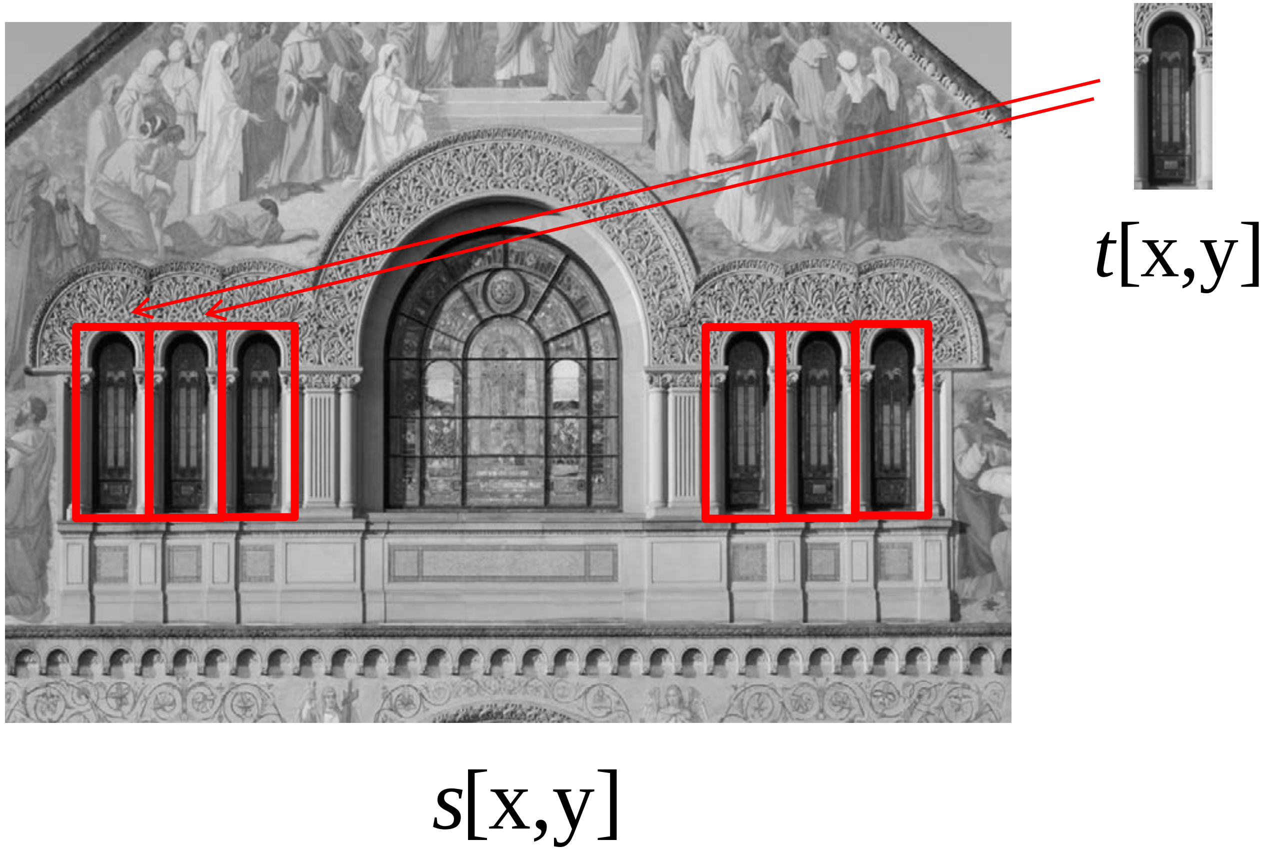
Template matching

- <u>Problem:</u> locate an object, described by a template $t[x,y]$, in the image $s[x,y]$
- Example



Template matching (cont.)

