

EE369C

LECTURE 3

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2D SAMPLING AND RECONSTRUCTION

NON-CARTESIAN MRI RECONSTRUCTION

NUFFT PROBLEMS

BASIC GRIDDING IDEA

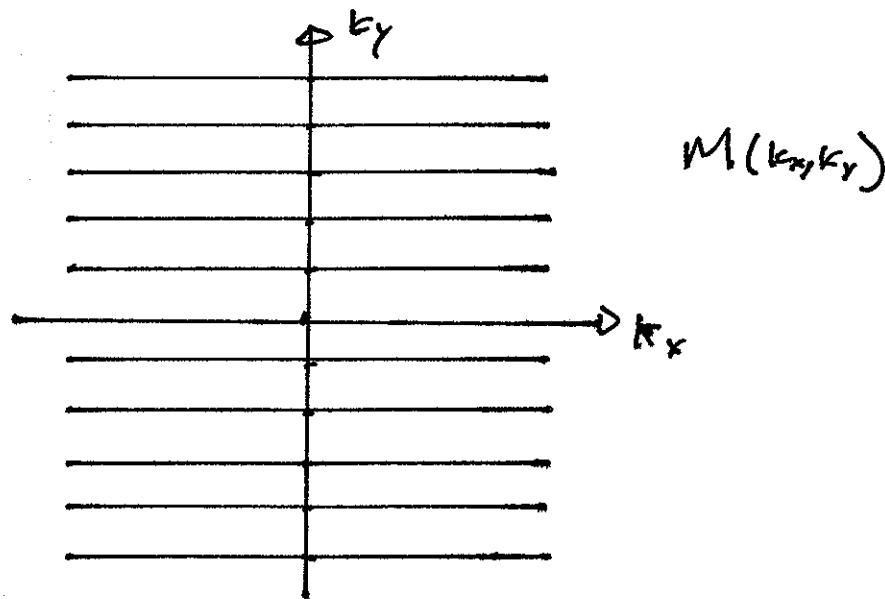
READ SECTION 13.2 IN BERNSTEIN

(L3)

## NON-CARTESIAN MRI

2

TYPICALLY MRI ACQUISITIONS COLLECT DATA ON A CARTESIAN GRID IN SPATIAL FREQUENCY



KNOWN AS 2DFT OR "SPIN WARP", DUE TO KUMAR, WECTI, AND ERNST (YES, THAT ERNST!)

RECONSTRUCTION IS JUST A 2D DFT.

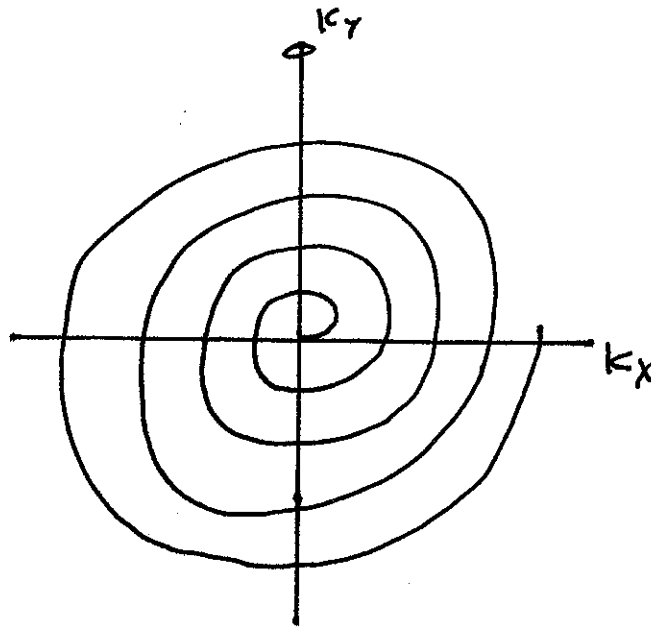
MUCH OF THE SUCCESS OF MRI IS DUE TO SPIN-WARP

