

PARALLEL MRI

FREQUENCY DOMAIN ALGORITHMS

SMASH

GRAPPA (PART 1)

SMASH

FIRST SUCCESSFUL PARALLEL IMAGING METHOD

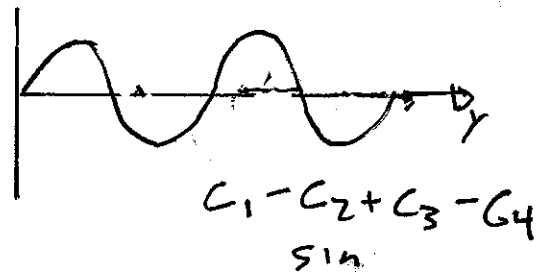
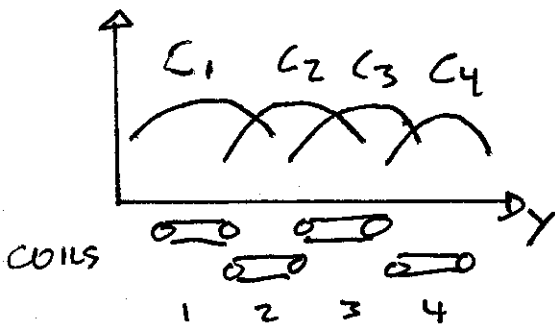
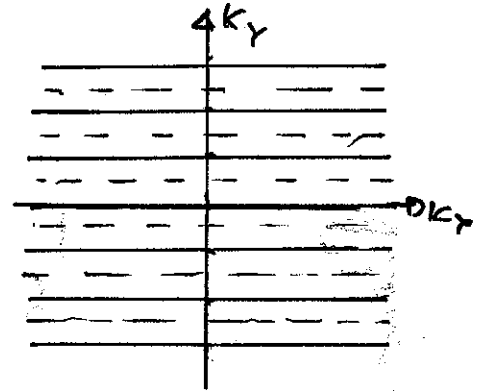
IDEA: ACQUIRE A REDUCED

SET OF PHASE ENCODES

USE COIL SENSITIVITIES

TO FILL IN MISSING

SPATIAL FREQUENCIES



THE LINEAR COMBINATION OF COIL SENSITIVITIES

LOOK LIKE SINUSOIDS

ONLY EASY TO DO IN ONE DIMENSION

LINEAR ARRAYS

HOW DO WE USE THIS?

MEASUREMENT OF j^{th} COIL

$$M_j(\underline{k}_m) = \int_{\underline{x}} C_j(\underline{x}) m(\underline{x}) e^{-i 2\pi \underline{k}_m \cdot \underline{x}} d\underline{x}$$

FOR SMASH, RECONSTRUCTION IS IN y , DO x DFT FIRST

LET k_m BE THE m^{th} PHASE ENCODE

$$M_j(k_m) = \int_y C_j(y) m(y) e^{-i 2\pi k_m y} dy$$

FOR AN ACCELERATION OF R WE NEED TO SYNTHESIZE R MEASUREMENTS

$$\hat{M}(k_m) = \int_y m(y) e^{-i 2\pi k_m y} dy$$

$$\hat{M}(k_m + \Delta k) = \int_y m(y) e^{-i 2\pi (k_m + \Delta k) y} dy$$

$$\hat{M}(k_m + (R-1)\Delta k) = \int_y m(y) e^{-i 2\pi (k_m + (R-1)\Delta k) y} dy$$

NOTE: THIS WOULD BE DATA FROM FULL FOV ACQUISITION

WE WANT TO SYNTHESIZE

$$e^{-i2\pi(p\Delta k)y} = \sum_{j=0}^{L-1} a_{j,p} C_j(y)$$

APPROXIMATES p^{th} PHASE ENCODING BY LINEAR COMBINATION OF $C_j(y)$

ONCE WE HAVE $a_{j,p}$ (LATER)