- Search for the best match by minimizing mean-squared error

$$E[p,q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left[s[x,y] - t\left[x-p, y-q \right] \right]^2$$

$$= \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| s[x,y] \right|^2 + \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| t[x,y] \right|^2 - 2 \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x,y] \cdot t\left[x-p, y-q \right]$$

- Equivalently, maximize *area correlation*

$$r[p,q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x,y] \cdot t[x-p, y-q] = s[p,q] * t[-p,-q]$$

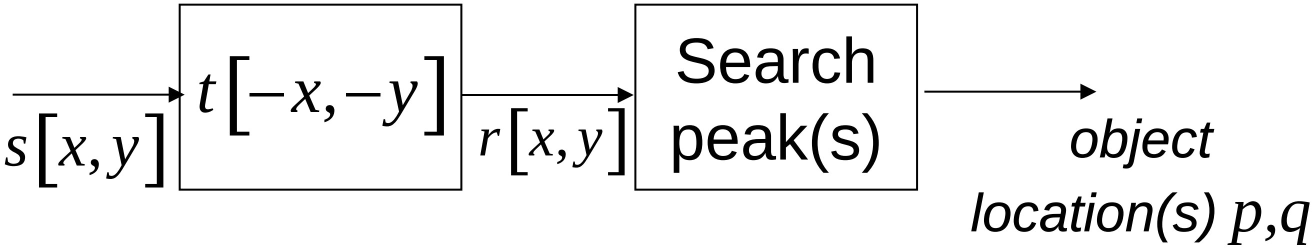
- Area correlation is equivalent to convolution of image $s[x,y]$ with impulse response $t[-x,-y]$

Template matching (cont.)

- From Cauchy-Schwarz inequality

$$r[p,q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x,y] \cdot t[x-p, y-q] \leq \sqrt{\left(\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| s[x,y] \right|^2 \right) \left(\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| t[x,y] \right|^2 \right)}$$

- Equality, iff $s[x,y] = \alpha \cdot t[x-p, y-q]$ with $\alpha \geq 0$
- Block diagram of template matcher

