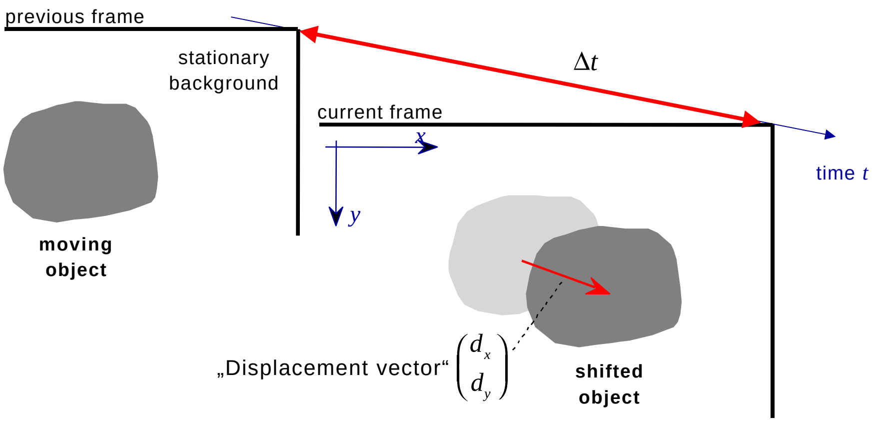
# Overview: motion-compensated coding

- Motion-compensated prediction
- Motion-compensated hybrid coding
- Power spectral density of the motion-compensated prediction error
- Rate-distortion analysis
- Loop filter
- Motion compensation with sub-pel accuracy



# Motion-compensated prediction




Prediction for the luminance signal $S(x,y,t)$ within the moving object:

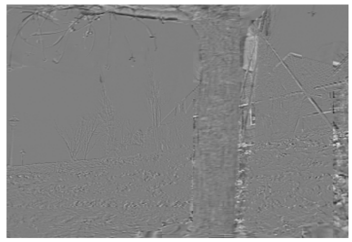
$$\hat{S}(x,y,t) = S(x-d_x, y-d_y, t-\Delta t)$$

