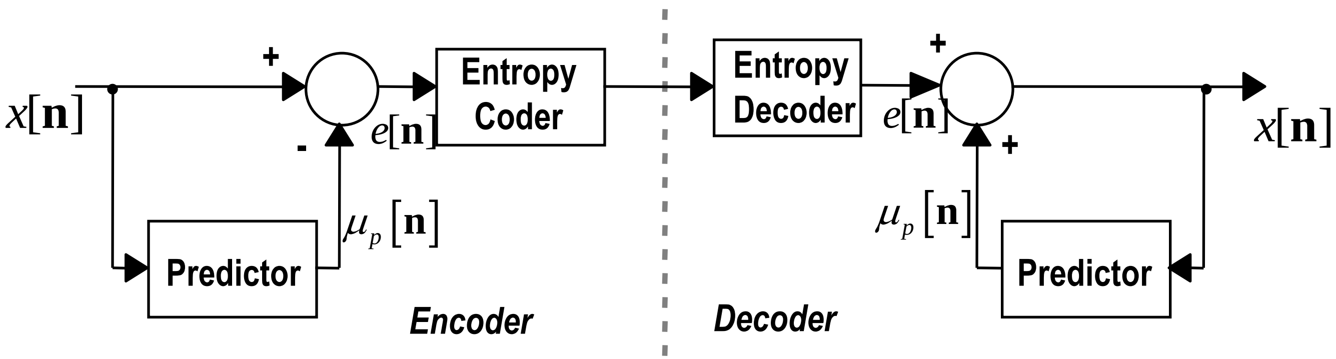
Predictive Coding

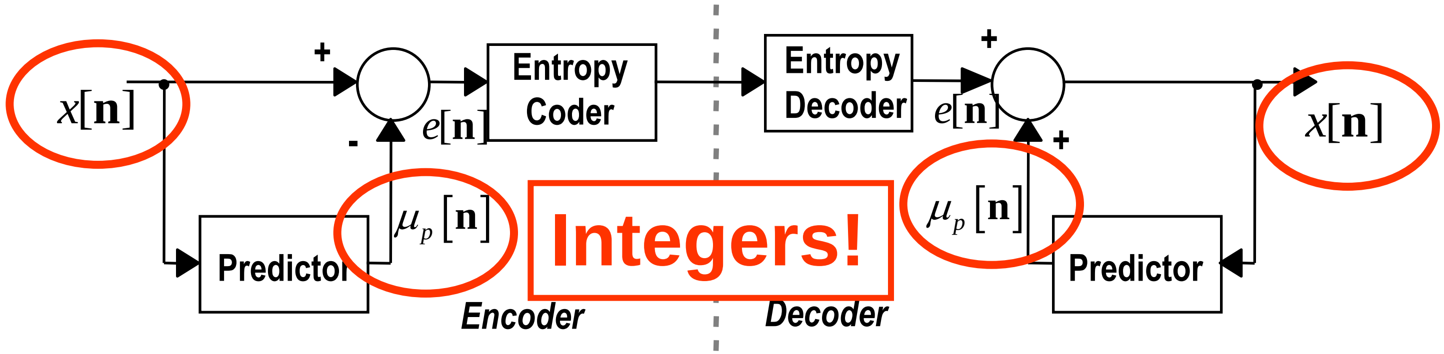
- Lossless predictive coding
- Optimum predictors
- JPEG-LS lossless compression standard
- Lossy predictive coding: DPCM
- Rate distortion performance of DPCM

Lossless Predictive Coding



- Prediction $\mu_p[\mathbf{n}]$ is calculated for $x[\mathbf{n}]$ from previous samples $\mathbf{x}_{\mathcal{N}+\mathbf{n}}$
- $e[\mathbf{n}]$ is prediction error, with greatly reduced statistical dependencies between adjacent samples
- Entropy coder may assume i.i.d. prediction error $e[\mathbf{n}]$
- Receiver can reconstruct $x[\mathbf{n}]$ without loss for amplitude-discrete signals $x, \mu_p, e \in \mathbb{Z}$
- Much simpler than context-adaptive coder