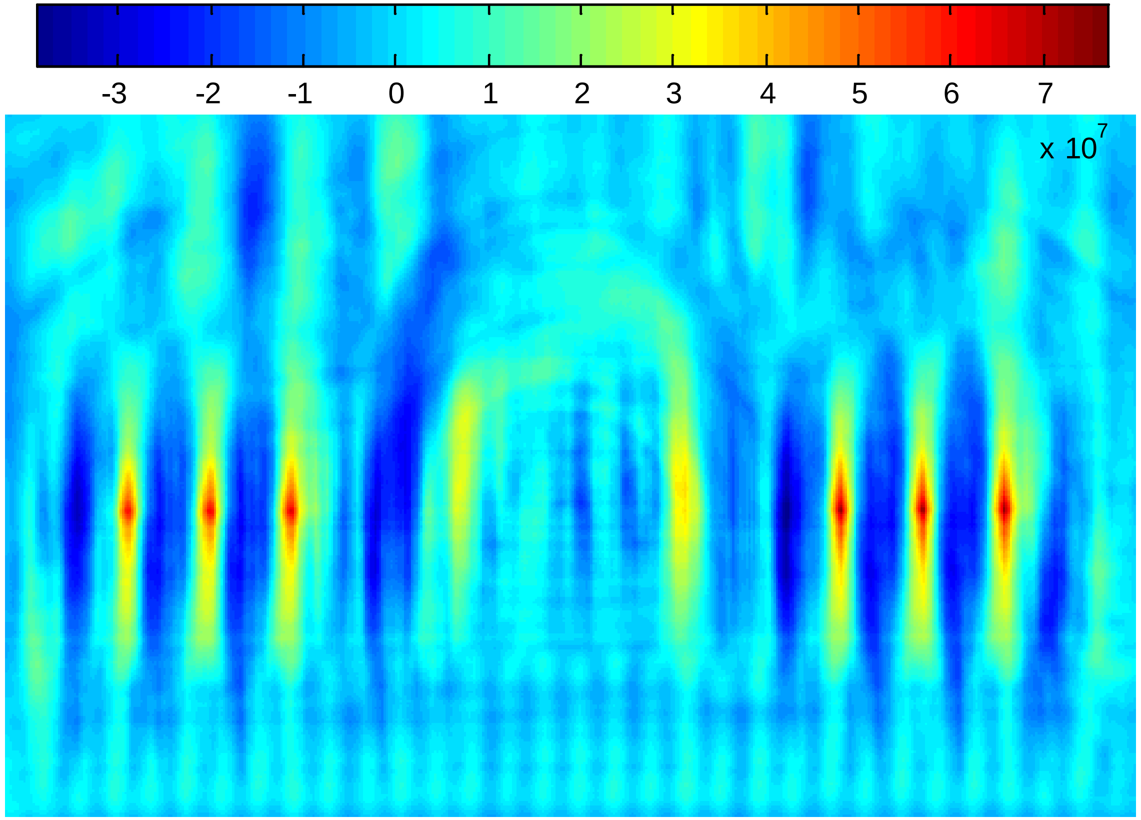


- Remove mean before template matching to avoid bias towards bright image areas

Template matching example

$t[x,y]$

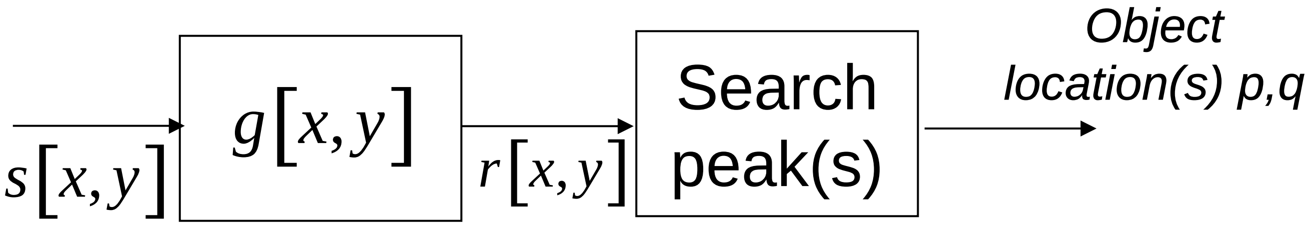
$s[x,y]$



$r[p,q]$

Matched filtering

- Consider signal detection problem



- Signal model

shifted template

$$s[x,y] = t[x-p, y-q] + n[x,y]$$

Other objects: "noise" or "clutter"
psd $\Phi_{nn}\left(e^{j\omega_x}, e^{j\omega_y}\right)$

- Problem: design filter $g[x,y]$ to maximize

$$SNR = \frac{\left| r[p,q] \right|^2}{E\left\{ \left| n[x,y] * g[x,y] \right|^2 \right\}}$$

correct peak

false readings

Matched filtering (cont.)

- Optimum filter has frequency response

$$G\left(e^{j\omega_x}, e^{j\omega_y}\right) = \frac{T^*\left(e^{j\omega_x}, e^{j\omega_y}\right)}{\Phi_{nn}\left(e^{j\omega_x}, e^{j\omega_y}\right)}$$

- Proof:

$$SNR = \frac{\left|r[p,q]\right|^2}{E\left\{\left|n[x,y]*g[x,y]\right|^2\right\}} = \frac{\left|\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} G\left(e^{j\omega_x}, e^{j\omega_y}\right) T\left(e^{j\omega_x}, e^{j\omega_y}\right) d\omega_x d\omega_y\right|^2}{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left|G\left(e^{j\omega_x}, e^{j\omega_y}\right)\right|^2 \Phi_{nn}\left(e^{j\omega_x}, e^{j\omega_y}\right) d\omega_x d\omega_y}$$

$$= \frac{\left|\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left[G\Phi_{nn}^{1/2}\right]\left[\Phi_{nn}^{-1/2} T\right] d\omega_x d\omega_y\right|^2}{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |G|^2 \Phi_{nn}\, d\omega_x d\omega_y} \le \frac{\left[\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |G|^2 \Phi_{nn}\, d\omega_x d\omega_y\right]\left[\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |T|^2 \Phi_{nn}^{-1}\, d\omega_x d\omega_y\right]}{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |G|^2 \Phi_{nn}\, d\omega_x d\omega_y}$$

$$= \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |T|^2 \Phi_{nn}^{-1}\, d\omega_x d\omega_y$$