# Motion-compensated prediction: example

Previous frame

Current frame

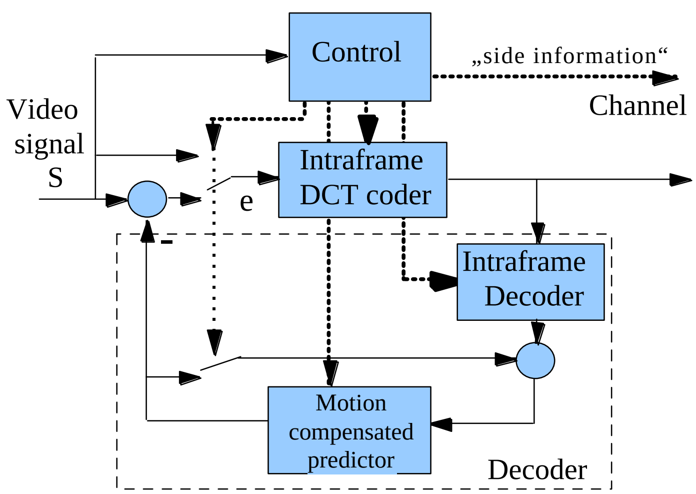
Current frame with displacement vectors



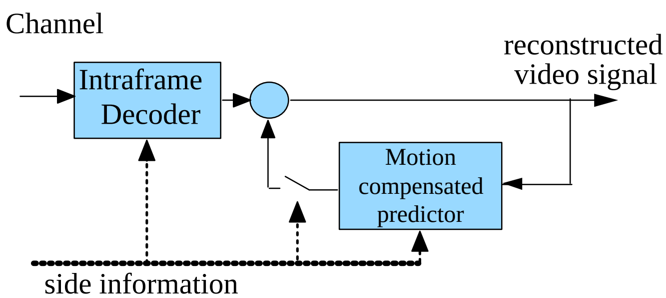
Motion-compensated Prediction error



# Motion-compensated hybrid coder

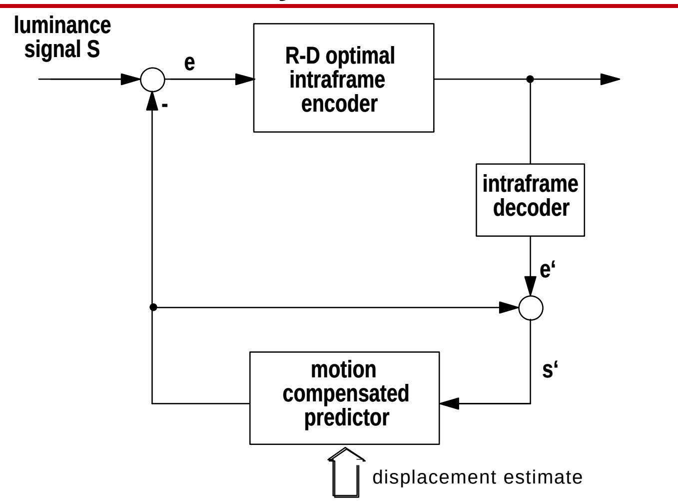


## Motion-compensated hybrid decoder



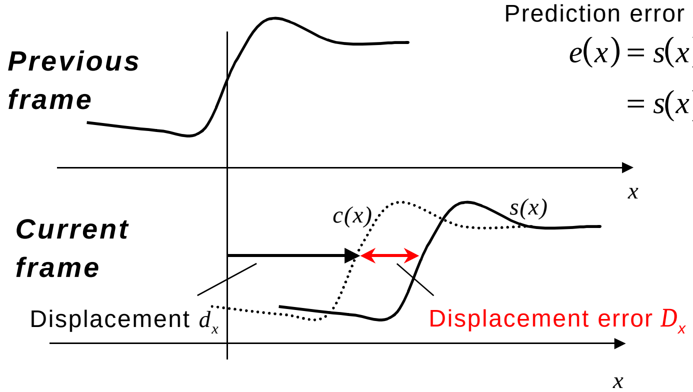



## Model for performance analysis of an MCP hybrid coder




$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} + \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix}$$

## Analysis of the motion-compensated prediction error




Motion-compensated signal

$$c(x) = s(x - \Delta_x) - n(x)$$

Prediction error

$$e(x) = s(x) - c(x)$$
$$= s(x) - s(x - \Delta_x) + n(x)$$



## Analysis of m.c. prediction error (cont.)

- Motion-compensated prediction error

$$e(x) = s(x) - c(x) = s(x) - s(x - \Delta_x) + n(x) = \left(d(x) - d(x - \Delta_x)\right) * s(x) + n(x)$$

- Power spectrum of prediction error, assuming constant displacement error $\Delta_x$, statistical independence of $s$ and $n$

$$\Phi_{ee}(w) = \Phi_{ss}(w)\left(1 - e^{-jw\Delta_x}\right)\left(1 - e^{jw\Delta_x}\right) + \Phi_{nn}(w)$$
$$= 2\Phi_{ss}(w)\left(1 - \mathrm{Re}\left\{e^{-jw\Delta_x}\right\}\right) + \Phi_{nn}(w)$$

- Random displacement error $\Delta_x$, statistically independent from $s$, $n$

$$\Phi_{ee}(w) = E\left\{2\Phi_{ss}(w)\left(1 - \mathrm{Re}\left\{e^{-jw\Delta_x}\right\}\right) + \Phi_{nn}(w)\right\}$$
$$= 2\Phi_{ss}(w)\left(1 - \mathrm{Re}\left\{E\left\{e^{-jw\Delta_x}\right\}\right\}\right) + \Phi_{nn}(w)$$
$$= 2\Phi_{ss}(w)\left(1 - \mathrm{Re}\left\{P(w)\right\}\right) + \Phi_{nn}(w)$$

## Analysis of m.c. prediction error (cont.)

- What is *P(w)*? $P(w) = E\{e^{-jw\Delta_x}\}$

$$= \int_{-\infty}^{\infty} p_{\Delta_x}(\Delta) e^{-jw\Delta} d\Delta = F\{p_{\Delta_x}(\Delta_x)\}$$

Fourier transform of the displacement error pdf!

