LOW RANK FOR TIME SERIES RECONSTRUCTION

BASIC PROBLEM

SERIES OF IMAGES
SAME OBJECT
DIFFERENT CONTRAST - OR -
STATIC IMAGE + DYNAMIC COMPONENT

EXAMPLES

DYNAMIC CONTRAST STUDIES
T1, T2 PARAMETER ESTIMATION
4D FLOW
IMAGE GUIDED INTERVENTIONS
THERMAL THERAPIES
INTERVENTIONAL DEVICES
CARDIAC IMAGING

QUESTION

HOW DO YOU EXPLOIT TEMPORAL REDUNDANCY FOR FASTER IMAGING?
Examine Dynamic Contrast Imaging

Pre-Contrast  First Pass  Vascular Early  Late Organs

Same Object, Different Contrast

Example T_2 Mapping

Pulse Sequence

RF: 90 180 180 180 180 180 180 180

Again, Same Object, Different Contrast, Signal Levels
We have a stack of $N$ images

$$m = \{m_i\} \quad (nx\times ny, N)$$

We'd like to reconstruct these all together.

Let $C$ be an operator that vectorizes each image

$$C_m = \begin{pmatrix} m_1 & m_2 & \cdots & m_N \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} \quad (nx\times ny, N)$$

This is the Casorati matrix.

What we want is the simplest representation of this data.

In practice, this means we take the minimization

$$\min_{m} \| C_m \|_*$$
WHERE \( \|C_m\|_1 \) is the nuclear norm of \( C_m \), which is the sum of the singular values of \( C_m \).

**WHY NUCLEAR NORM?**

Singular values are non-negative.
Singular values are the square root of the eigenvalues of \( C^*C \) or \( CC^* \).

Effectively \( L_1 \) norm of \( SVD \)
Promotes sparsity
Solved with FISTA (Iterative Thresholding Shrinkage)

**WHAT DOES THIS DO IN PRACTICE?**

\[
C_m = U \Sigma V^T
\]

Left singular vectors & singular values & right singular vectors

\[
\begin{bmatrix}
\tilde{U} \\
\tilde{\Sigma} \\
\tilde{V}
\end{bmatrix}
\]
Specifically for T2 mapping

\[
\begin{pmatrix}
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
= 
\begin{pmatrix}
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]

Source Images

AVG first diff noise

Image basis

Singular values significance of each computed

Temporal basis sets

In practice, the singular values will fall off rapidly

This can be exploited for denoising

Compressed sensing
DENOISING

Assume \( m \) is fully sampled

If we zero out singular values below a threshold we can reject noise and improve SNR.

Compressed Sensing

Assume each image has a unique random undersampling pattern.

Random sampling = incoherent artifacts. These will be independent from image to image.

Undersampling looks like higher noise background.

Higher threshold.
Compressed Sensing Reconstruction

\[
\text{minimize} \quad \text{trade off between LR+CS} \\
\min \left\{ \| C m \|_\infty + \lambda \| D F m - y \|_2 \right\} \\
\text{low rank} \quad \text{data consistency}
\]

where \( F \) is Fourier transform operator, and \( D \) is subsampling.

We do this with the same iterative compressed sensing reconstruction.

Iterate over

(1) Compute SVD of casorati matrix, and zero out low values, shrink the rest (FISTA)

(2) Enforce data consistency in spatial frequency
GLOBAL LOW RANK vs LOCAL

LOW RANK

SO FAR, WE HAVE INCLUDED THE WHOLE IMAGE IN THE LOW RANK DECOMPOSITION

GLOBAL LOW RANK (GLR)

THIS WORKS, BUT WE CAN DO BETTER BY BREAKING THE IMAGE INTO BLOCKS

BETTER REPRESENTS LOCAL VARIATIONS

LOCAL LOW RANK (LLR)

Block 1: Nothing changes we can use the entire time series

Block 2: we have several time courses
1) Background
2) First pass
3) Delayed enhancement

Different representations as a function of position
EXTENSION TO PARALLEL IMAGING

IN PRACTICE WE HAVE MULTIPLE RECEIVE CHANNELS

THIS ALLOWS GREATER SUBSAMPLING

THE SOURCE IMAGES ARE NOW FROM EACH CHANNEL, AND THE \( m_i \) ARE \((m_x, m_y, m_z)\)

THE CASOROTTI OPERATOR VECTORIZES ALL OF THE CHANNELS

\[
C_m = \begin{pmatrix}
\begin{array}{ccc}
M_{11} & M_{12} & \cdots & M_{1M} \\
M_{21} & M_{22} & \cdots & M_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
M_{M1} & M_{M2} & \cdots & M_{MM}
\end{array}
\end{pmatrix}
\]

PARALLEL IMAGING ALLOWS MUCH GREATER SUBSAMPLING
Compressed Sensing, Parallel Imaging

Locally Low Rank Reconstruction

This is solved iteratively by cycling between

1) PI Reconstruction

2) LLR Expansion, Thresholding

3) Data Consistency

This works well when the underlying object is the same, and only the contrast varies.
Low Rank + Sparse (LR+CS)

We frequently have a relatively static background component and a dynamic foreground component.

Examples:
- Cardiac Imaging
  - Chest wall is constant, heart is moving.
- Interventional Imaging
  - Anatomy is constant, device is moving.

How do we reconstruct these time series?

Solution: Represent the image sequence as a combination of a low rank component (background) and a sparse component (dynamic).

After eliminating the background low rank component, the dynamic data is very sparse!
Example: Cardiac Imaging

Eliminating LR component makes residual very sparse
Supports a large acceleration factor
Solution is to minimize

\[ \min \left\{ \| D \tilde{F} n - y \|_2^2 + \lambda_2 \| L \|_* + \lambda_3 \| T m \| \right\} \]

Data Consistency \quad Low Rank \quad Sparsity

The values \( \lambda_2 \) and \( \lambda_3 \) trade off sparsity, low rank, and data consistency.

Reconstruction algorithm cycles through the various constraints.

Again, this extends to parallel imaging.
CONCLUSION

IF WE HAVE A TIME SERIES OF IMAGES FROM THE SAME OBJECT, WE CAN GREATLY ACCELERATE RECONSTRUCTION.

THE LOW RANK CONSTRAINT ADDS ANOTHER ACCELERATION FACTOR.

\[ R_T = R_p \cdot R_c \cdot R_{LR} \]

WE'VE SEEN THAT \( R_p \) AND \( R_c \) ARE ON THE ORDER OF 2-4 FOR STANDARD CLINICAL IMAGES.

THE ADDITION OF LOW RANK BRINGS THE TOTAL ACCELERATION TO 15-20!

THIS IS ENABLING FOR STUDIES LIKE 4D FLOW, THAT WOULD OTHERWISE TAKE AN HOUR OR MORE.