Lossless Predictive Coding



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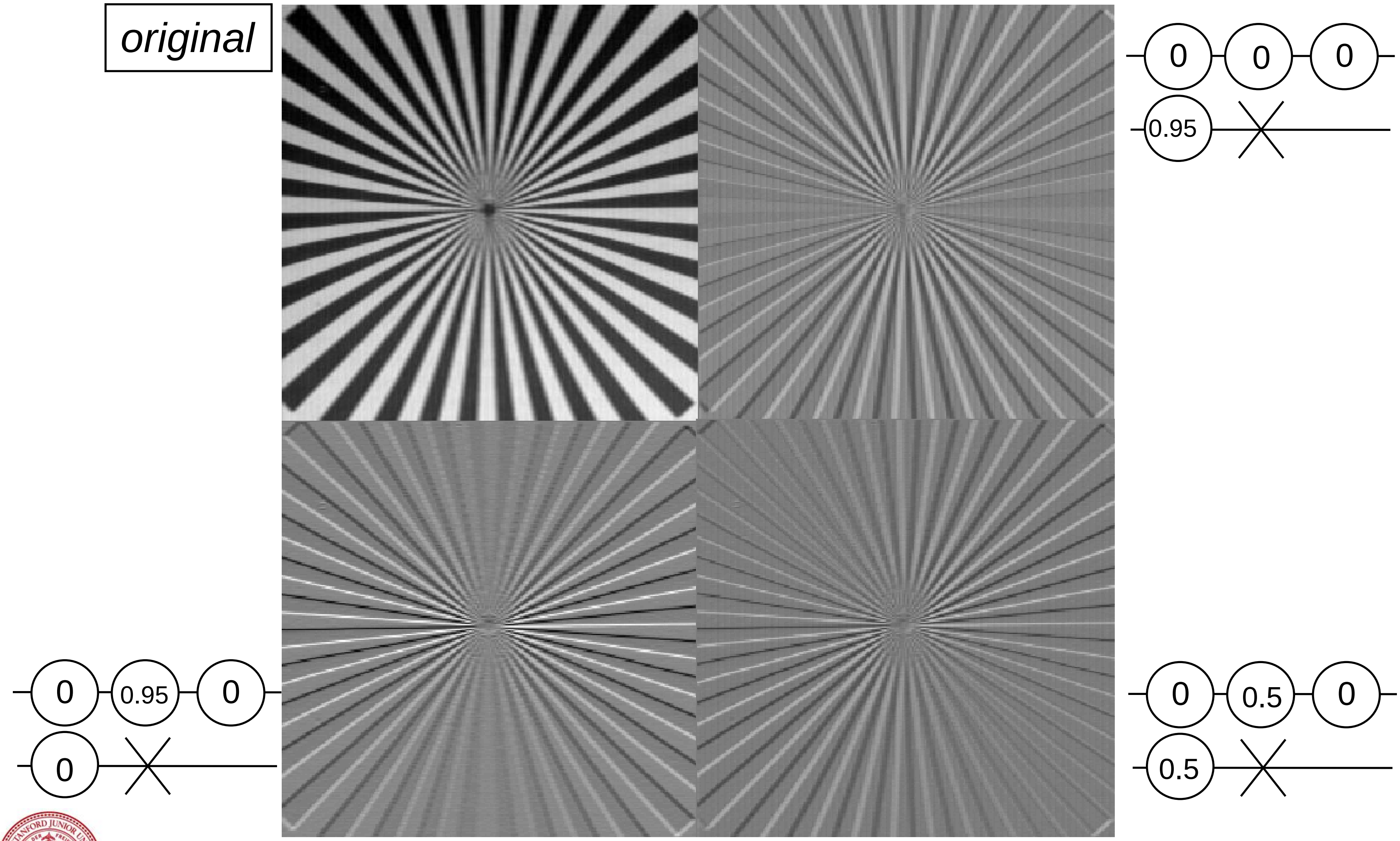
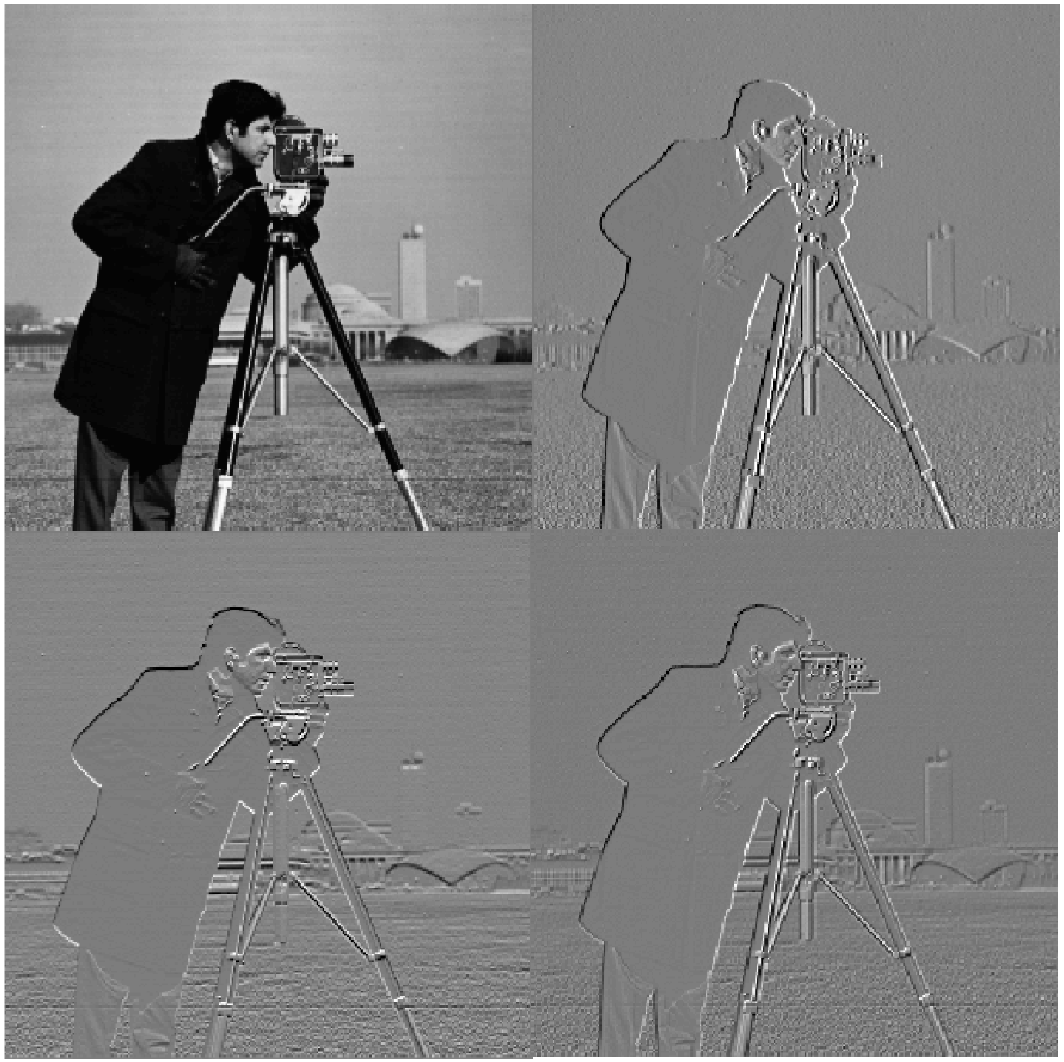
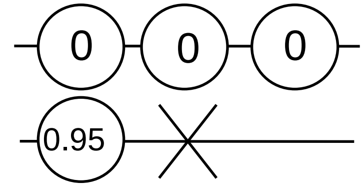
Prediction example: test pattern

