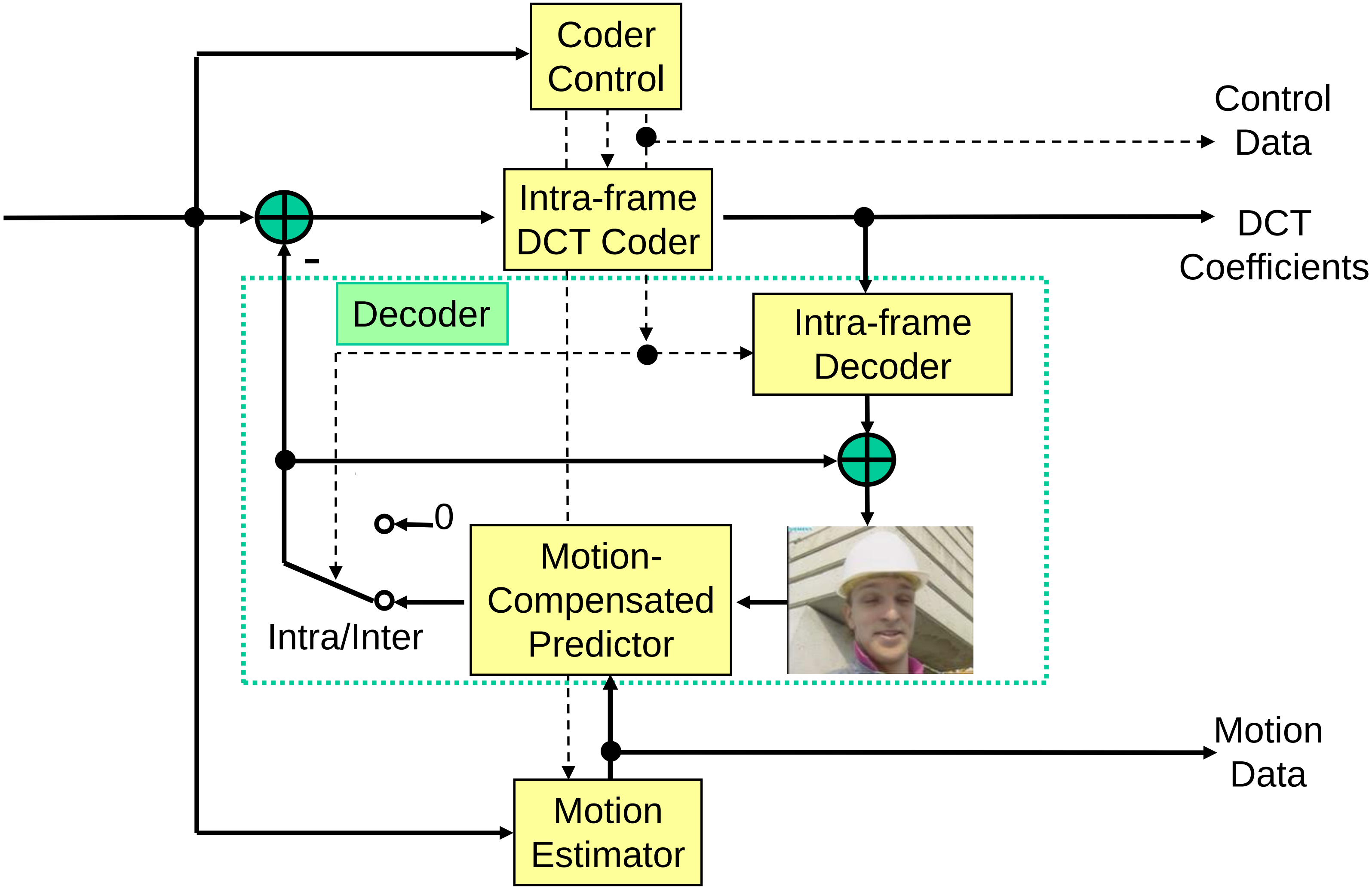
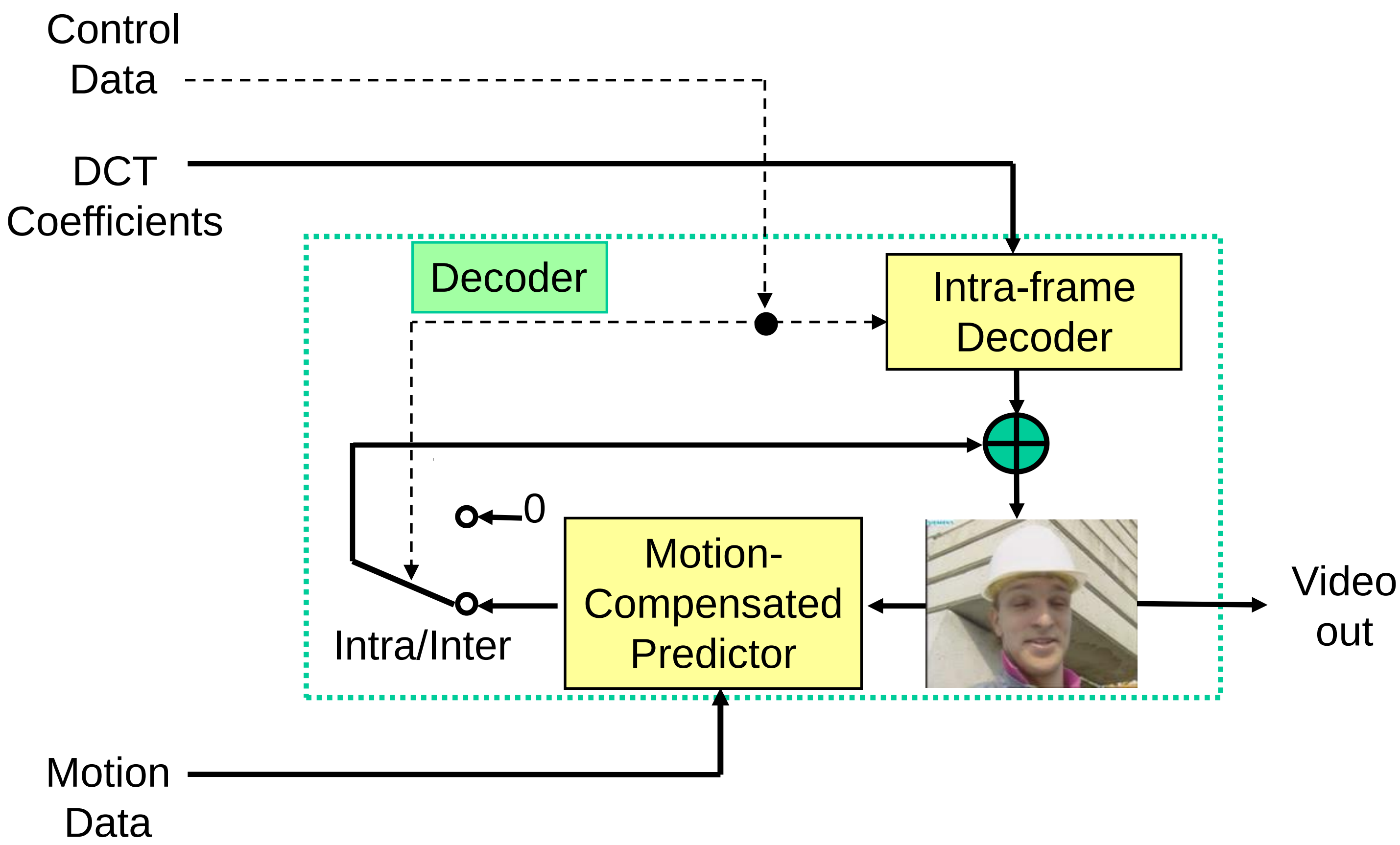
Motion-compensated hybrid coding

- Power spectral density of motion-compensated prediction error
- Rate-distortion analysis of m.c. hybrid coding
- RD performance vs. motion compensation accuracy
- Motion-compensated coding with loop filter