- Same as characteristic function of displacement error, except for sign
- Extension to 2-d

$$\Phi_{ee}(w_x, w_y) = 2\Phi_{ss}(w_x, w_y)\left(1 - \mathrm{Re}\{P(w_x, w_y)\}\right) + \Phi_{nn}(w_x, w_y)$$

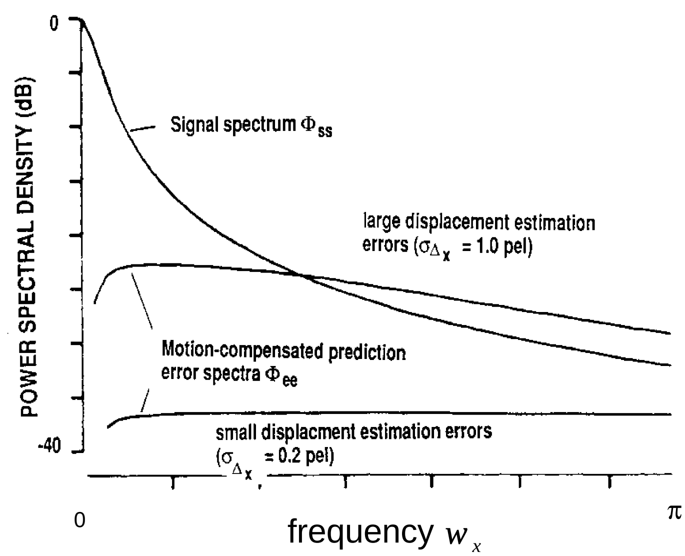
power spectrum of luminance signal

Fourier transform of the displacement error pdf $p(\Delta_x, \Delta_y)$

noise spectrum



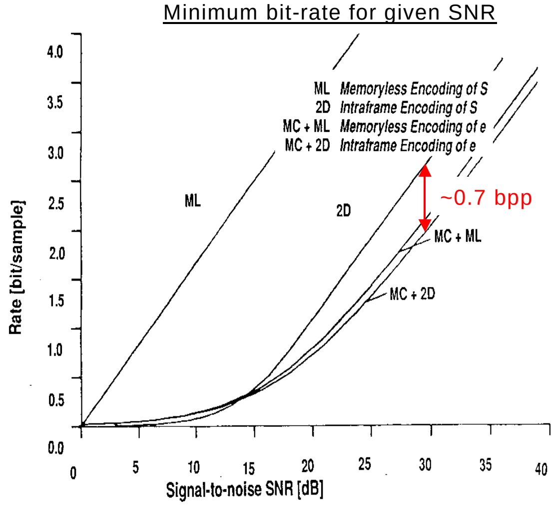
## Power spectrum of motion-compensated prediction error



## R-D function for MCP with integer-pixel accuracy

- $(\Delta_x, \Delta_y)^T$ assumed uniformly distributed between
  $$\Delta_x = \pm\frac{1}{2} \text{ pel}$$
  $$\Delta_y = \pm\frac{1}{2} \text{ line}$$
- Gaussian signal model
  $$\Phi_{ss}(w_x, w_y) = A\left(1 + \frac{w_x^2 + w_y^2}{w_0^2}\right)^{-\frac{3}{2}}$$
- Typical parameters for CIF resolution (352 x 288 pixels)

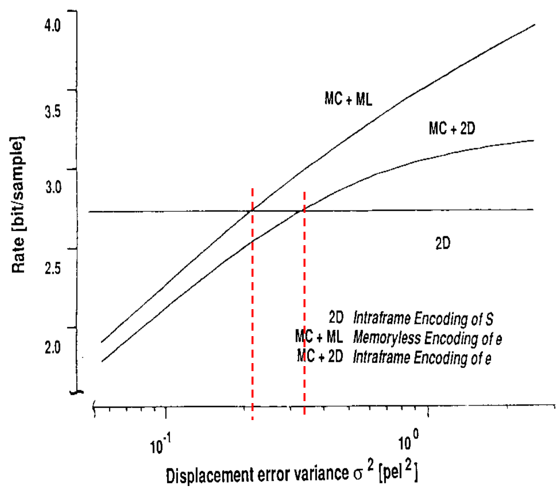
Minimum bit-rate for given SNR



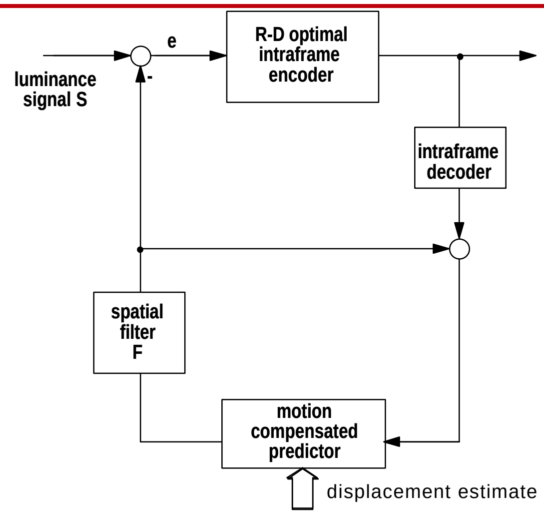



## Required accuracy of motion compensation

- $p(\Delta_x, \Delta_y)$ isotropic Gaussian pdf with variance $s^2$
- $\Phi_{ss}(w_x, w_y) = A\left(1 + \frac{w_x^2 + w_y^2}{w_0^2}\right)^{-\frac{3}{2}}$
- Typical parameters for CIF resolution (352 x 288 pixels)
- Minimum bit-rate for SNR = 30 dB