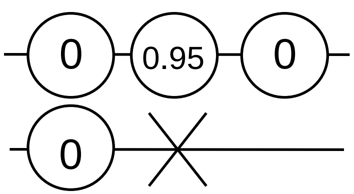


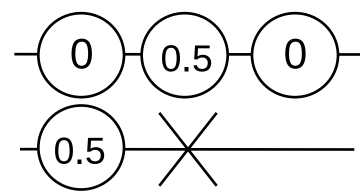
Prediction example: Cameraman

original

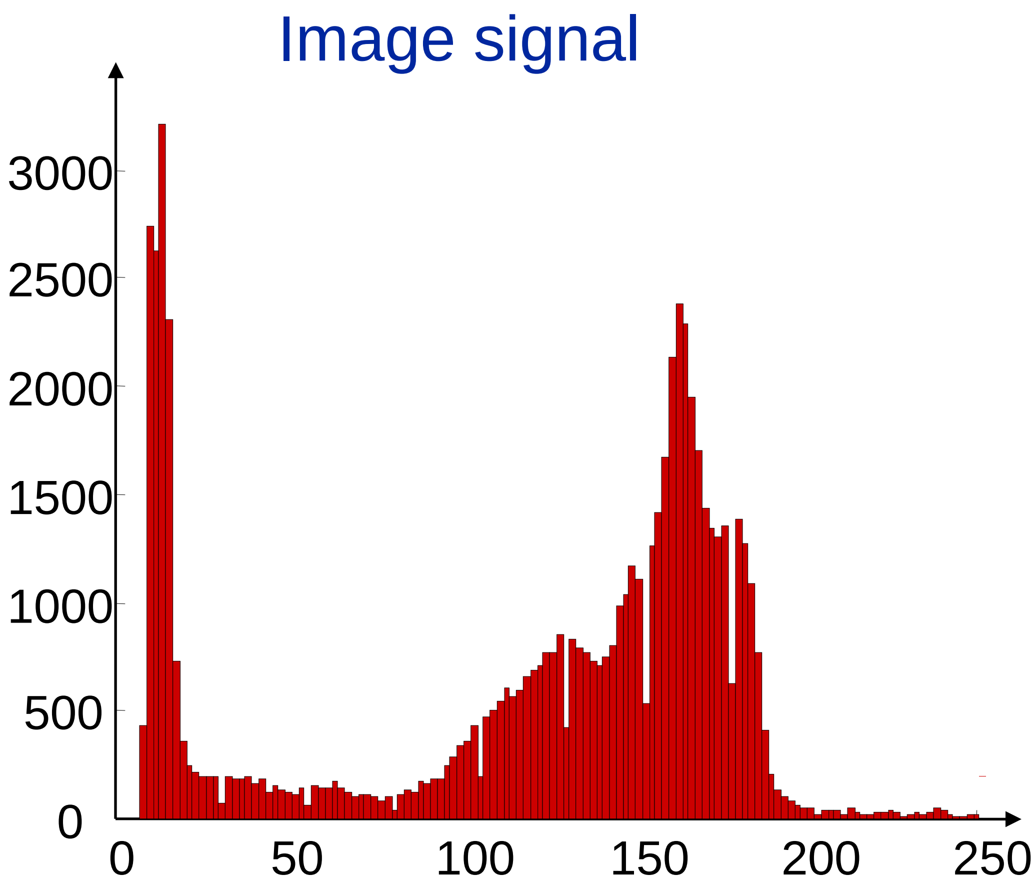


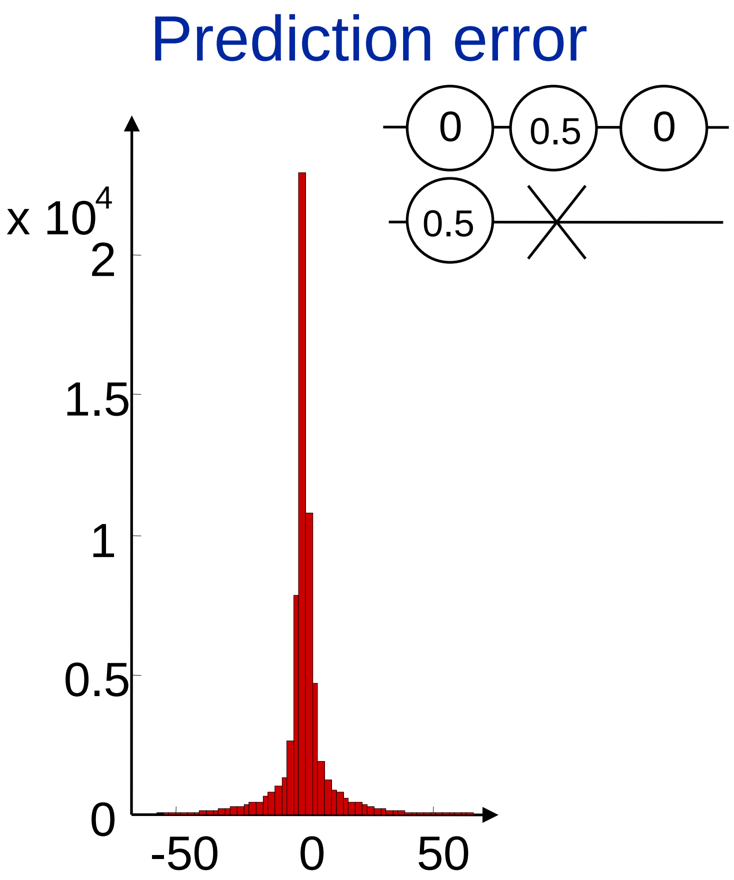






Histograms: Cameraman





Entropy and variance of the prediction error

- Approximation of the entropy of the prediction error E

standard deviation of E

constant that depends on the shape of the underlying PDF

$$H(E) \approx \log_2 \frac{\sigma_E}{\Delta} + c_{pdf} \quad \text{for} \quad \Delta << \sigma_E$$

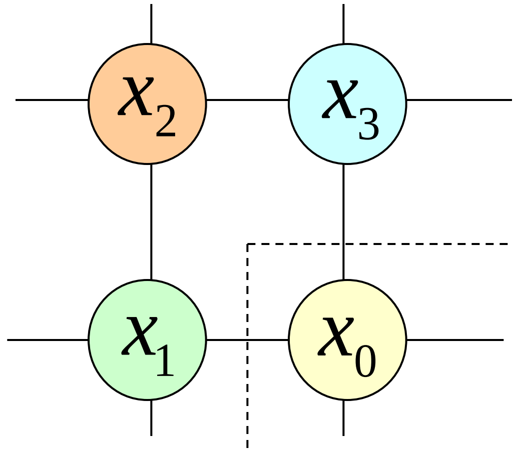
quantization step size

- Shape constant

Gaussian PDF: $c_{pdf} = 2.047 \text{ bit}$ Laplacian PDF: $c_{pdf} = 1.943 \text{ bit}$

- With linear prediction of image signals the prediction error PDF is typically Laplacian.
- Minimization of prediction error variance or prediction error entropy typically lead to very similar results.