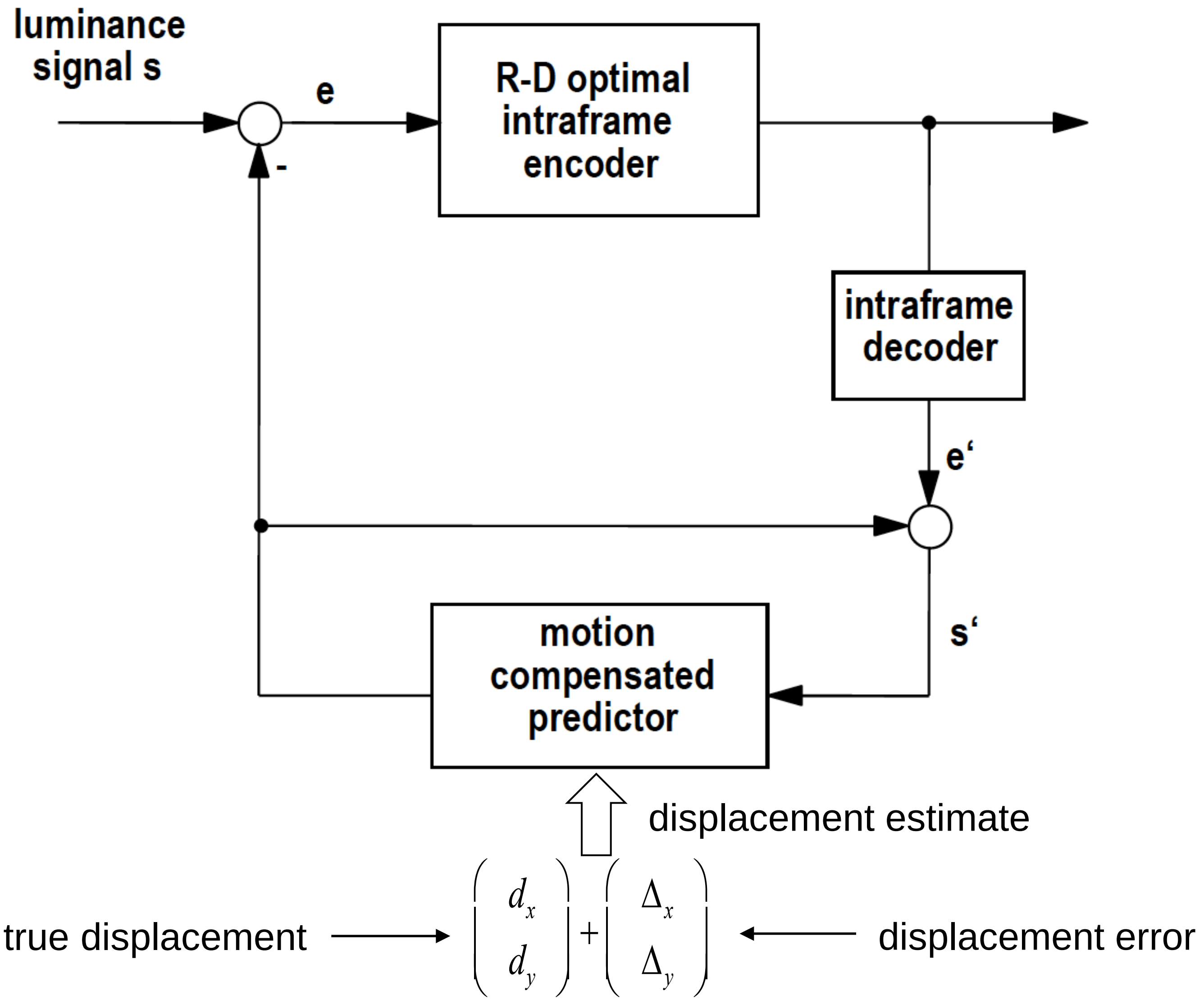
Motion-compensated hybrid coder



Motion-compensated hybrid decoder



Model for RD analysis of motion-compensated hybrid coding



$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} + \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix}$$

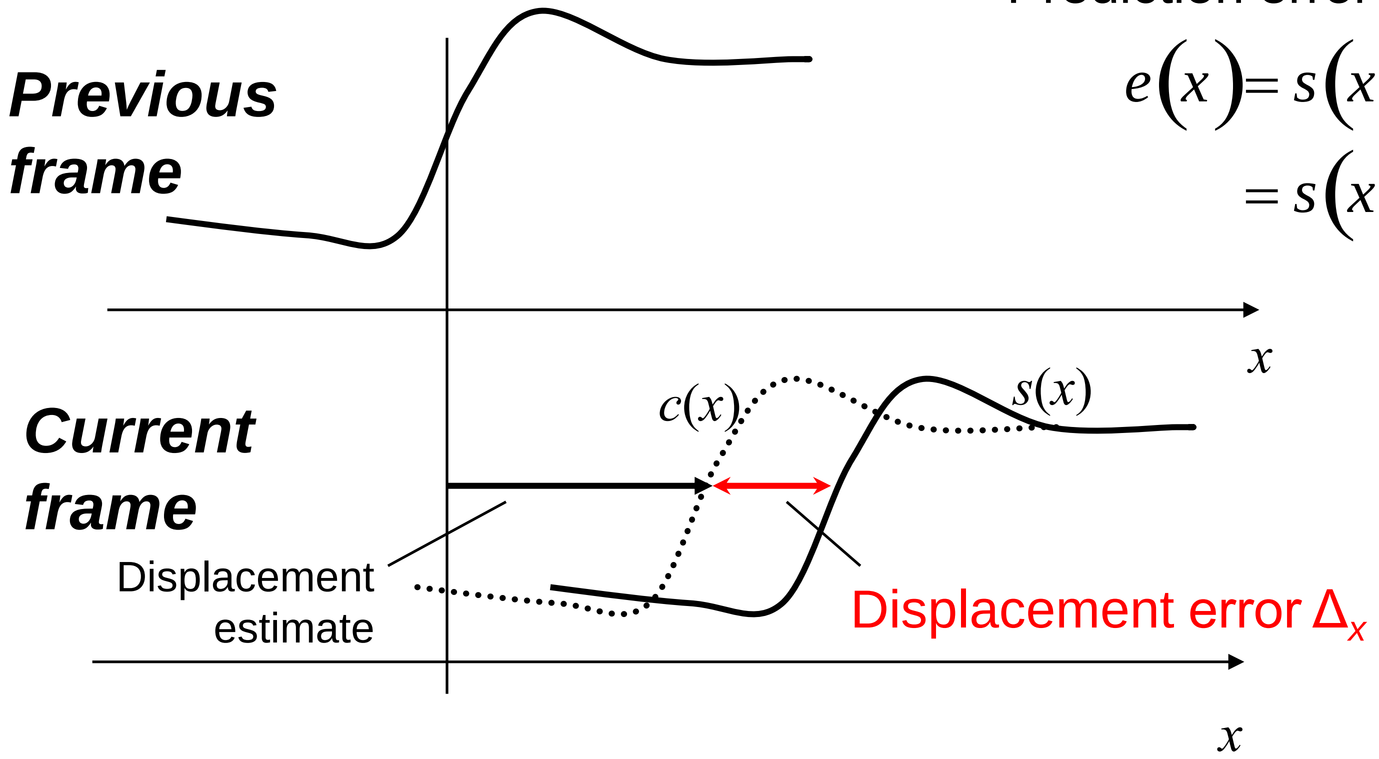
Analysis of the motion-compensated prediction error

Motion-compensated signal

$$c(x) = s(x - \Delta_x) - n(x)$$

Prediction error

$$e(x) = s(x) - c(x)$$
$$= s(x) - s(x - \Delta_x) + n(x)$$



Analysis of m.c. prediction error (cont.)

- Motion-compensated prediction error

$$e(x) = s(x) - c(x) = s(x) - s(x - \Delta_x) + n(x) = \left(\delta(x) - \delta(x - \Delta_x)\right) * s(x) + n(x)$$

- Power spectrum of prediction error, assuming constant displacement error Δ_x, statistical independence of s and n

$$\Phi_{ee}(\omega) = \Phi_{ss}(\omega)\left(1 - e^{-j\omega\Delta_x}\right)\left(1 - e^{j\omega\Delta_x}\right) + \Phi_{nn}(\omega)$$
$$= 2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{e^{-j\omega\Delta_x}\right\}\right) + \Phi_{nn}(\omega)$$

- Random displacement error Δ_x, statistically independent from s, n

$$\Phi_{ee}(\omega) = E\left\{2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{e^{-j\omega\Delta_x}\right\}\right) + \Phi_{nn}(\omega)\right\}$$
$$= 2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{E\left\{e^{-j\omega\Delta_x}\right\}\right\}\right) + \Phi_{nn}(\omega)$$
$$= 2\Phi_{ss}(\omega)\left(1 - \mathrm{Re}\left\{P(\omega)\right\}\right) + \Phi_{nn}(\omega)$$

Analysis of m.c. prediction error (cont.)

