Acknowledgements

This talk is based on:

Doc, What Are My Chances?


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For more information, see:

[comap.com](http://comap.com) (UMAP Journal publisher)

[modernnomograms.com](http://modernnomograms.com) (nomograms)
Medicine Today

- Intensive use of high-tech
  - Diagnostic imaging
  - Surgical advances (e.g. laparoscopy)
  - Robotics (e.g. daVinci)
  - Genome sequencing
  - Pharmaceuticals

- Science and mathematics lagging
  - Evidence-based medicine (1990)
  - Education of clinicians
  - Patient empowerment
Road Map of the Talk

Medical Diagnostics

Bayes’ Theorem  Nomography
Road Map of the Talk

Medical Diagnostics
It’s All About Probabilities

Bayes’ Theorem
Nomography
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Medical Diagnostics
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Bayes’ Theorem
Math for Updating Probabilities Based on Test Results

\[ P_{\text{OLD}} + \text{Test} = P_{\text{NEW}} \]
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Nomography
Graphical Technique for Doing the Math
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Medical Diagnostics
It’s All About Probabilities

Bayes’ Theorem
Math for Updating Probabilities Based on Test Results
\[ P_{OLD} + \text{Test} = P_{NEW} \]

Nomography
Graphical Technique for Doing the Math

\[ P_{NEW}, \text{Test}, P_{OLD} \]
Example: PSA Screening

We agree
- Screening misses 83% with prostate cancer
  - 15 false negatives out of 18 total
- > 50% negative biopsy rate
  - 5 false alarms out of 8 positives

Wrong cutoff?
- PSA = 4 is best trade-off

80% “overall accuracy” misleading

Test characterization problem
Back to Square One

- Test characterization depends on usage
- Test only one part of diagnostic protocol
- Reasonable diagnostic protocol
  - Keeps physician in the loop
  - Employs scientific method
  - Uses historical data
  - Simple
  - Easy to explain
- Candidate: Bayes’ Theorem
**The Scientific Method**

1. Initial probability theory is valid
2. Do experiment
3. Result either tends to confirm or refute theory
4. **Update**: New probability theory is valid depends on initial probability, experimental result and *strength* of experiment
5. New probability becomes the initial probability for next cycle
6. Go to 1 (iterative approach)

**Diagnostic Protocol**

1. Doctor’s estimate of probability of disease based on history, symptoms, examination, etc.
2. Perform diagnostic test
3. Test comes back positive or negative
4. **Update**: New estimate of probability of disease based on prior estimate, test result and *test strength*
5. New estimate is starting point for next test, if necessary
6. Go to 2, if necessary
Pretest Probability Below TT

Only a Positive Test Result Helps

Test too weak

Test Strong Enough

Probability of Disease

0 % 100 %

Pretest Test Posttest Pretest Test Posttest

Treatment Threshold
Only a **Negative** Test Result Helps

**Pretest Probability Above TT**

- **Test too weak**
  - Pretest: 100%
  - Test: -
  - Posttest: 100%

- **Test Strong Enough**
  - Pretest: 100%
  - Test: -
  - Posttest: 0%
These two numbers, called Likelihood Ratios (LR), completely characterize the test.
Waltzing with Bayes
1. Pretest Probability \(\rightarrow\) (algebra) \(\rightarrow\) Pretest Odds
2. Apply Bayes: Posttest Odds = Pretest Odds \times\ LR
3. Posttest Odds \(\rightarrow\) (algebra) \(\rightarrow\) Posttest Probability

Many tests specified by sensitivity and specificity
- (Sensitivity, Specificity) \(\rightarrow\) (algebra) \(\rightarrow\) (LR+, LR-)

Nomograms eliminate mechanical problem
Nomography in a Nutshell

- Developed in 1890’s by Maurice d'Ocagne
- Bespoke paper Slide Rule (w/o slide)
- Algebraic relationships rendered geometrically
- Provides insight through visualization
- Draw straight line from one scale to another; answer is found where line intersects 3rd scale
- 75-year run in engineering and medicine
- Went out of fashion with advent of pocket calculators and computers (circa 1970’s)
- Very easy to use, can be very difficult to craft
Enter Dr. Fagan

- Arrives end of nomographic era
- Letter to NEJM, July 1975
- Elegant
- Minimalist
- Became “Fagan Nomogram”
- Cited hundreds of times over the following 35 years
- Not widely used
- Not improved upon
- Until 2011

↩ Please respect NEJM’s copyright
A New Bayes’ Nomogram
1. This test has an LR+ = 2.8

2. Absent any other info, Pretest Probability = 18%

3. Assume test comes back positive

4. Find 18% on Pretest Probability on scale at bottom

5. Find 2.8 on LR+ scale in middle

6. Determine Posttest Probability of 38% on scale at top

**Conclusion:** If treatment threshold is 50%, this test is not useful for routine screening
1. This test has
   - Sensitivity = 0.17
   - Specificity = 0.94
2. Locate 0.17 on lower right ellipse (blue)
3. Locate 0.94 on upper left ellipse (blue)
4. Connect these two points
5. LR+ value is now determined: 2.8
6. Proceed as before

Conclusion: If treatment threshold is 50%, this test is not useful for routine screening
Similar analysis for LR-
Nomogram has other scales to determine all parameters from the 2x2 matrix
In practice, sensitivity and specificity are the most commonly listed parameters
Test parameters are cohort-dependent and evolve over time as more data are obtained
Thus protocol is doubly-Bayesian
- Individual’s posttest probability is a Bayesian update based on test parameters
- Over time, new data cause a Bayesian update on the test parameters themselves
There is a second new nomogram for rare diseases (very low pretest probabilities requiring high LR+)
Problem: Construction of good nomograms is non-trivial

Solution: PyNomo

- Python package for creating nomograms
- **Input:** Python script describing all aspects of the relationships and the display
- **Output:** Camera-ready PDF or EPS
- Vector graphic precision
- Written by Leif Roschier
- Open source
- Get information at [pynomo.org](http://pynomo.org)

All our nomograms were created using PyNomo

Ask me about iPad support
THANK YOU

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