- FAST RECONSTRUCTION
- IMMUNE TO SYSTEM IMPERFECTIONS
- VERY ROBUST

THERE ARE MANY OTHER POSSIBILITIES, AND THESE CAN HAVE ADVANTAGES

EXAMPLES

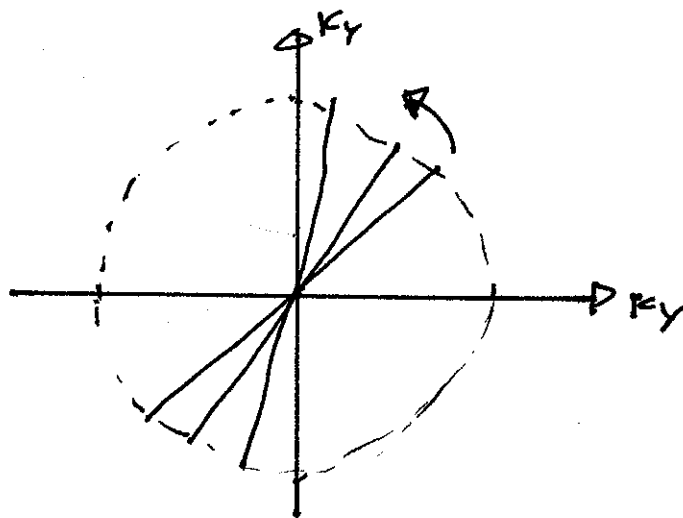
SPIRAL



USEFUL WHEN MOTION IS AN ISSUE

- CARDIAC
- fMRI

PROJECTION



USEFUL FOR UNDERSAMPLING

- REAL-TIME IMAGING
- CONTRAST ENHANCED IMAGING
- SHORT T2
- CT, PET

# PROBLEM

DATA DOES NOT FALL ON 2D CARTESIAN GRID.

HOW CAN WE RECONSTRUCT THE DATA?

## GENERAL PROBLEM: NON-UNIFORM FFT

### NUFFT

FOUR GENERAL PROBLEMS, DEPENDING ON WHETHER THE SOURCE OR DESTINATION IS NON-UNIFORM, (OR BOTH, OR NEITHER)

#### SOURCE DOMAIN

#### DESTINATION DOMAIN

UNIFORM

UNIFORM

DFT

UNIFORM

NON-UNIFORM

"PROBLEM 2"  
NUFFT  
INVERSE GRIDDING

NON-UNIFORM

UNIFORM

"PROBLEM 1"  
GRIDDING

NON-UNIFORM

NON-UNIFORM

"PROBLEM 3"

PROBLEM CLASSIFICATION DUE TO DUTT + ROKHLIN  
SIAM J. SCI. COMPUT. (1993) PP 1368-1393

PROBLEM 2 TURNS UP IN ITERATIVE RECONSTRUCTION OF PET + CT DATA

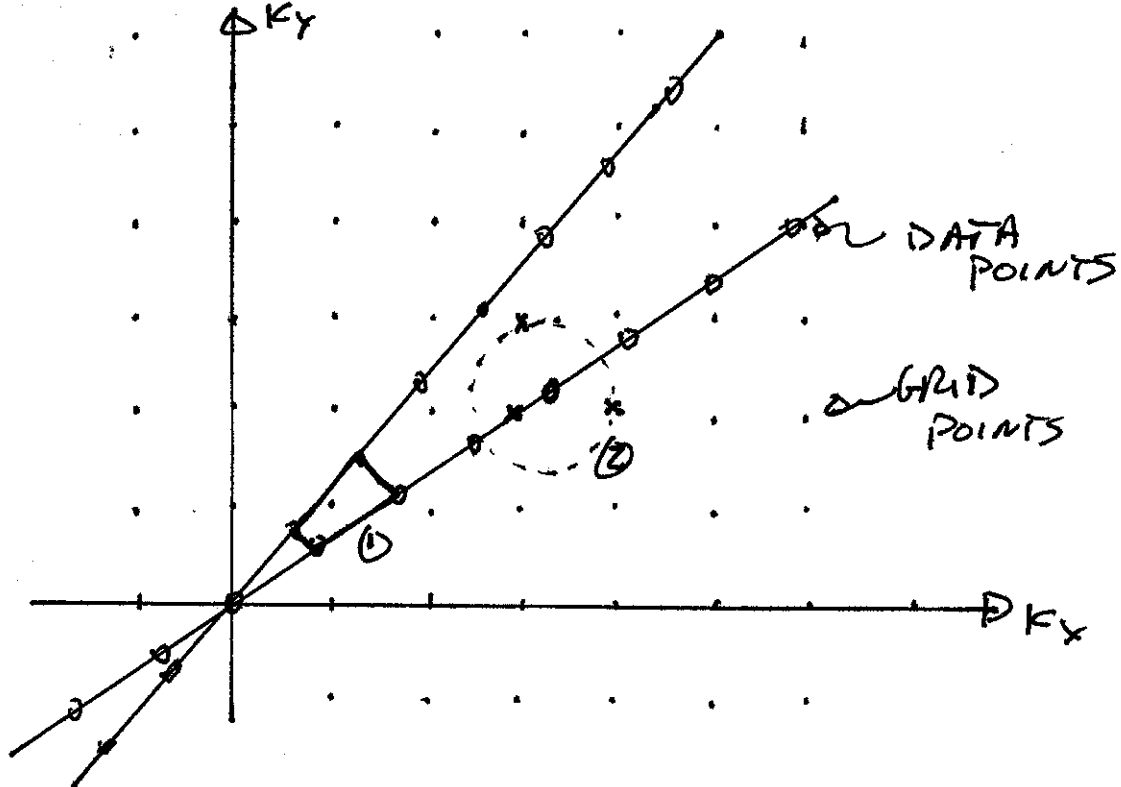
PROBLEM 1 IS NON-CARTESIAN MRI RECONSTRUCTION GRIDDING

# APPROACHES TO PROBLEM 1

CONSIDER THE PROJECTION CASE.