- What is $P(\omega)$?

$$P(\omega) = E\left\{e^{-j\omega\Delta_x}\right\}$$

$$= \int_{-\infty}^{\infty} p_{\Delta_x}(\Delta) e^{-j\omega\Delta} \, d\Delta = F\left\{p_{\Delta_x}(\Delta_x)\right\}$$

Fourier transform of the displacement error pdf!

- Same as characteristic function of displacement error, except for sign
- Extension to 2-d

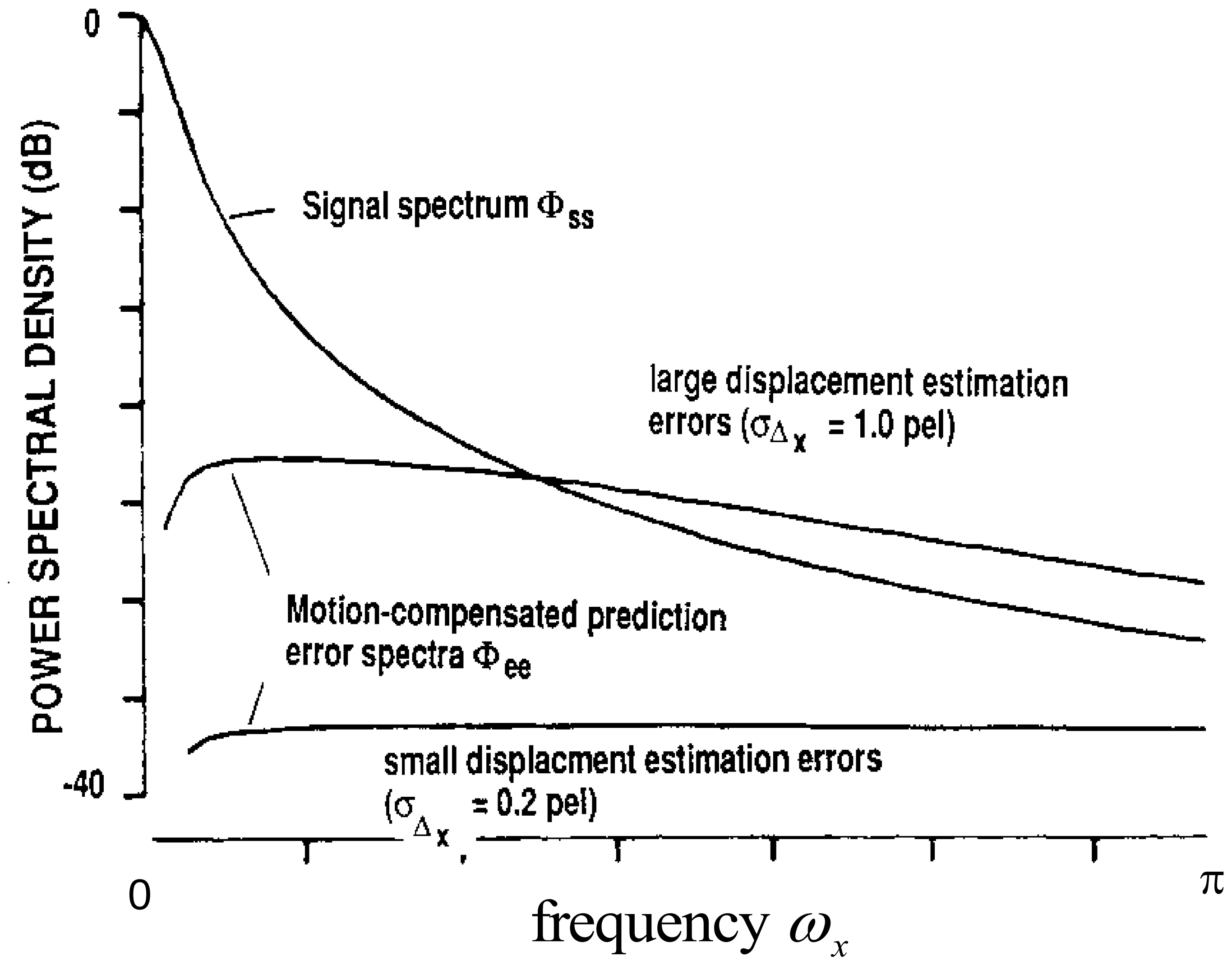
$$\Phi_{ee}(\omega_x,\omega_y) = 2\Phi_{ss}(\omega_x,\omega_y)\left(1 - \mathrm{Re}\left\{P(\omega_x,\omega_y)\right\}\right) + \Phi_{nn}(\omega_x,\omega_y)$$

power spectrum of luminance signal

Fourier transform of the displacement error pdf $p(\Delta_x,\Delta_y)$

noise spectrum

Power spectrum of motion-compensated prediction error



R-D function for MCP with integer-pixel accuracy

- $(\Delta_x, \Delta_y)^T$ assumed uniformly distributed between

$$\Delta_x = \pm\frac{1}{2} \text{ pel}$$

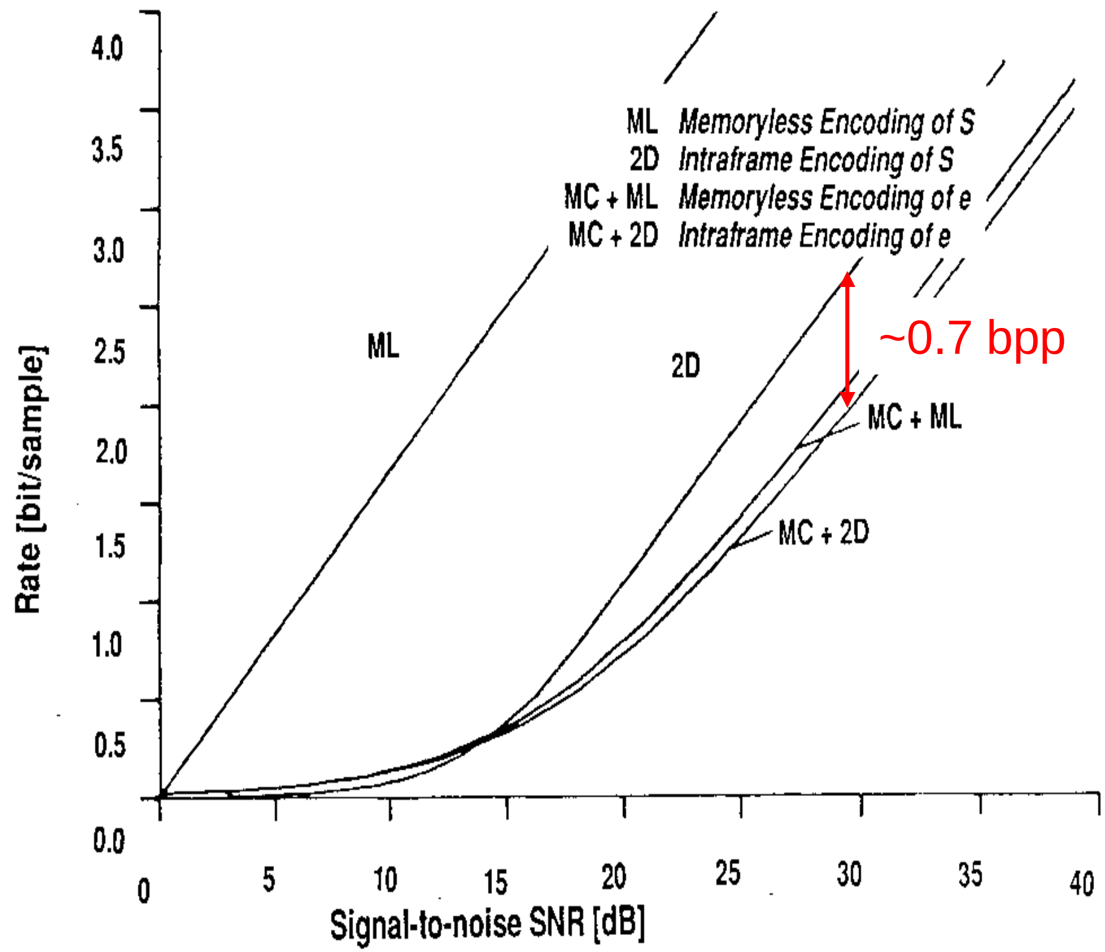
$$\Delta_y = \pm\frac{1}{2} \text{ line}$$