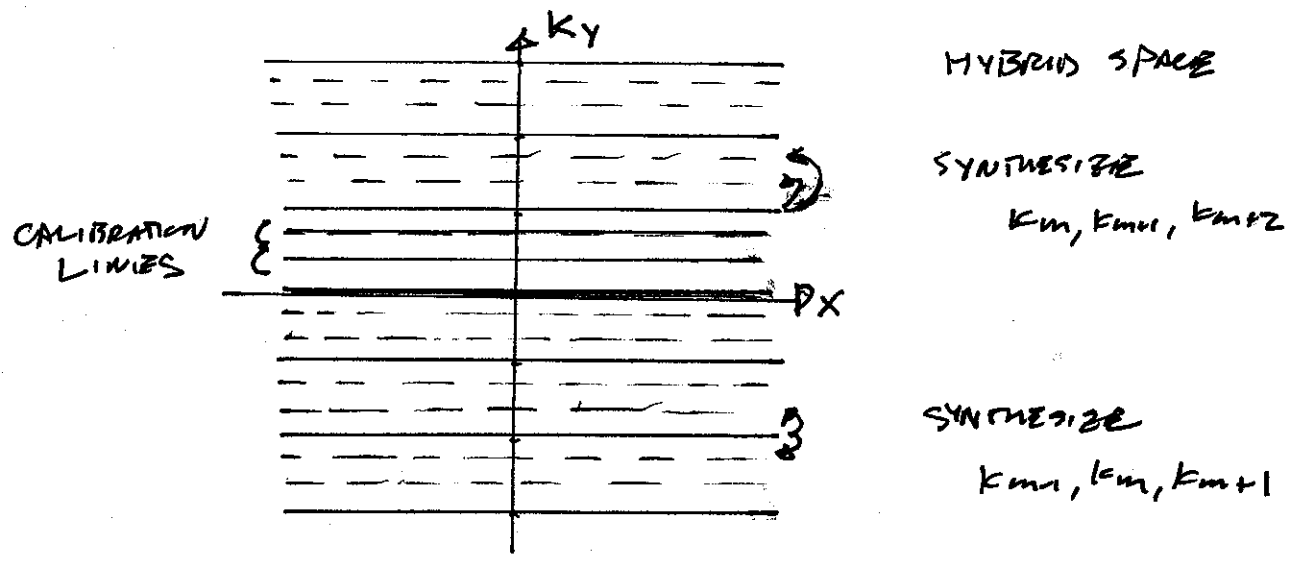
$$\hat{M}(k_m + p\Delta k) = \int_y m(y) e^{-i2\pi k_m y} \underbrace{e^{-i2\pi(p\Delta k)y}}_{\sum_{j=0}^{L-1} a_{j,p} C_j(y)} dy$$

$$= \int_y m(y) e^{-i2\pi k_m y} \sum_{j=0}^{L-1} a_{j,p} C_j(y) dy$$

$$= \sum_{j=0}^{L-1} a_{j,p} \int_y m(y) C_j(y) e^{-i2\pi k_m y} dy$$

$m_j(k_m)$ ACQUIRED DATA

$$\hat{M}(k_m + p\Delta k) = \sum_{j=0}^{L-1} a_{j,p} m_j(k_m)$$



COMPUTING $a_{j,p}$

DEPENDS ON X, WE'LL IGNORE THIS

WE WANT

$$e^{-jz\alpha(p\alpha k)y} = \sum_{j=0}^{L-1} a_{j,p} c_j(y)$$

WHERE $p = 0..R-1$ AND y IS SAMPLED $0..N_y-1$

OPTIONS

- 1) MEASURE $c_j(y)$, DO LEAST SQUARES FIT FOR $a_{j,p}$
- 2) DERIVE $a_{j,p}$ DIRECTLY FROM DATA

AUTOCALIBRATION

AUTO-SMASH

ESTIMATE $a_{j,p}$ DIRECTLY

SMASH SYNTHESIS EQU

$$\hat{M}(k_m + p\Delta k) = \sum_{p=0}^{L-1} a_{j,p} m_j(k_m)$$

SYNTHESIS
 ←
 L-1
 →
 CALIBRATION

FOR THE CALIBRATION LINES WE KNOW

$$\hat{M}(k_m + p\Delta k), m_j(k_m)$$

SOLVE FOR $a_{j,p}$ FROM CALIBRATION DATA

SYNTHESIZE ALL THE MISSING DATA WITH $a_{j,p}$.