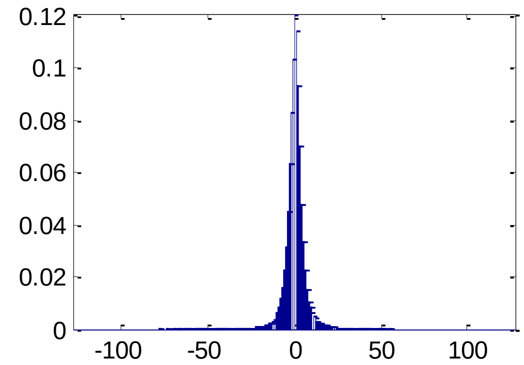
Optimum linear predictors for the luminance signal Y



line of pixels above

current line of pixels

$f_E(e)$ approx. Laplacian



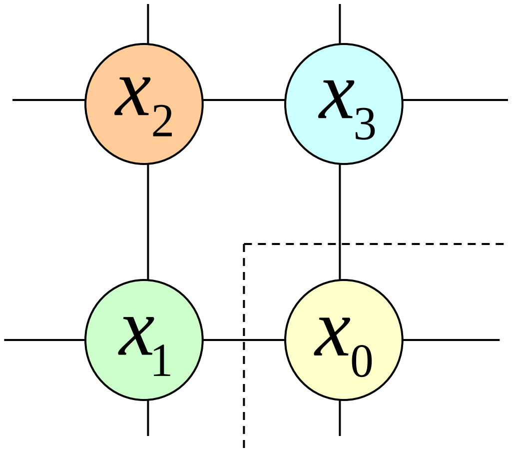
$$\Delta << \sigma_E \Rightarrow H(E) \cong \log_2\left(\sqrt{2}\, e\, \sigma_E\right) - \log_2 \Delta \Rightarrow \arg\min \sigma_E \cong \arg\min H(E)$$

$H(X_0)$ [bit]	Predictor			MSE	$H(E)$ [bit]	Criterion
	a_1	a_2	a_3			
7.23	0.595	-0.434	0.831	33.810	4.301	minimum variance
	0.464	-0.264	0.799	35.075	4.2813	minimum entropy

Image: 'Lena', 512 x 512 pixels, 8 bpp

Δ =1, 2^8 levels (-128..127)

Optimum linear predictors for the luminance signal Y



line of pixels above

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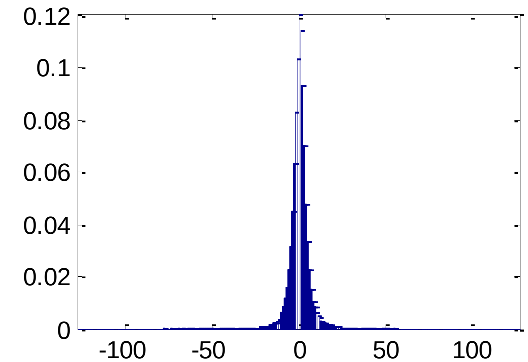


<u>Constraint:</u> 3 bit word length of the prediction coefficients, +1 bit for sign

$H(X_0)$ [bit]	Predictor			MSE	$H(E)$ [bit]	Criterion
	a_1	a_2	a_3			
7.23	5/8	-1/2	7/8	34.161	4.318	minimum variance
	1/2	-1/4	3/4	35.395	4.285	minimum entropy