WE HAVE DATA ALONG DIAMETERS

WE WANT DATA ON 2D GRID



## TWO APPROACHES

1) FOR EVERY GRID POINT INTERPOLATE BETWEEN NEAREST DATA POINTS

2) FOR EVERY DATA POINT, ADD CONTRIBUTION TO NEARBY GRID POINTS

APPROACH (1) WORKS WELL

DOESN'T USE ALL THE DATA

APPROACH (2) USES ALL THE DATA

REQUIRES COMPENSATION FOR DENSITY

# BASIC GRIDDING IDEA

(6)

DATA FALLS ON NON-CARTESIAN POINTS

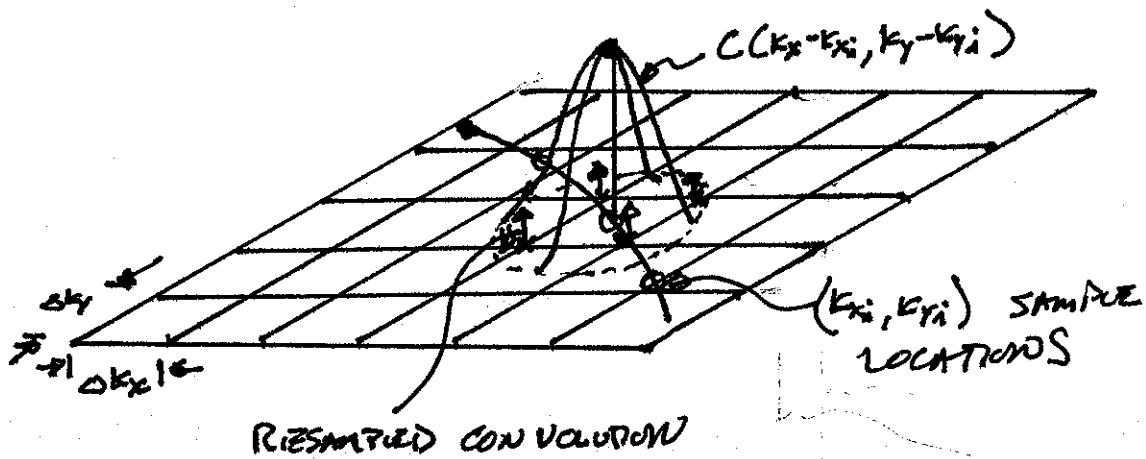
$$(k_{xi}, k_{yi})$$

CONVOLVE EACH POINT WITH A KERNEL

$$C(k_x, k_y)$$

RESAMPLE CONVOLUTION ON GRID POINTS

DFT TO RECONSTRUCT IMAGE



# SIMPLE VERSION

7

SAMPLING FUNCTION AT  $(k_{xi}, k_{yi})$

$$S(k_x, k_y) = \sum_i \delta(k_x - k_{xi}, k_y - k_{yi})$$

SAMPLED DATA,  $M(k_x, k_y)$  CONTINUOUS FREQUENCY DOMAIN DATA

$$M(k_x, k_y) S(k_x, k_y)$$

CONVOLVE WITH KERNEL

$$[M(k_x, k_y) S(k_x, k_y)] * C(k_x, k_y)$$

SAMPLE ON 2D GRID

$$\hat{M}(k_x, k_y) = \left( [M(k_x, k_y) S(k_x, k_y)] * C(k_x, k_y) \right) \Downarrow \left( \frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y} \right)$$

INVERSE TRANSFORM

$$\hat{m}(x, y) = \left[ (m(x, y) * s(x, y)) \cdot c(x, y) \right] * \Downarrow \left( \frac{x}{FOV_x}, \frac{y}{FOV_y} \right)$$

