## Introduction to Clos Networks

Clos network is a multistage switching network. Figure 1 shows an example of a 3-stage clos network. The advantage of such network is that connection between a large number of input and output ports can be made by using only small-sized switches. A bipartite matching between the ports can be made by configuring the switches in all stages. In figure 1, n represents the number of sources which feed into each of the m ingress stage crossbar switches. As can be seen, there is exactly one connection between each ingress stage switch and each middle stage switch. And each middle stage switch is connected exactly once to each egress stage switch.



Figure 1: 3-stage clos network.

It can be shown that with  $k \ge n$ , the clos network can be non-blocking like a crossbar switch. That is for each input-output matching we can find an arrangement of paths for connecting the inputs and outputs through the middle-stage switches. The following theorem shows that for adding a new connection, there won't be any need for rearranging the existing connections so long as the number of middle-stage switches is large enough.

**Clos Theorem:** If  $k \ge 2n-1$ , then a new connection can always be added without rearrangement.

Proof: Consider adding the *n*th connection between 1st stage  $I_a$  and 3rd stage  $O_b$  as shown in figure 2. We need to ensure that there is always some center-stage M available. If k > (n-1) + (n-1), then there is always an M available. i.e., we need  $k \ge 2n-1$ .



Figure 2: Adding the nth connection.