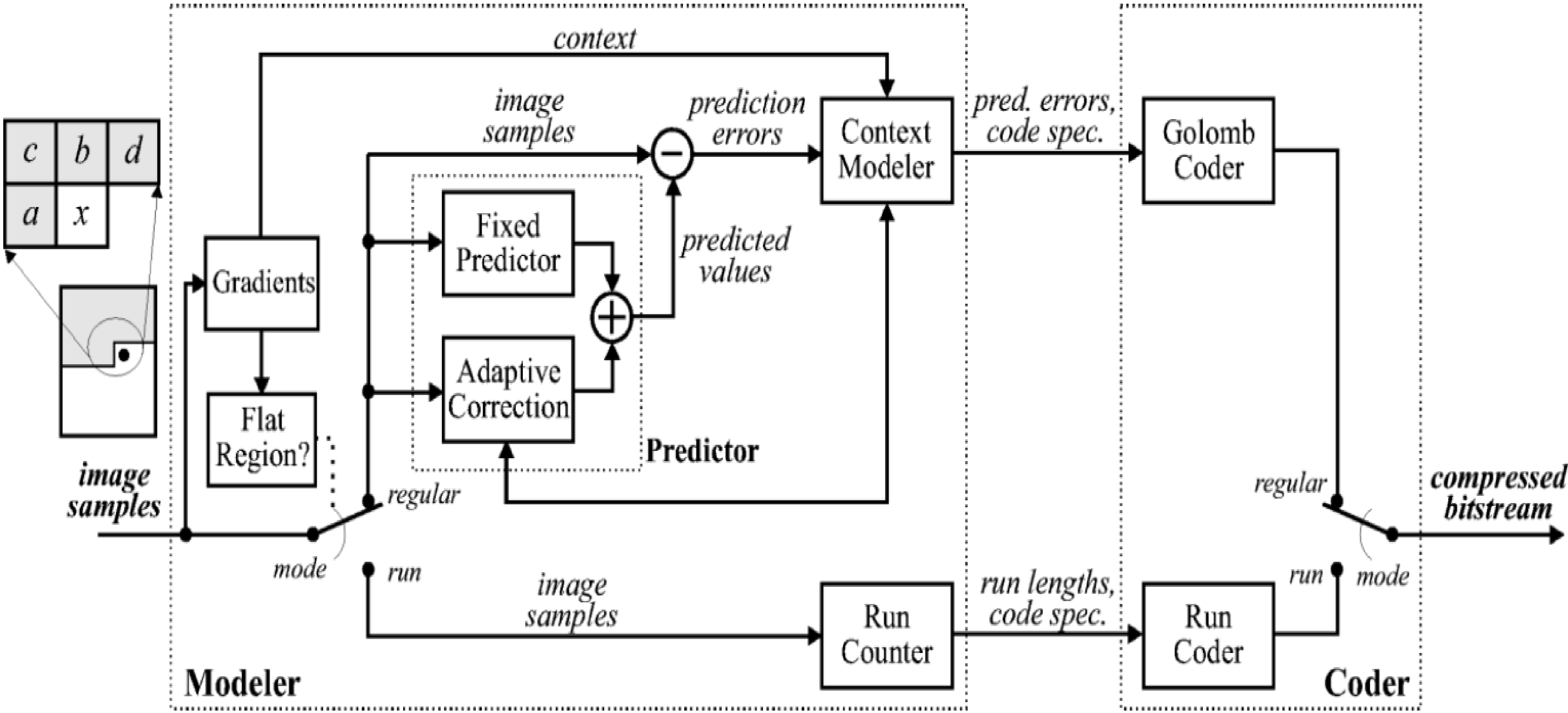
Image: 'Lena', 512 x 512 pixels, 8 bpp

Δ =1, 2^8 levels (-128..127)

JPEG-LS lossless compression standard

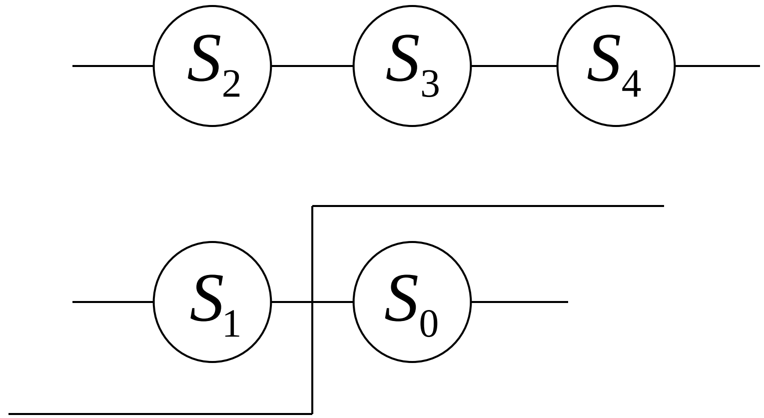
- Standards ISO/IEC 14495-1 and ITU-T T.87 *[1999]*
- Not to be confused with lossless mode of original JPEG
- Based on LOCO-I (Low Complexity Compression of Images) *[Weinberger, Seroussi, Sapiro, 1996]*
- Predictive coding with nonlinear predictor
- Context-adaptive Golomb coding of prediction error
- 365 different coding contexts, based on pixel differences in the causal neighborhood
- Switches to 1-d run-length coding for one context
- Run-lengths encoded by Golomb code
- “Near-lossless” mode extension

JPEG-LS blockdiagram



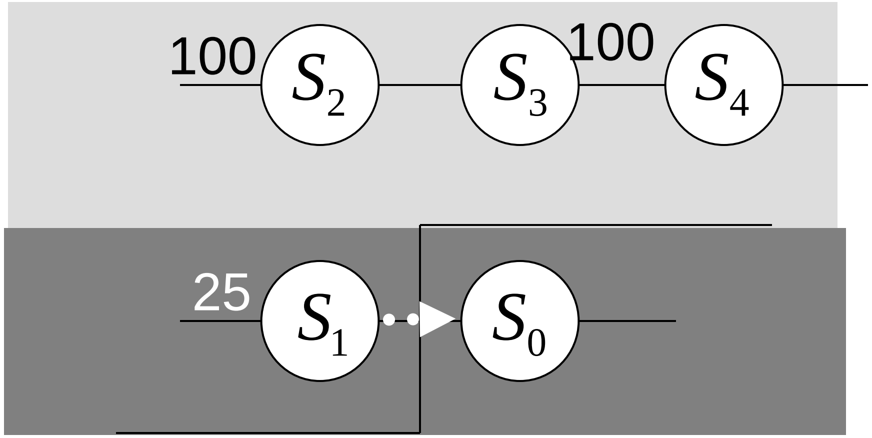
[Weinberger, Seroussi, Sapiro, 2000]

JPEG-LS nonlinear predictor



$$\mu_p = \begin{cases} \min\left(S_1, S_3\right) & \text{if } \mathrm{S}_2 = \max\left(S_1, S_2, S_3\right) \\ \max\left(S_1, S_3\right) & \text{if } \mathrm{S}_2 = \min\left(S_1, S_2, S_3\right) \\ S_1 - S_2 + S_3 & \text{else} \end{cases}$$

JPEG-LS nonlinear predictor



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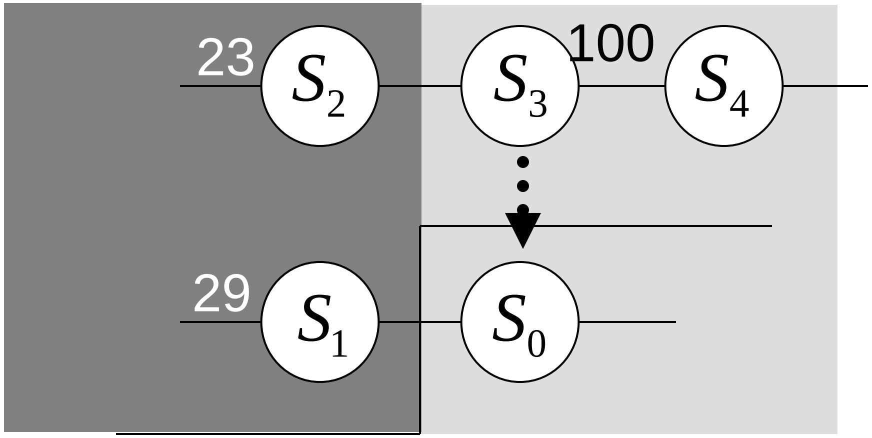
JPEG-LS nonlinear predictor