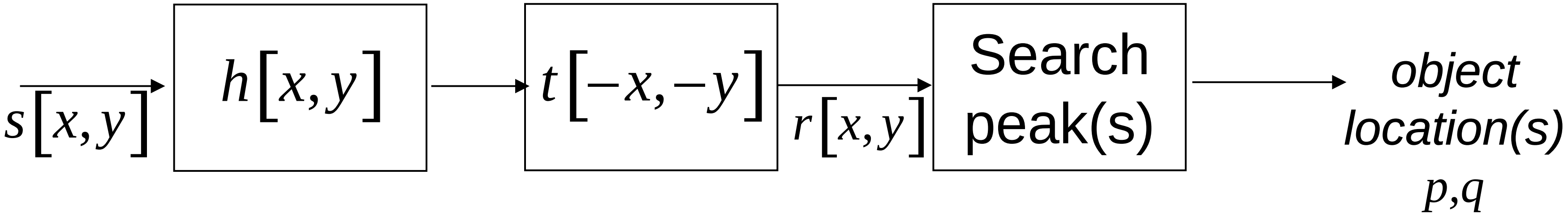
max. SNR

Cauchy-Schwarz inequality,

with equality, iff $G\Phi_{nn}^{1/2} = \alpha \cdot \left[\Phi_{nn}^{-1/2} T\right]^*$

Matched filtering (cont.)

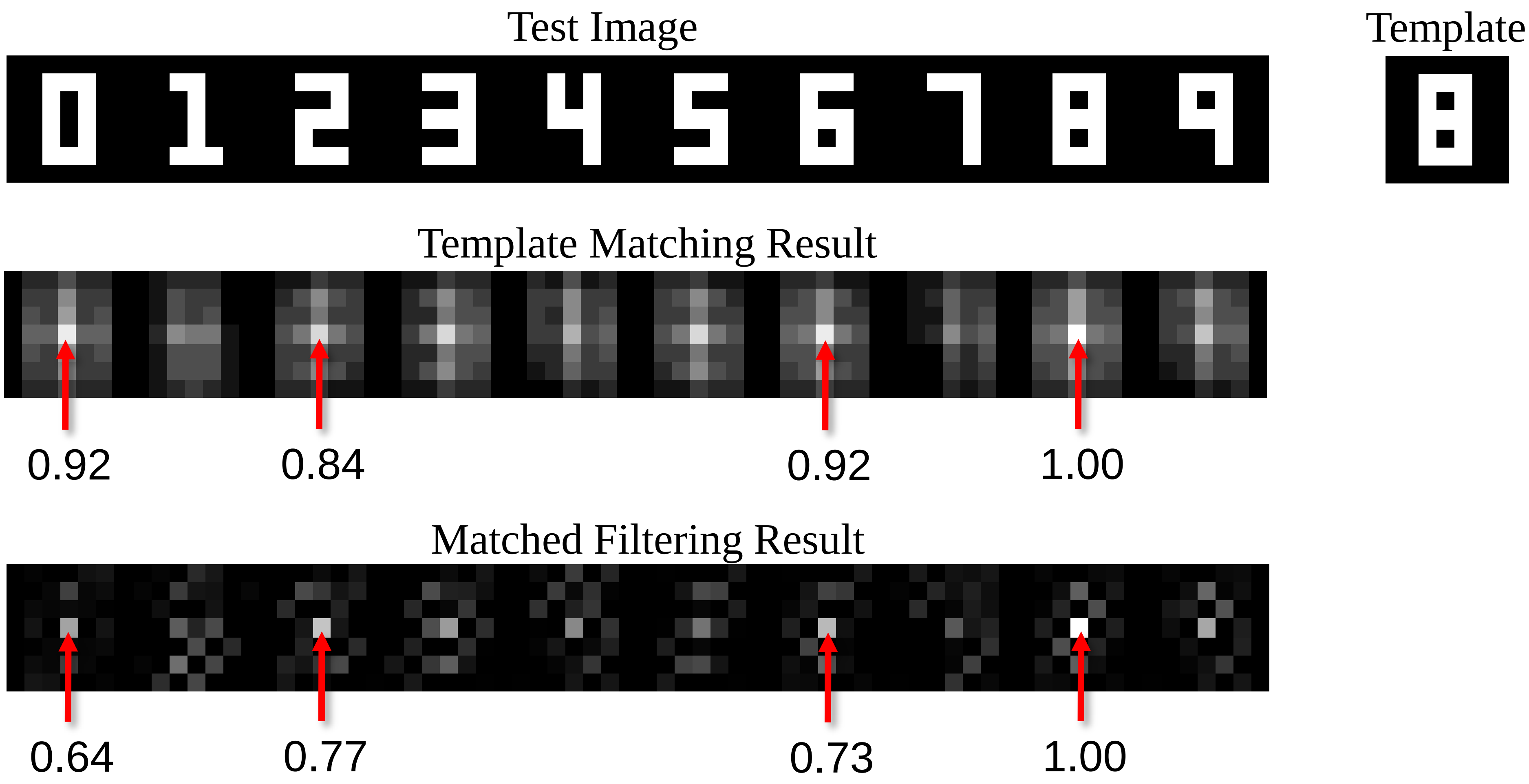
- Optimum detection: prefiltering & template matching