- Gaussian signal model

$$\Phi_{ss}(\omega_x, \omega_y) = A\left(1 + \frac{\omega_x^2 + \omega_y^2}{\omega_0^2}\right)^{-\frac{3}{2}}$$

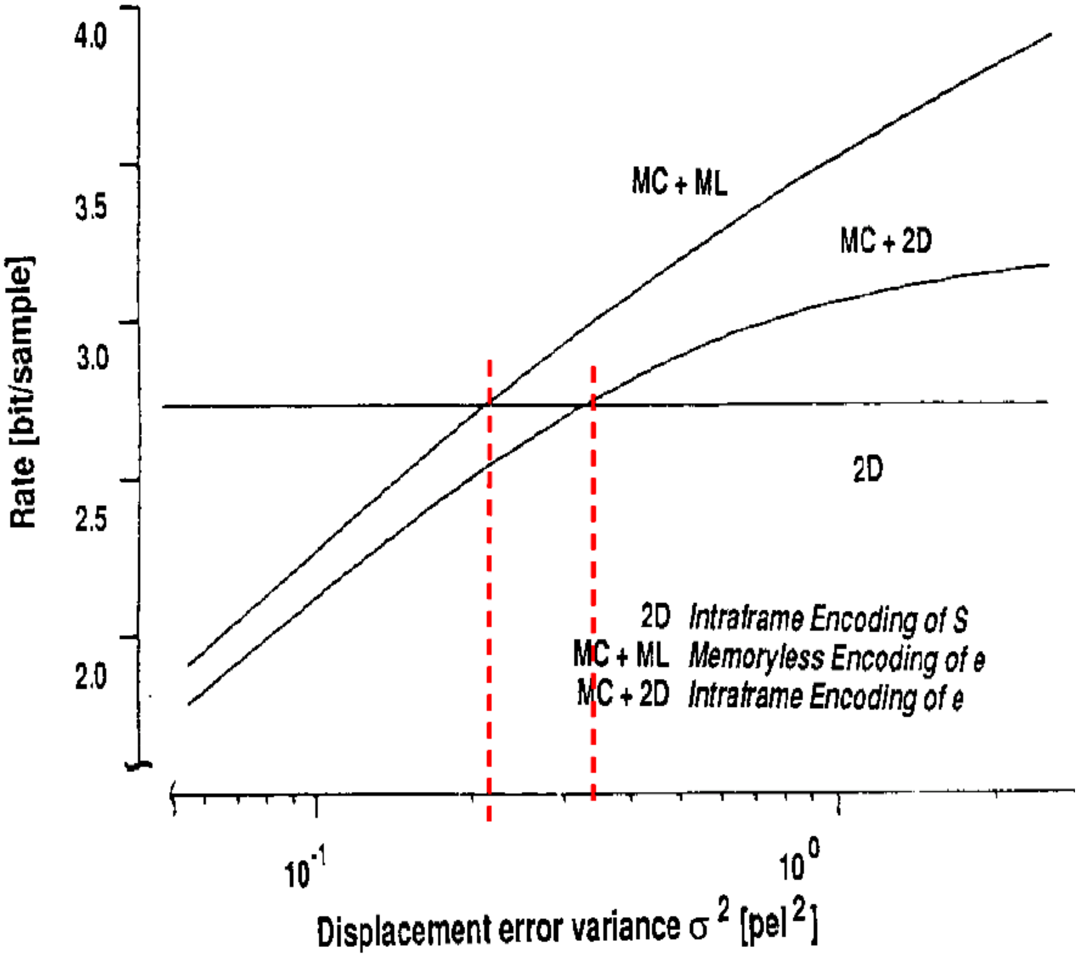
- Typical parameters for CIF resolution (352 x 288 pixels)

<u>Minimum bit-rate for given SNR</u>

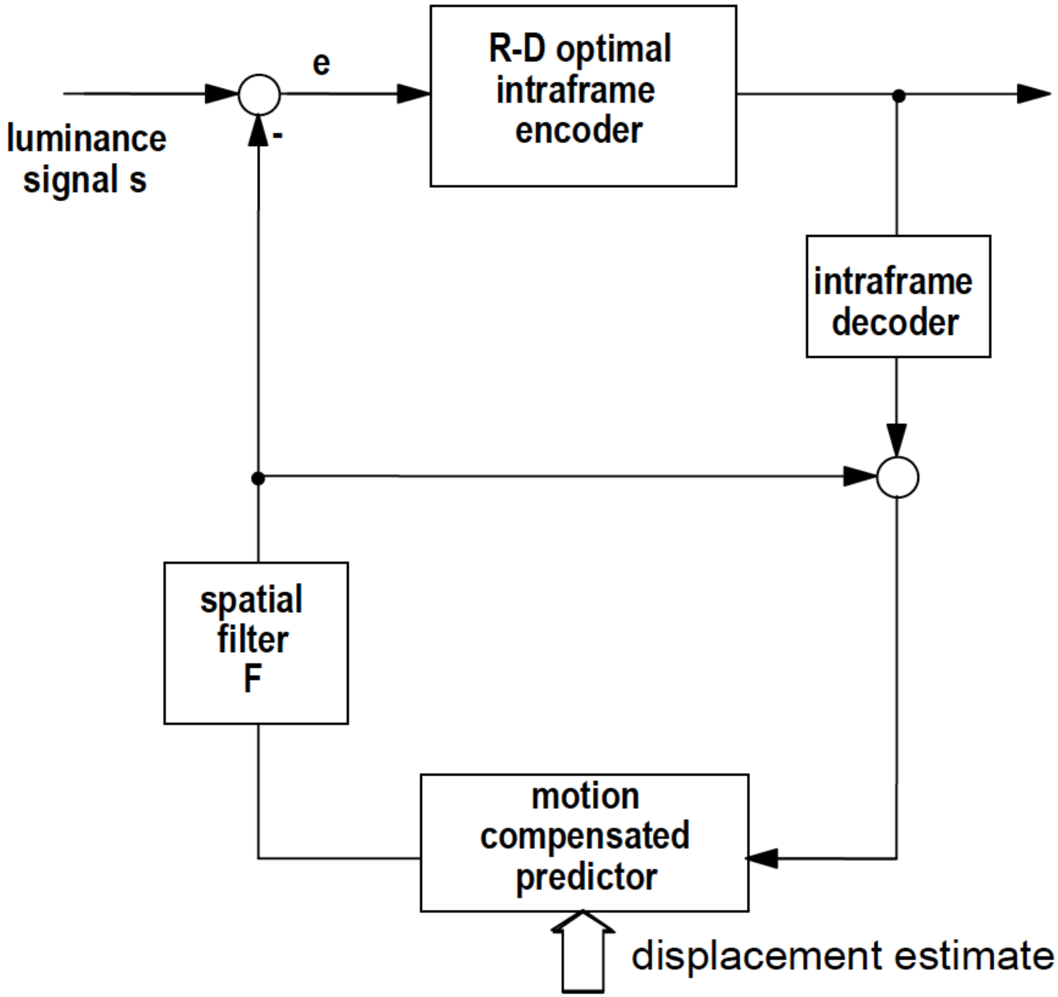


Required accuracy of motion compensation

- $p(\Delta_x, \Delta_y)$ isotropic Gaussian pdf with variance σ^2
- $$\Phi_{ss}(\omega_x, \omega_y) = A\left(1 + \frac{\omega_x^2 + \omega_y^2}{\omega_0^2}\right)^{-\frac{3}{2}}$$
- Typical parameters for CIF resolution (352 x 288 pixels)
- Minimum bit-rate for SNR = 30 dB



Model of MCP hybrid coder with loop filter



$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} + \begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix}$$