VARIATIONS

USE MULTIPLE SETS OF CALIBRATION LINES

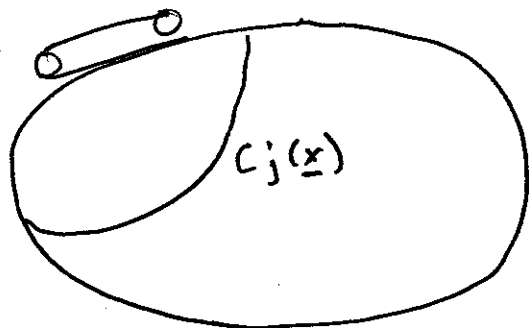
USE A BLOCK OF PHASE-ENCODE LINES

⇒ GRAPPA

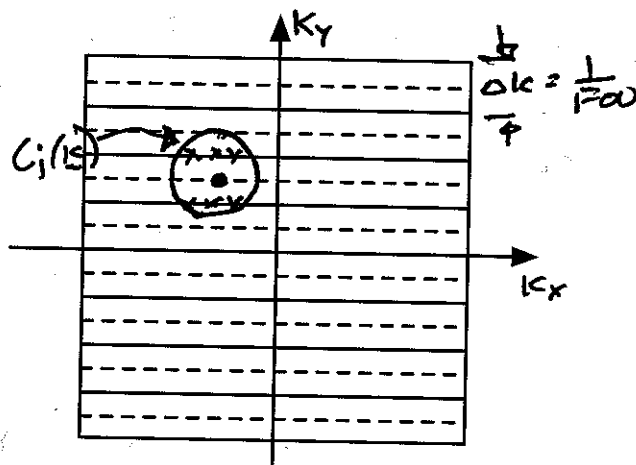
GRAPPA

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COIL SENSITIVITIES ARE
LOCAL IN IMAGE SPACE
EXTENDED IN k -SPACE

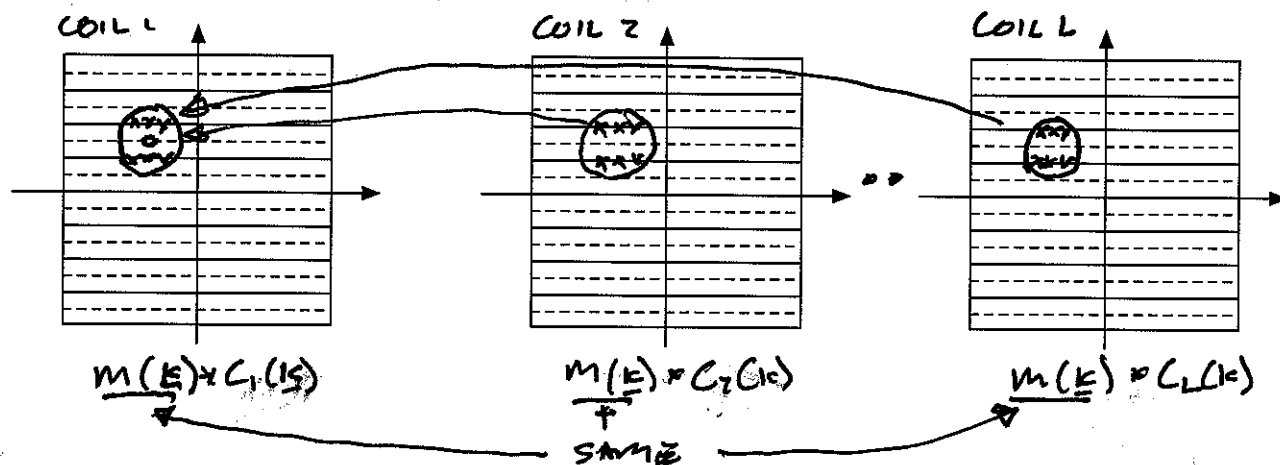


$$m(x) c_j(x)$$



$$M(k) * C_j(k)$$

MISSING INFORMATION IS IMPLICITLY CONTAINED
BY ADJACENT DATA

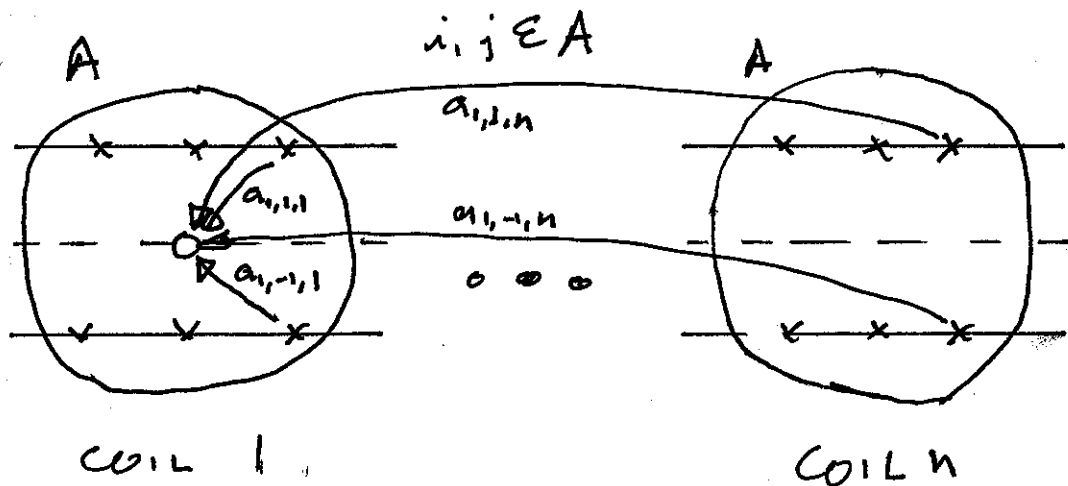


HOW DO WE FIND MISSING DATA FROM THESE SAMPLES?