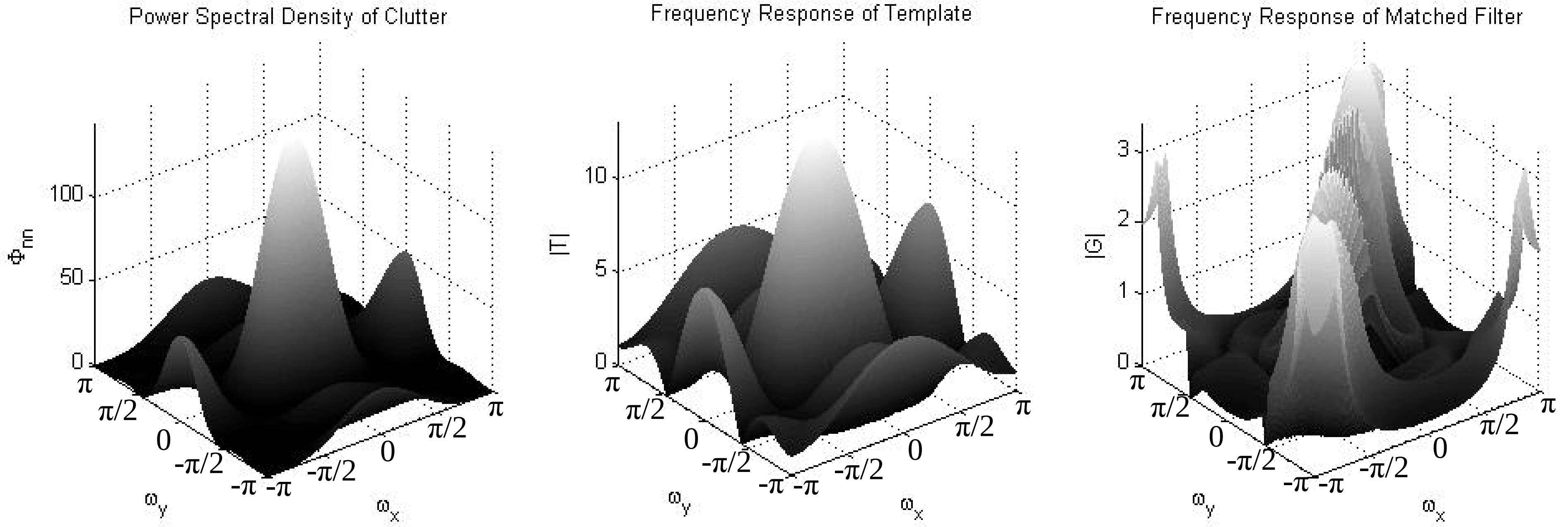
$$h[x,y]=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{1}{\Phi_{nn}\left(e^{j\omega_x},e^{j\omega_y}\right)}e^{j\omega_x x+j\omega_y y}\,d\omega_x d\omega_y$$