Motion-compensated prediction error with loop filter

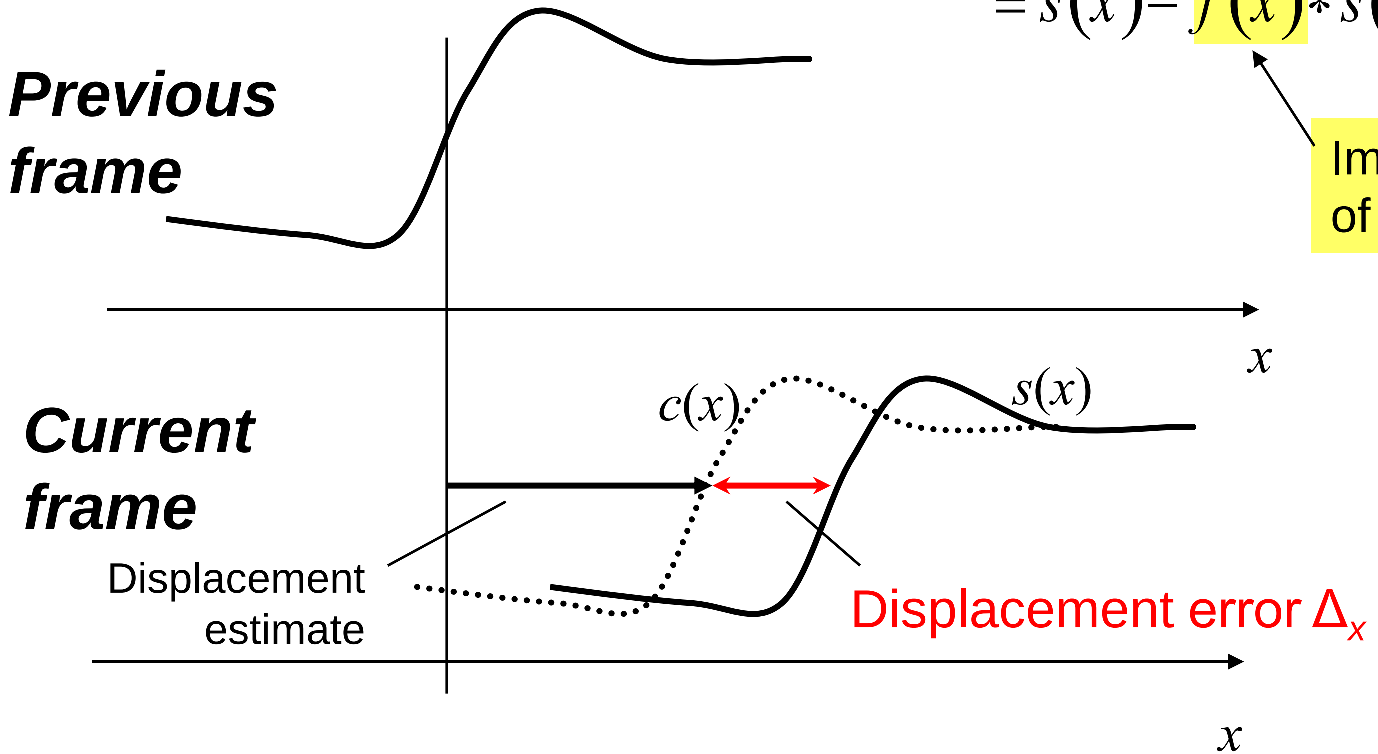
Motion-compensated signal

$$c(x) = s(x - \Delta_x) - n(x)$$

Prediction error

$$e(x) = s(x) - \boxed{f(x)} * c(x)$$
$$= s(x) - \boxed{f(x)} * s(x - \Delta_x) + \boxed{f(x)} * n(x)$$

Impulse response of loop filter



Spatial power spectrum of m.c. prediction error with loop filter

$$\boxed{\Phi_{ee}(\Lambda) = \Phi_{ss}(\Lambda)\left(1 + |F(\Lambda)|^2 - 2\,\mathrm{Re}\left\{F(\Lambda)P(\Lambda)\right\}\right) + \Phi_{nn}(\Lambda)\,|F(\Lambda)|^2}$$

$P(\Lambda)$	2-D Fourier transform of displacement error pdf
$F(\Lambda)$	2-D Fourier transform of $f(x,y)$
Φ_{uu}	spatial spectral power density of signal u
Λ	vector of spatial frequencies (ω_x, ω_y)
$n(x,y)$	noise

Optimum loop filter

- Wiener filter minimizes prediction error variance

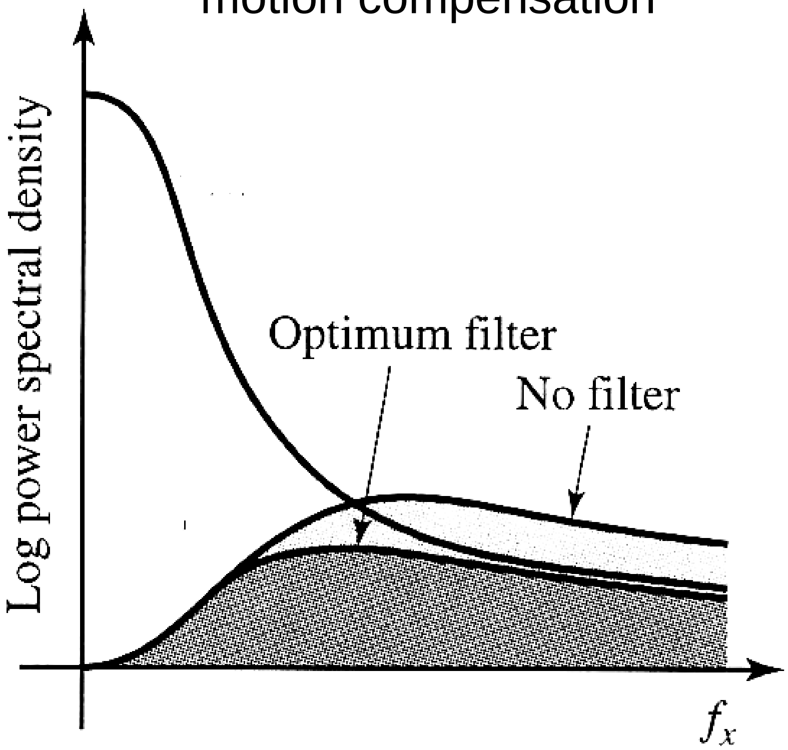
$$\boxed{F_{\text{opt}}(\Lambda) = \underbrace{P^*(\Lambda)}_{\text{accounts for accuracy of motion compensation}} \cdot \frac{\Phi_{ss}(\Lambda)}{\underbrace{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}_{\text{accounts for noise}}}}$$

- Resulting minimum prediction error spectrum

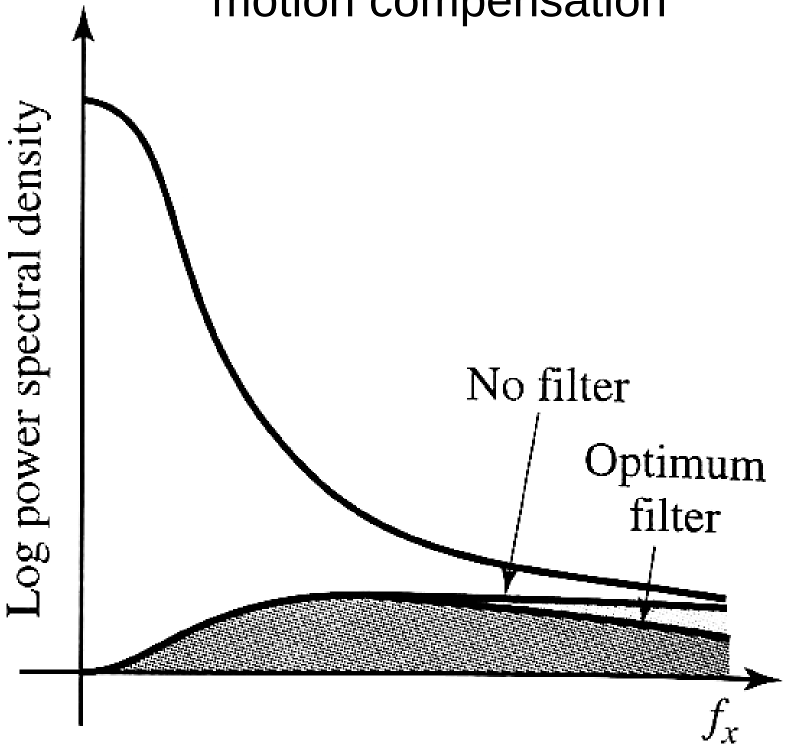
$$\boxed{\Phi_{ee}(\Lambda) = \Phi_{ss}(\Lambda)\left(1 - |P(\Lambda)|^2 \frac{\Phi_{ss}(\Lambda)}{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}\right)}$$