WE WANT TO FIND COEFFICIENTS SUCH THAT

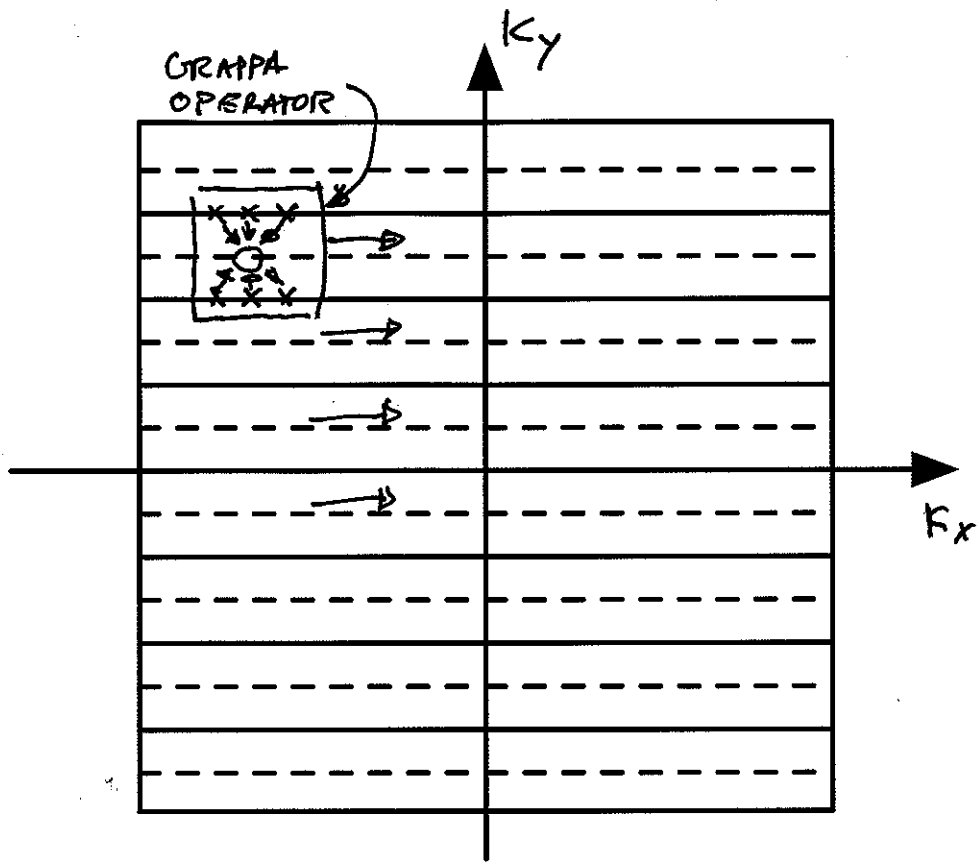
$$\underbrace{\vec{m}_{ic}(k_x, k_y)}_{\text{MISSING DATA}} = \sum_{i,j \in A} \underbrace{a_{i,j,k}}_{\text{WEIGHTS}} \underbrace{m_k(k_x + i \Delta k, k_y + j \Delta k)}_{\text{NEIGHBORHOOD DATA FOR EACH COIL}}$$

i, j ARE RELATIVE INDICES FOR ACQUIRED SAMPLES IN NEIGHBORHOOD



ONE COEFFICIENT FOR EACH NEIGHBORHOOD SAMPLE, FOR EACH COIL

SAME COEFFICIENTS WORK EVERYWHERE



FILL IN K-SPACE LINE BY LINE

FILL IN MISSING DATA FOR EACH COIL

RECONSTRUCT WITH ZDFT

GIVES SENSITIVITY WEIGHTED IMAGES

$$\hat{M}_k(\underline{x}) = C_k(\underline{x}) m(\underline{x})$$

COMBINE WITH

- SQUARE ROOT OF SUM OF SQUARES
- MULTICOIL RECON

HOW DO WE FIND $a_{i,jk}$?