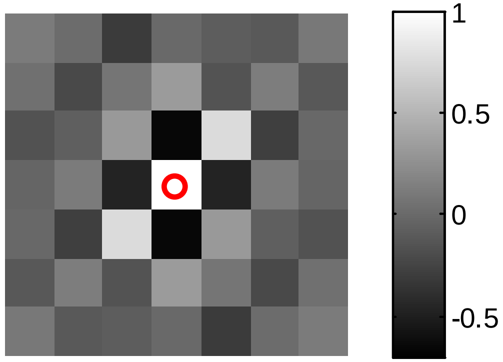
- For white noise $n[x,y]$, no prefiltering $h[x,y]$ required
- Low frequency clutter: highpass prefilter

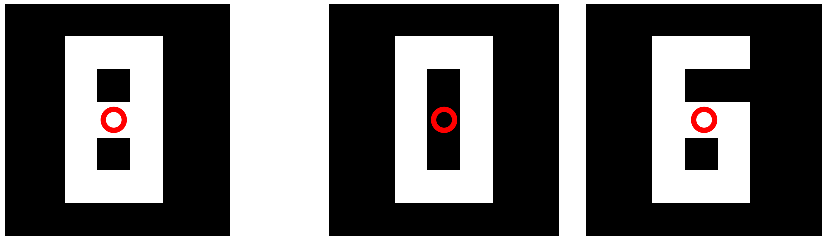
Matched filtering example



Matched filtering example (cont.)

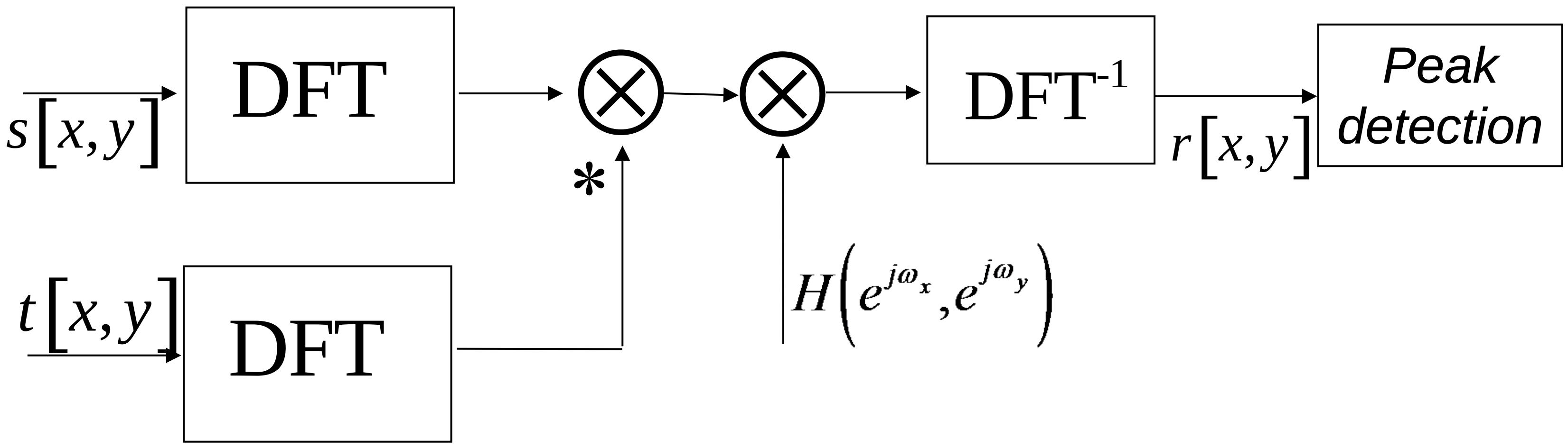






Phase correlation

- Efficient implementation employing the Discrete Fourier Transform



- Phase correlation

$$H\left(e^{j\omega_x}, e^{j\omega_y}\right) = \frac{1}{\left|S\left(e^{j\omega_x}, e^{j\omega_y}\right)\right|\left|T\left(e^{j\omega_x}, e^{j\omega_y}\right)\right|}$$