Effect of loop filter

Moderately accurate
motion compensation

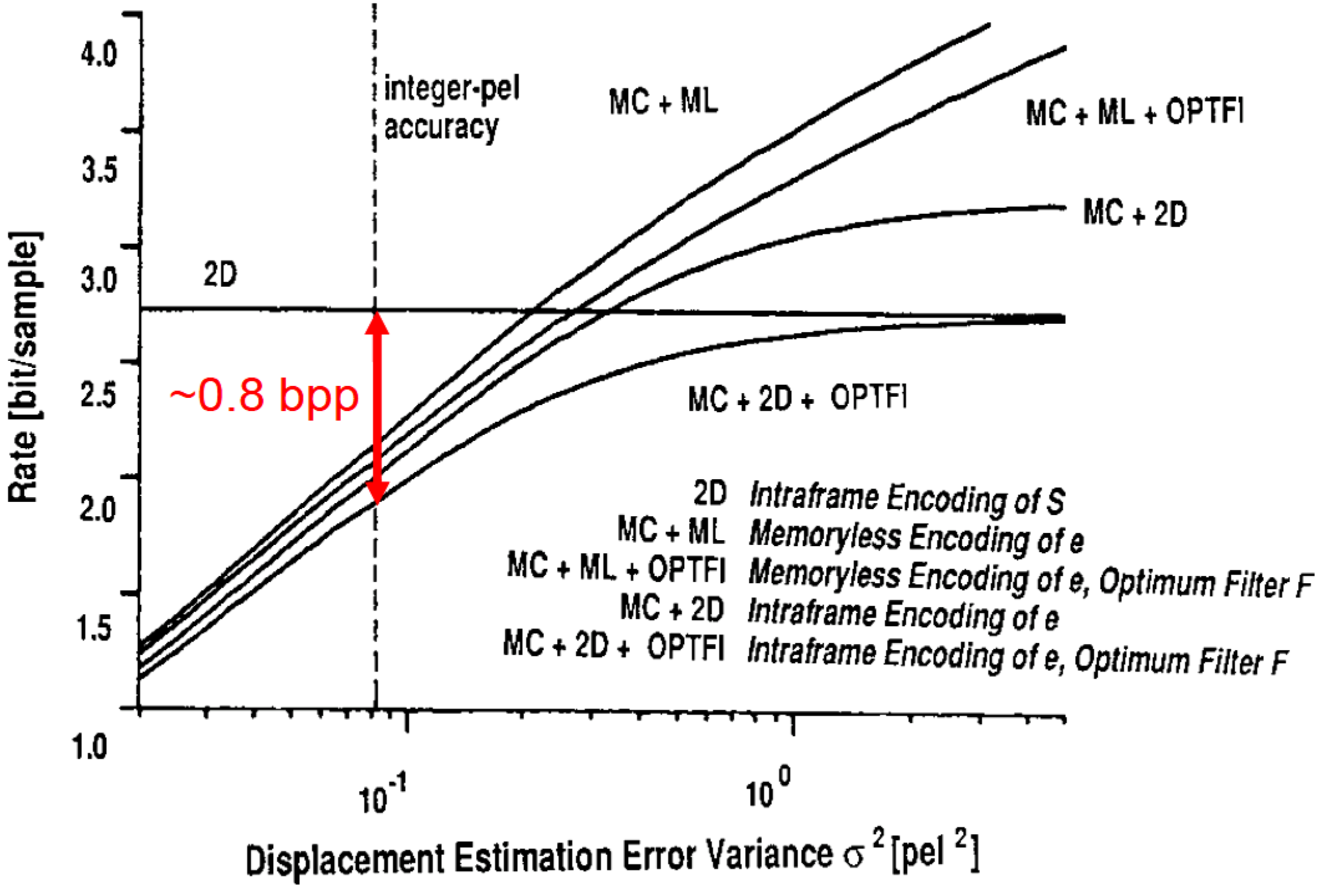


Very accurate
motion compensation



Required accuracy of motion compensation with loop filter

- $p(\Delta_x, \Delta_y)$ isotropic Gaussian pdf with variance σ^2
- Minimum bit-rate for SNR = 30 dB



Practical optimum loop filter design

- Not practical for loop filter design

$$F_{\text{opt}}(\Lambda) = \underbrace{P^*(\Lambda)}_{\text{Motion compensation accuracy not known}} \cdot \frac{\Phi_{ss}(\Lambda)}{\underbrace{\Phi_{ss}(\Lambda) + \Phi_{nn}(\Lambda)}_{\text{"Noise" psd not known}}}$$

- To determine Wiener filter from measurements:

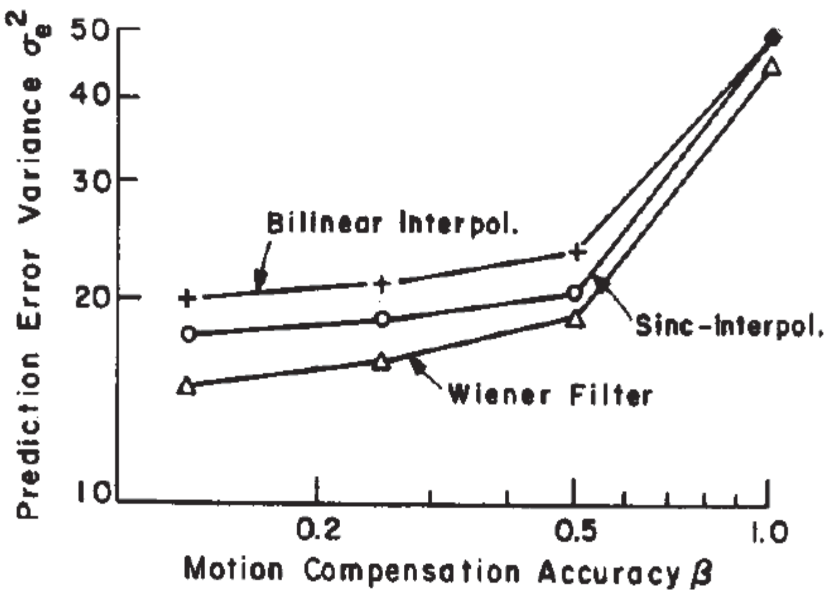
$$\boxed{F_{\text{opt}}(\Lambda) = \frac{\Phi_{sc}(\Lambda)}{\Phi_{cc}(\Lambda)}}$$

cross spectrum between $s(x,y)$ and the motion-compensated signal $c(x,y) = r(x-\hat{d}_x, y-\hat{d}_y)$

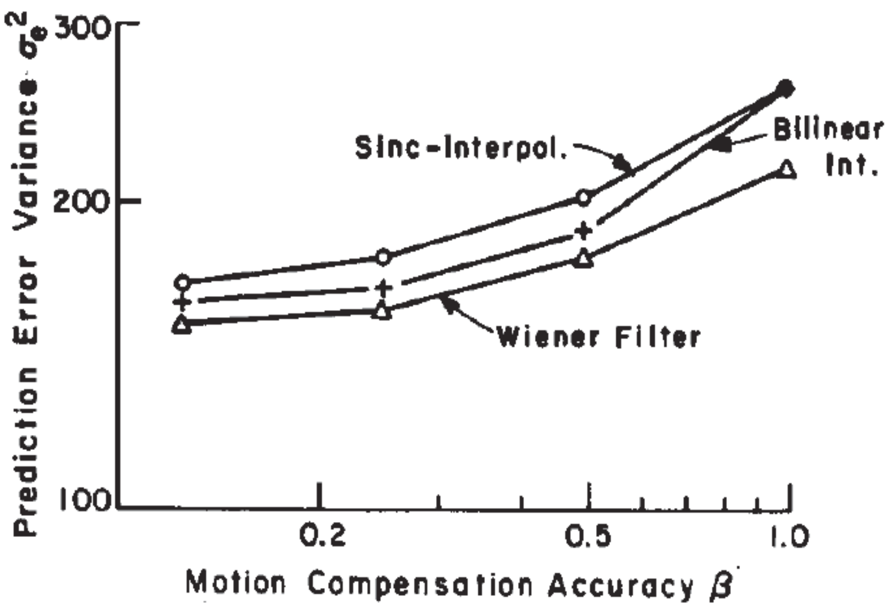
Experimental evaluation of fractional-pixel motion compensation