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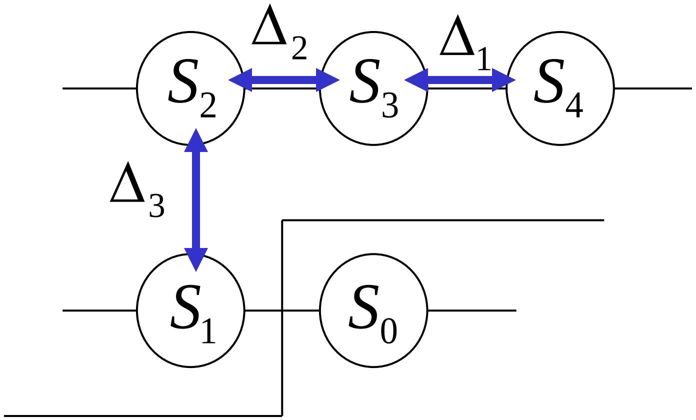
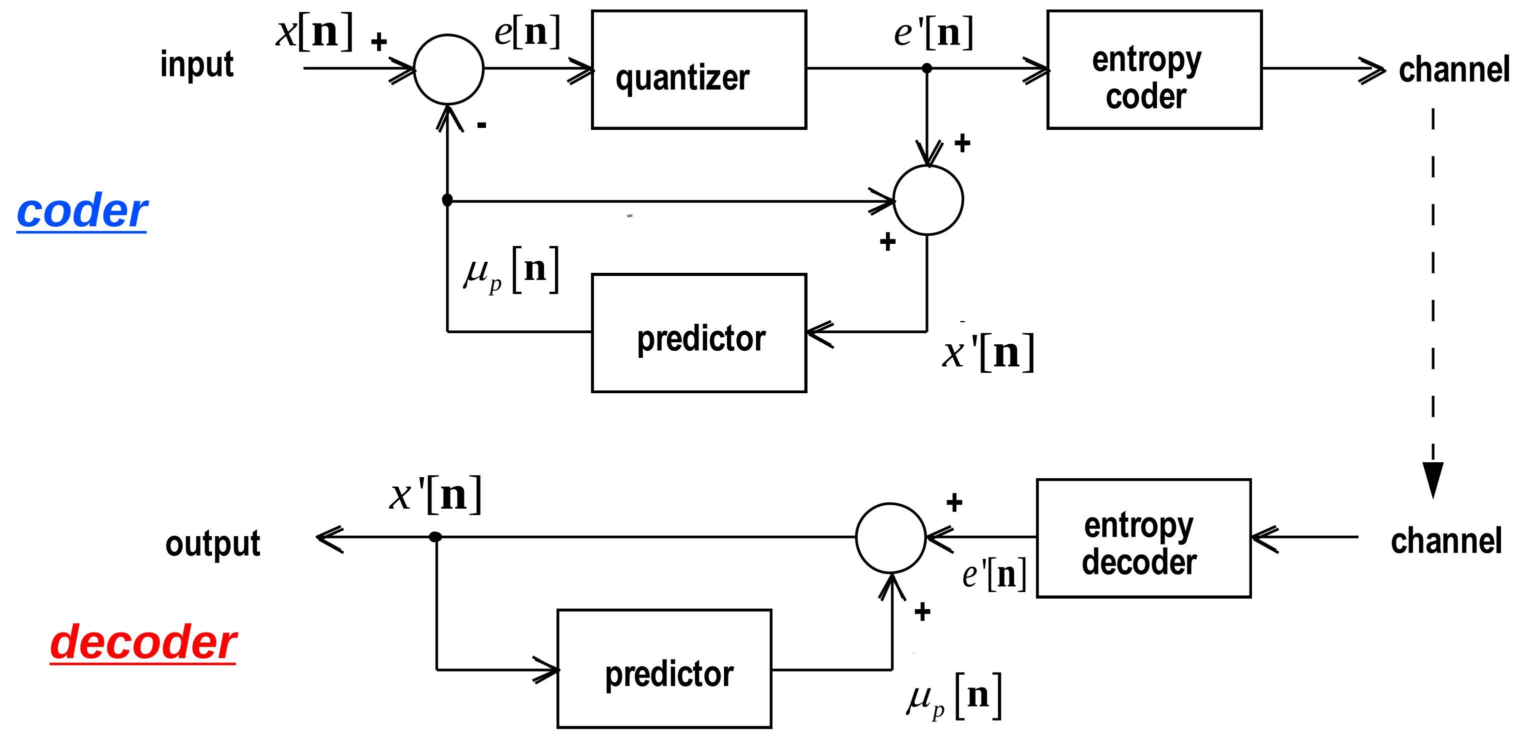
JPEG-LS context labeling



- Quantize each Δ_i to 1 out of 9 different indices: $9^3=729$ distinct contexts, default thresholds 3,7,21
- Further reduce to 365 contexts by exploiting sign symmetries
- For $\Delta_1 = \Delta_2 = \Delta_3 = 0$, switch to run-length coding

Lossy Predictive Coding: Differential Pulse Code Modulation (DPCM)



Reconstruction error = quantization error

$$x' - x = e' - e = q$$

Quantization error feedback in the DPCM coder

- For a linear predictor, the DPCM coder is equivalent to:

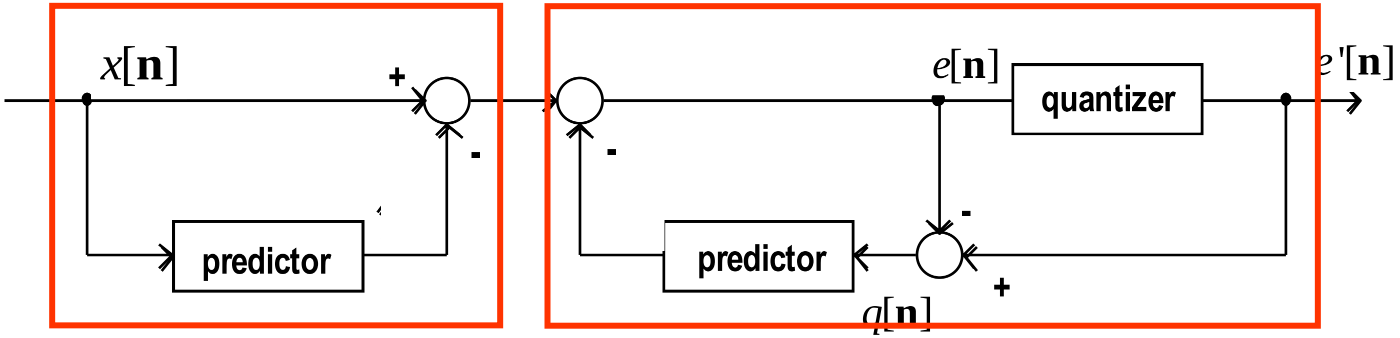
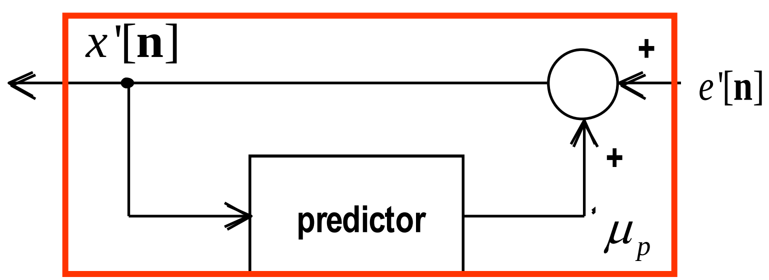