# Model of MCP hybrid coder with loop filter




$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} + \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix}$$



# Motion-compensated prediction error with loop filter

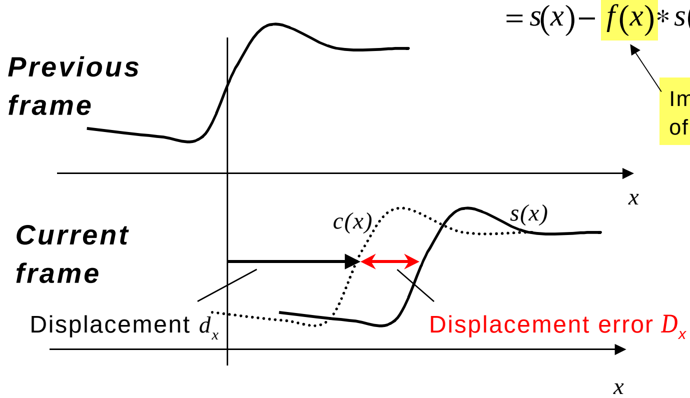
Motion-compensated signal

$$c(x) = s(x - \Delta_x) - n(x)$$

Prediction error

$$e(x) = s(x) - f(x) * c(x)$$
$$= s(x) - f(x) * s(x - \Delta_x) + f(x) * n(x)$$

Impulse response of loop filter



## Spatial power spectrum of m.c. prediction error with loop filter

$$\Phi_{ee}(\Lambda) = \Phi_{ss}(\Lambda)\left(1 + |F(\Lambda)|^2 - 2\,\mathrm{Re}\{F(\Lambda)P(\Lambda)\}\right) + \Phi_{nn}(\Lambda)\,|F(\Lambda)|^2$$

| | |
|---|---|
| $P(\Lambda)$ | 2-D Fourier transform of displacement error pdf |
| $F(\Lambda)$ | 2-D Fourier transform of $f(x,y)$ |
| $\Phi_{uu}$ | spatial spectral power density of signal $u$ |
| $\Lambda$ | vector of spatial frequencies $(w_x, w_y)$ |
| $n(x,y)$ | noise |



## Optimum loop filter

- Wiener filter minimizes prediction error variance

$$F_{\text{opt}}(\Lambda) = \underbrace{P^*(\Lambda)}_{\text{accounts for accuracy of motion estimation}} \cdot \underbrace{\frac{\Phi_{ss}(\Lambda)}{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}}_{\text{accounts for noise}}$$

accounts for accuracy of motion estimation

accounts for noise

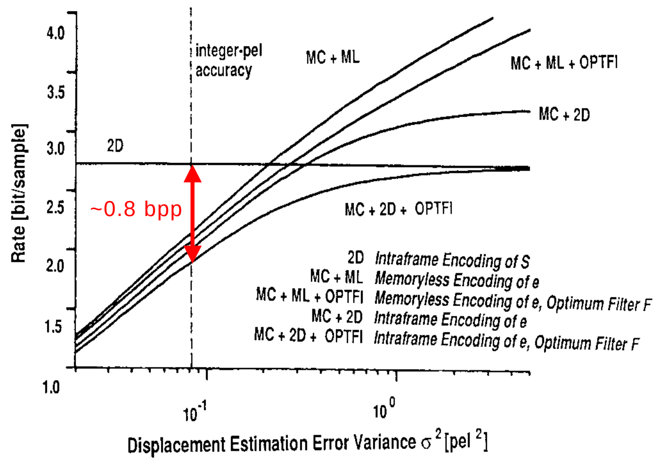
- To determine Wiener filter from measurements:

$$F_{\text{opt}}(\Lambda) = \frac{\Phi_{sc}(\Lambda)}{\Phi_{cc}(\Lambda)}$$

cross spectrum between *s(x,y)* and the motion-compensated signal $c(x,y) = r(x - \hat{d}_x, y - \hat{d}_y)$

