Note: HW0 has NO effect on your grade of EE376A. There is no requirement to hand it in. Those are warm-up exercises in probability. However, we are happy to get your feedback or hear if you have some cute problems in probability.

1. Independence.

(a) Let $Z$ be a bernoulli random variables (r.v.) with parameter $1/2$, (Bern(1/2)), i.e. $P(Z = 0) = 1/2, P(Z = 1) = 1/2$. Let $X$ be a bernoulli random variables with parameter $p$ and $X$ is independent of $Z$. Show that the modulo 2 sum of $X$ and $Z$ is independent of $X$ for any $p$ value.

(b) Let $Z_1, \cdots, Z_n$ be independent Bern(1/2) r.vs. Suppose $(X_1, \cdots, X_n)$ is a random binary sequence and independent of $(Z_1, \cdots, Z_n)$. Let $Y_i = X_i + Z_i \pmod{2}$. What is the distribution of $(Y_1, \cdots, Y_n)$?

(c) Consider a more general case. Let $Z$ be uniformly distributed over $\{0, 1, \cdots, M - 1\}$. $X$ is independent of $Z$, with arbitrary probability mass function over $\{0, 1, \cdots, M - 1\}$. Show that the modulo $M$ sum of $X$ and $Z$ is independent of $X$.

2. Expectation

(a) We flip a fair coin until we see a head. What is the expected number of flips? What is the variance of the number of flips?

(b) If we keep flipping until we see $m$ heads. What is the expected number of flips? What is the variance of the number of flips?

3. Conditional probability

Suppose $X$ has an exponential distribution with parameter 1 and $Y$ is an arbitrary non-negative random variable, independent of $X$. Given the event $X > Y$, what is the distribution of $X - Y$. [hint: calculate $P(X - Y > t|X > Y)$].

4. Gaussian random variables

Let $X$ be a Gaussian random variable (r.v) with mean 0 and variance 1. Set

$$Y = \begin{cases} -X & \text{if } |X| \leq c, \\ X & \text{if } |X| > c. \end{cases}$$

(a) What is the distribution of $Y$?

(b) What is the joint distribution of $(X, Y)$? Is it a multivariate Gaussian distribution?

(c) Can you find a value of $c$ such that $\mathbb{E}[XY] = 0$ (not necessary in closed form)? For this $c$ value, is $Y$ independent of $X$?
(d) For $n > 2$, can you find a random vector $(X_1, \cdots, X_n)$ such that the marginal distribution for any $m$ ($m < n$) of them is a $m$ dimensional Gaussian distribution, but the joint distribution for $(X_1, \cdots, X_n)$ is not a $n$ dimensional Gaussian distribution?

5. Watch the documentary movie Claude Shannon  Father of the Information Age. 
   http://www.youtube.com/watch?v=z2Whj_nL-x8