Image filtered by $\left[1-P(\omega_x,\omega_y)\right]$

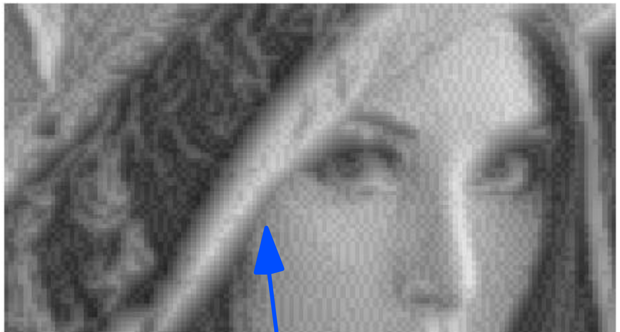
Quantization error filtered by $\left[1-P(\omega_x,\omega_y)\right]$

- Linear DPCM decoder



$$\frac{1}{1-P(\omega_x,\omega_y)}$$

Example of intraframe DPCM coding



1 bit/pixel
prediction error coding

2 bit/pixel

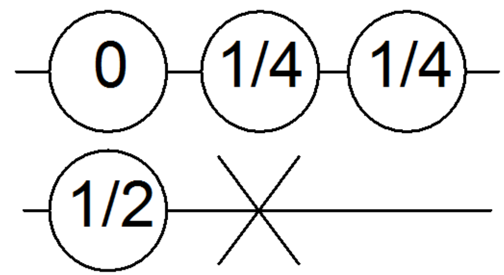
3 bit/pixel

slope overload

edge busyness

granular noise

- Linear predictor:



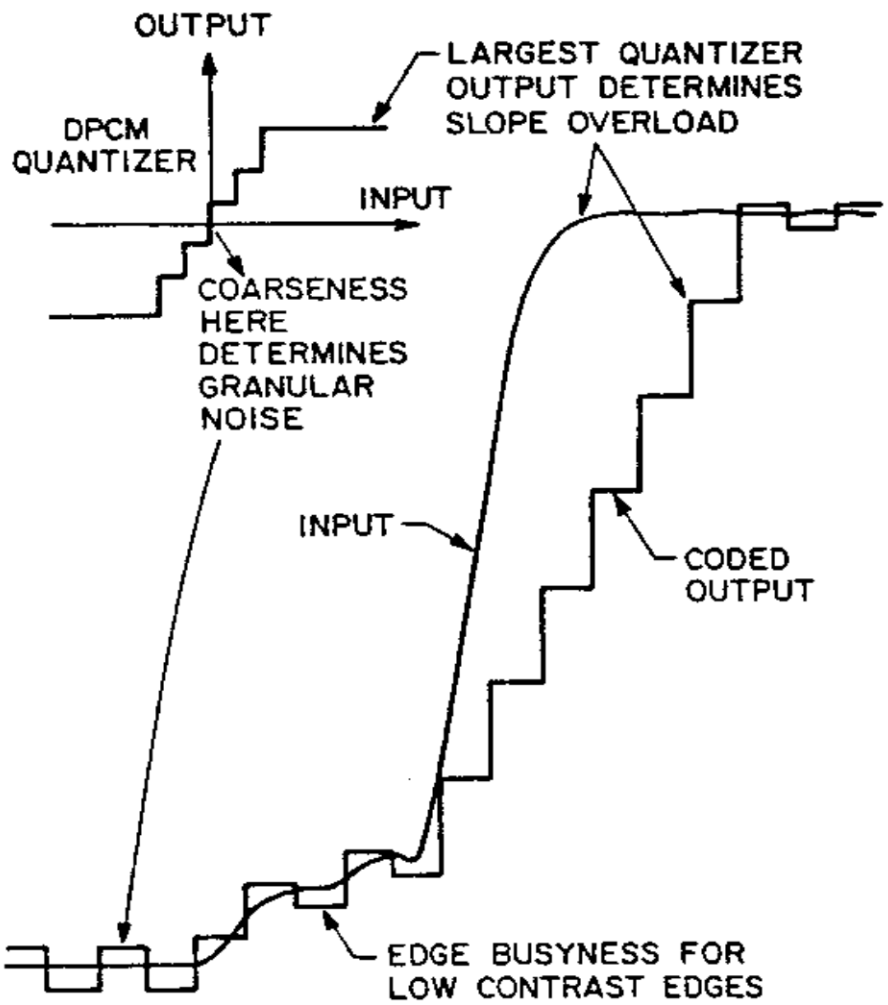
- Lloyd-Max quantizers
- Fixed-length coding

4 bit/pixel

original

Signal distortions due to intraframe DPCM coding

- <u>Granular noise</u>: random noise in flat areas of the picture
- <u>Edge busyness</u>: jittery appearance of edges (for video)
- <u>Slope overload</u>: blur of high-contrast edges, Moire patterns in periodic structures.