WHERE

$m(x, y)$  IS IMAGE DATA

$s(x, y)$  IMPULSE RESPONSE OF SAMPLING PATTERN

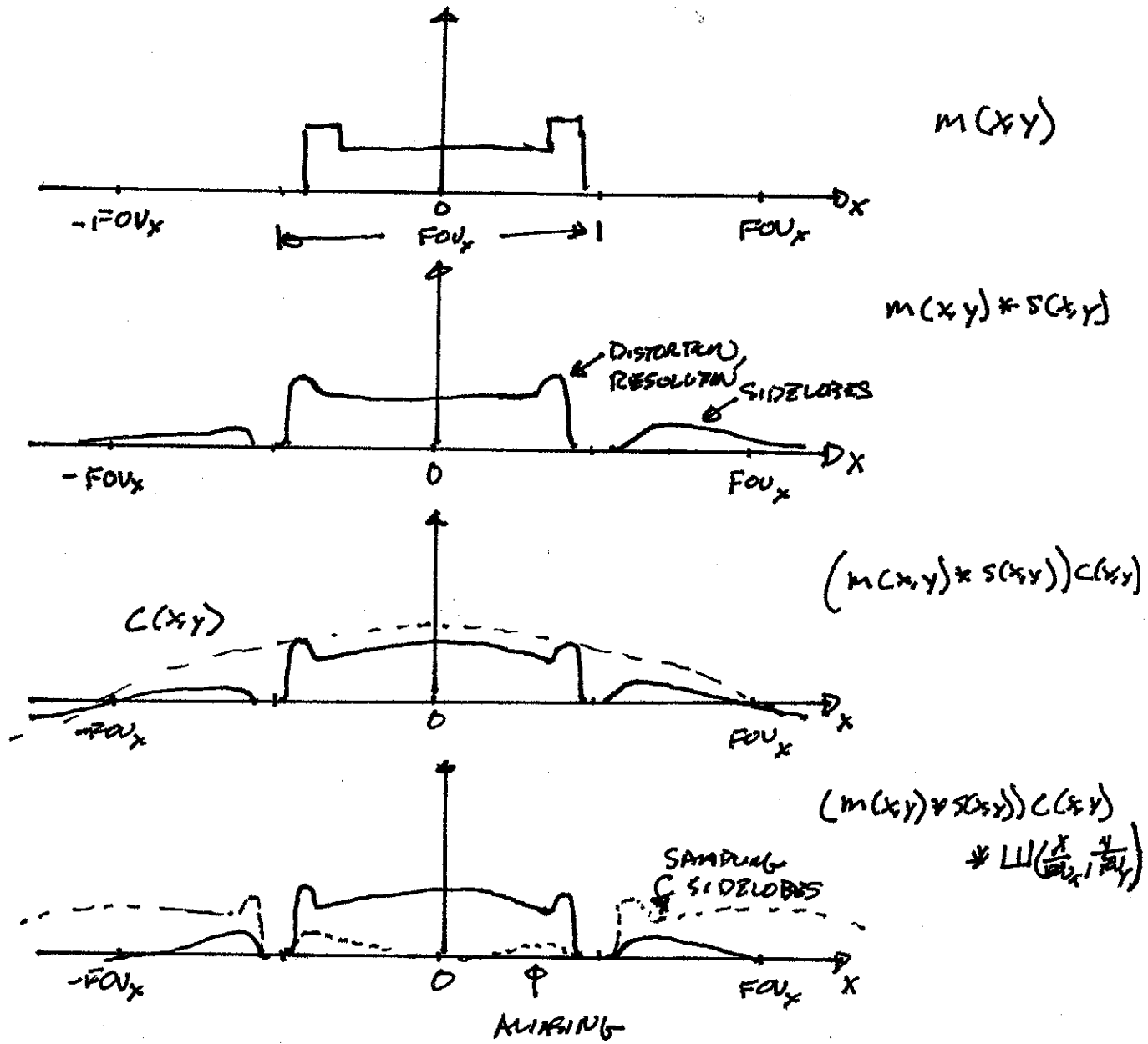
$c(x, y)$  APPROXIMATION

$FOV_x, FOV_y$  ARE  $\frac{1}{\Delta k_x}, \frac{1}{\Delta k_y}$

# GRAPHICAL INTERPRETATION

8

## 1D, IMAGE DOMAIN





RESULT

THE RECONSTRUCTION  $\hat{m}(x,y)$  IS

BLURRED, DISTORTED

ALIASED (DATA SAMPLING IN SPATIAL FREQUENCY)

APODIZED

ALIASED (GRID SAMPLING)

HOW CAN WE FIX THIS? IDEALLY

NO ALIASING

NO APODIZATION

FAST

DESIGN ISSUES

CONVOLUTION KERNEL  $(k_x, k_y)$

APODIZATION

ALIASING

GRID DENSITY  $\Delta k_x, \Delta k_y$

ALIASING

APODIZATION

SAMPLING PATTERN

IMPULSE RESPONSE

SIDELOBES, ALIASING

DENSITY