- ITU-R 601 TV signals, 13.5 MHz sampling rate, interlaced, blockwise motion compensation with blocksize16x16

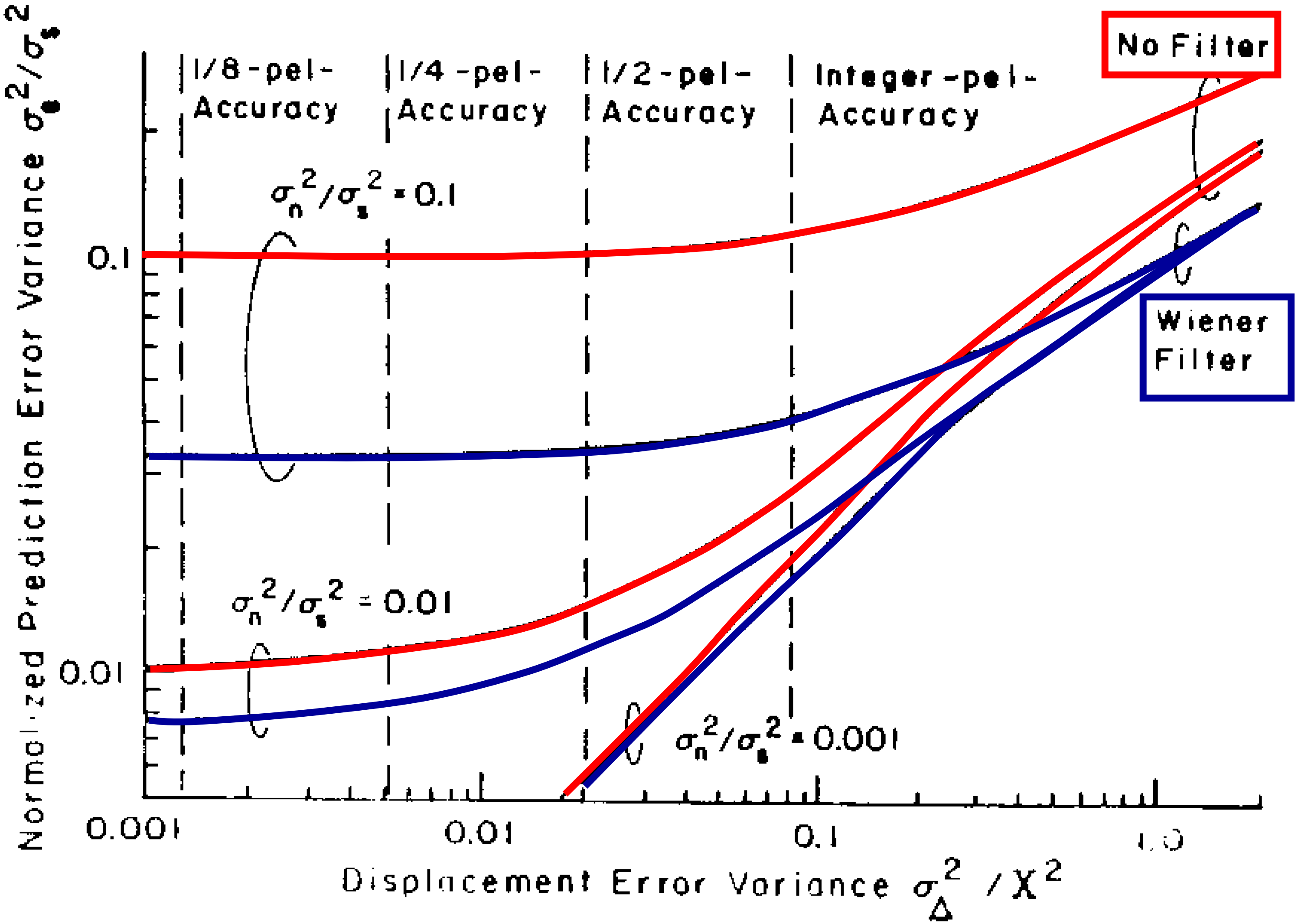
Zoom



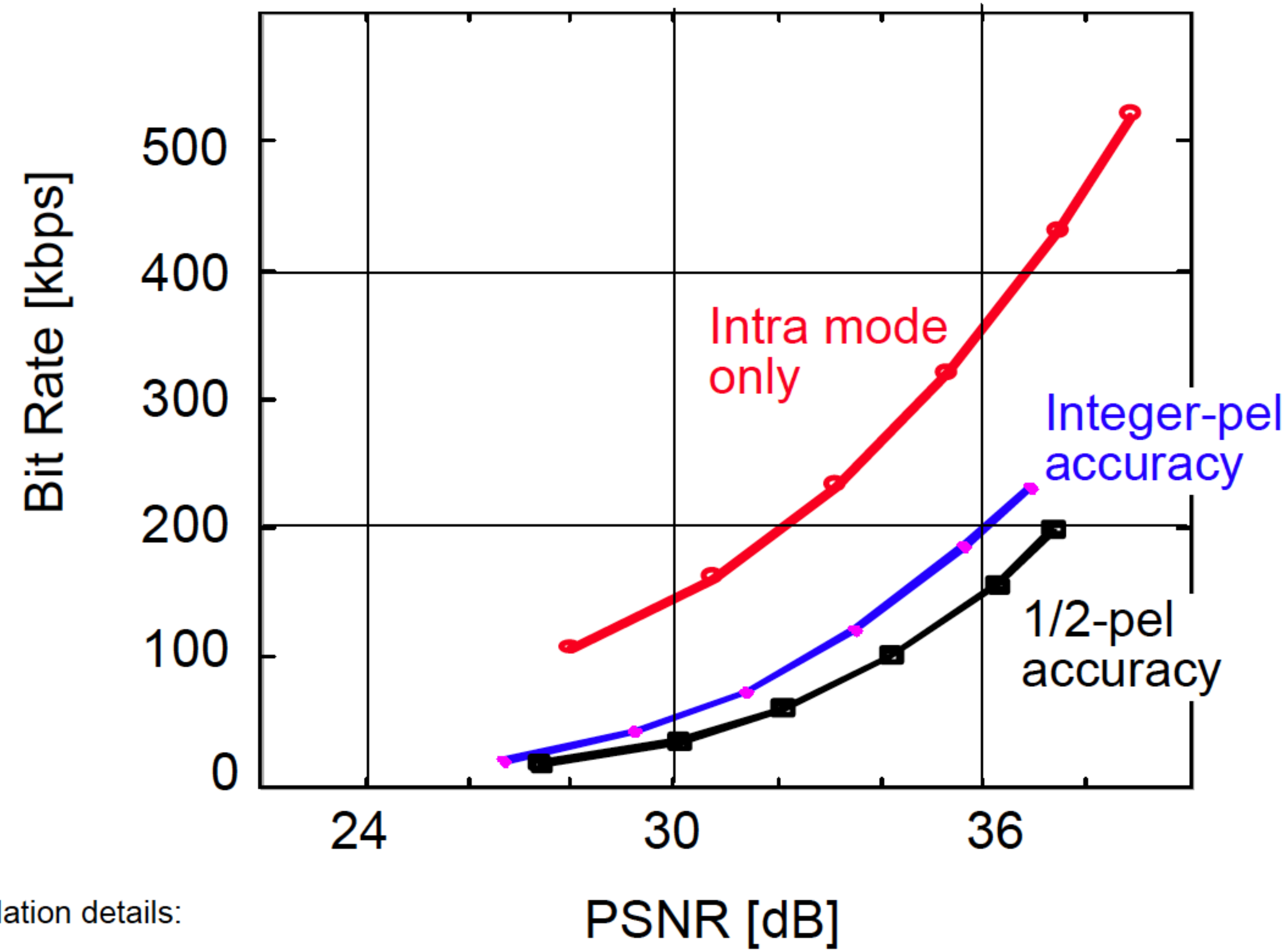
Voiture



Influence of noise on the performance of MCP



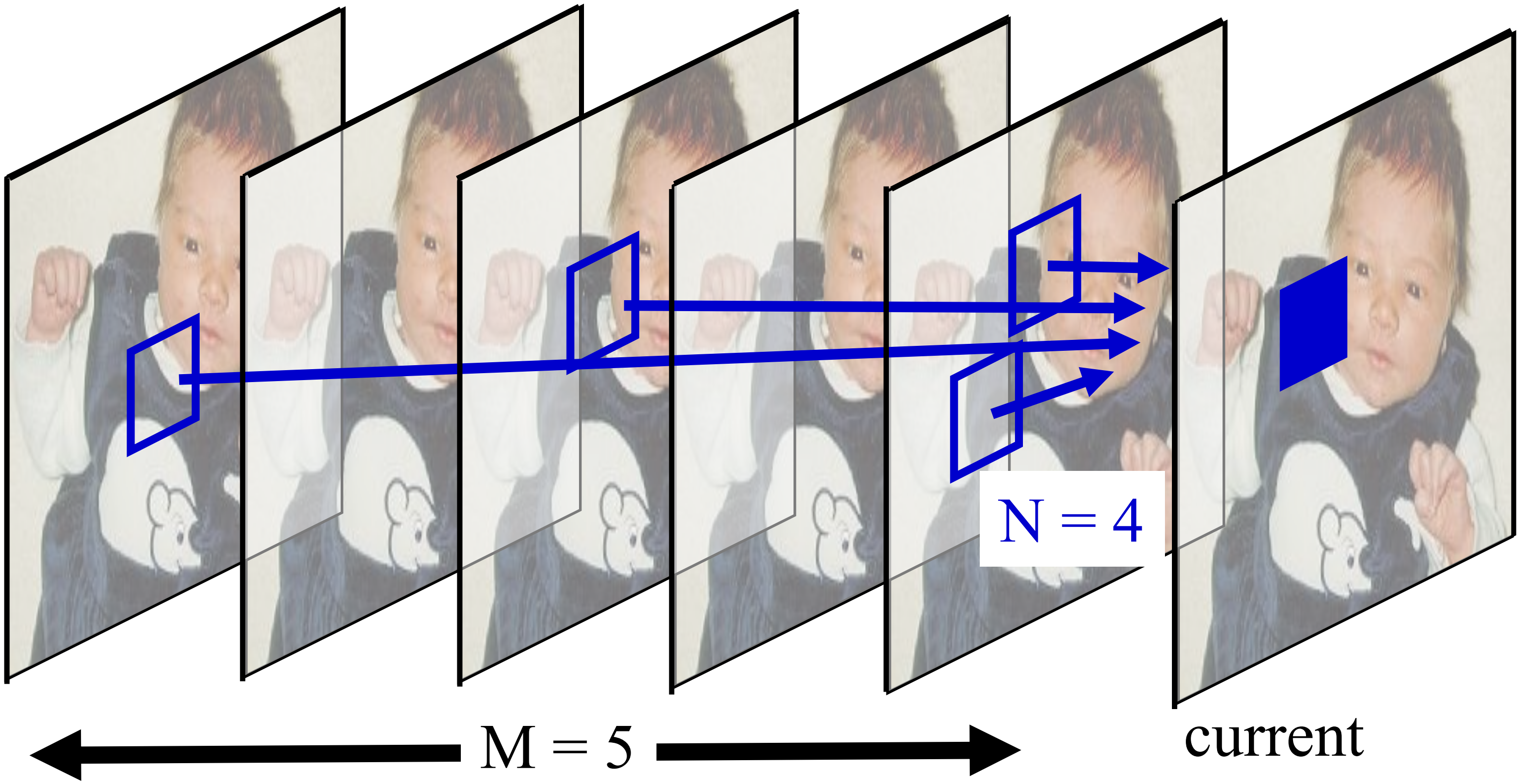
Motion Compensation Performance in H.263



Simulation details:

Foreman, QCIF, SKIP=2
Q=4,5,7,10,15,25

Multihypothesis motion-compensated prediction



Superposition of N hypotheses from M past frames

Multihypothesis motion-compensated prediction

Prediction error power spectrum for multi-input loop filter $F(\omega_x, \omega_y)$