[Netravali + Haskell]

DPCM with entropy-constrained quantization

K=511
$H(e')=H(e)$=4.79 bpp

K=15
$H(e')$=1.98 *bpp*

K=3
$H(e')$=0.88 *bpp*

K ... number of reconstruction levels,
$H(e')$ ... entropy of quantized prediction error

[J. R. Ohm]

Recall from Chapter "Quantization" High-rate performance of scalar quantizers

- High-rate distortion-rate function

$$d(R) \cong \varepsilon^2 \sigma_X^2 2^{-2R}$$

- Scaling factor ε^2

	Shannon LowBd	Lloyd-Max	Entropy-coded
Uniform	$\frac{6}{\pi e} \cong 0.703$	1	1
Laplacian	$\frac{e}{\pi} \cong 0.865$	$\frac{9}{2} = 4.5$	$\frac{e^2}{6} \cong 1.232$
Gaussian	1	$\frac{\sqrt{3}\pi}{2} \cong 2.721$	$\frac{\pi e}{6} \cong 1.423$

Predictive coding gain

- High-rate distortion-rate function with DPCM

$$d_{DPCM}\left(R\right) \cong \varepsilon_E^2 \sigma_E^2 2^{-2R}$$

Variance of prediction error

- Prediction gain

$$G_{DPCM} = \frac{\varepsilon_X^2 \sigma_X^2}{\varepsilon_E^2 \sigma_E^2}$$

- Linear prediction: smallest achievable prediction error variance for N-dimensional signal determined by *spectral flatness*

$$\frac{\sigma_E^2}{\sigma_X^2} = \frac{1}{\sigma_X^2} \exp\left(\frac{1}{(2\pi)^N} \int_{\Omega} \ln\left(\Phi_{XX}\left(\Omega\right)\right) d\Omega \right)$$

Predictive coding gain (cont.)

Example

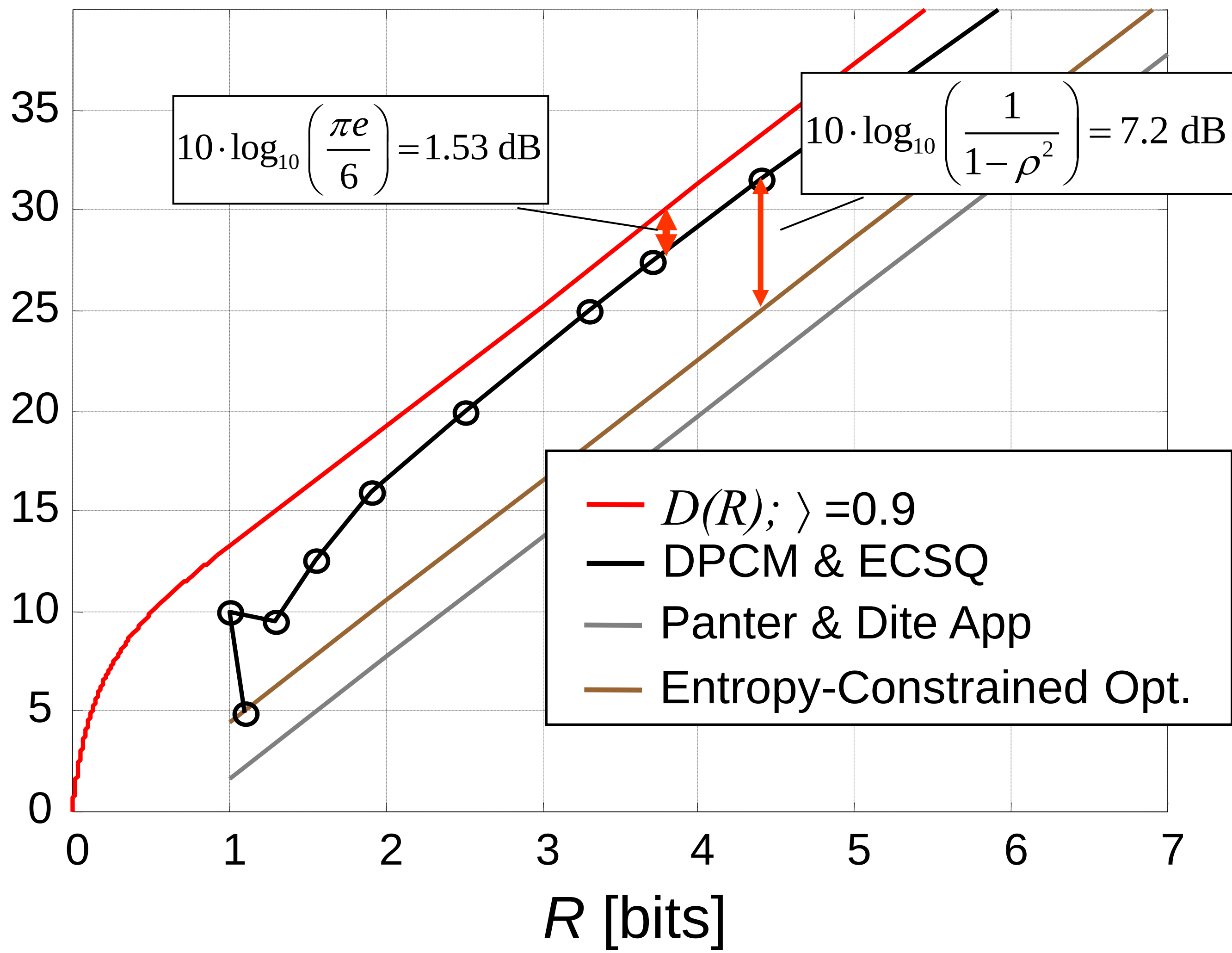
- 1-D Gaussian Markov-1 process with correlation coefficient ρ
- Autocorrelation function $E\left[X_n X_{n-k}\right] = \sigma_X^2 \rho^{|k|}$
- Prediction gain $G_{DPCM} = \dfrac{1}{1-\rho^2}$

R-D curves for Gauss-Markov-1 source

SNR [dB]

$$= 10 \cdot \log_{10} \frac{\sigma_X^2}{D}$$

- Linear predictor, order N=1, a=0.9
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied



Reading

- Wiegand, Schwarz, Chapter 6
- Taubman, Marcellin, 2.4.2, 3.3, Chapter 20 (JPEG-LS)
- S. K. Goyal, J. B. O'Neal, "Entropy Coded Differential Pulse-Code Modulation Systems for Television Systems," IEEE Trans. Communications, pp. 660-666, June 1975.
- N. Farvadin, J. W. Modestino, "Rate-distortion performance of DPCM schemes for autoregressive sources," IEEE Trans. Information Theory, vol. 31, no. 3, pp. 402-418, May 1985.