## Required accuracy of motion compensation with loop filter

- $p(\Delta_x, \Delta_y)$ isotropic Gaussian pdf with variance $s^2$
- Minimum bit-rate for SNR = 30 dB

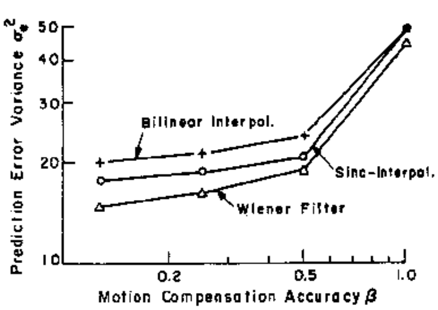




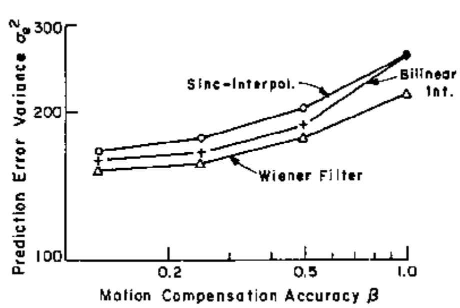

## Experimental evaluation of sub-pixel motion compensation

- ITU-R 601 TV signals, 13.5 MHz sampling rate, interlaced, blockwise motion compensation with blocksize16x16

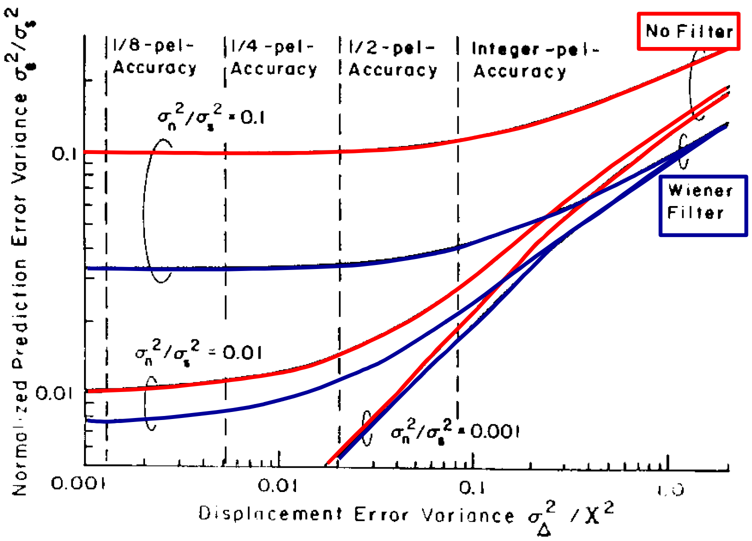
**Zoom**




**Voiture**

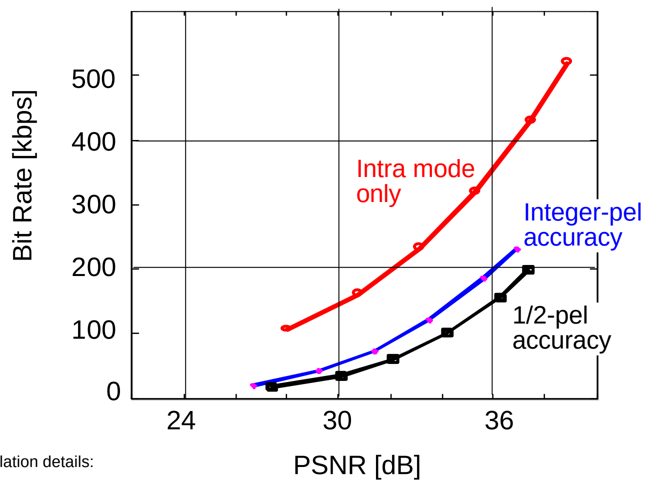


# Influence of noise on the performance of MCP






# Motion Compensation Performance in H.263




Simulation details:

Foreman, QCIF, SKIP=2
Q=4,5,7,10,15,25

# Summary: motion-compensated coding

- Motion-compensated prediction exploits similarity of successive frames of a video sequence.
- Hybrid coder combines motion compensation and spatial 2D coding.
- Power spectral density of motion-compensated prediction error is flat.
- Loop filter  improves the prediction.
- Maximum gain by motion compensation with integer-pel accuracy ~0.8 bit/sample.
- Sub-pixel accuracy of motion compensation  can improve prediction
- "Noise" in the image signal limits the accuracy of motion compensation.