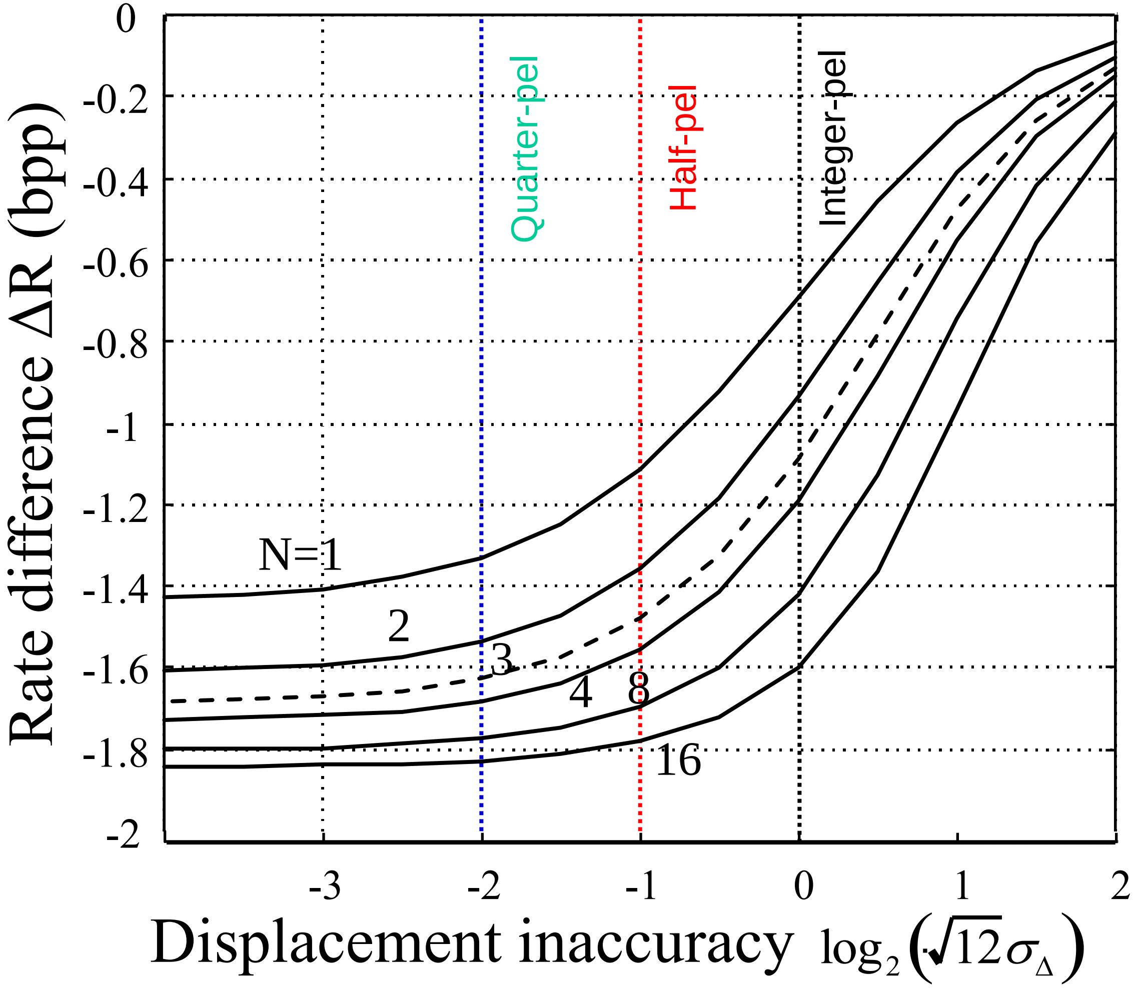
$$\frac{\Phi_{ee}}{\Phi_{ss}} = 1 + (1+\alpha_0)^{-1}\left(-2\Re\left\{F\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{pmatrix}\right\} + F\begin{pmatrix} 1+\alpha_1 & P_1 P_2^* & P_1 P_3^* & \cdots & P_1 P_N^* \\ P_2 P_1^* & 1+\alpha_2 & P_2 P_3^* & \cdots & P_2 P_N^* \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_N P_1^* & P_N P_2^* & P_N P_3^* & \cdots & 1+\alpha_N \end{pmatrix} F^H\right)$$

2D Fourier transform
of displacement error pdf's

spectral SNR

$$\alpha_i = \frac{\Phi_{nni}}{\Phi_{vv}}$$

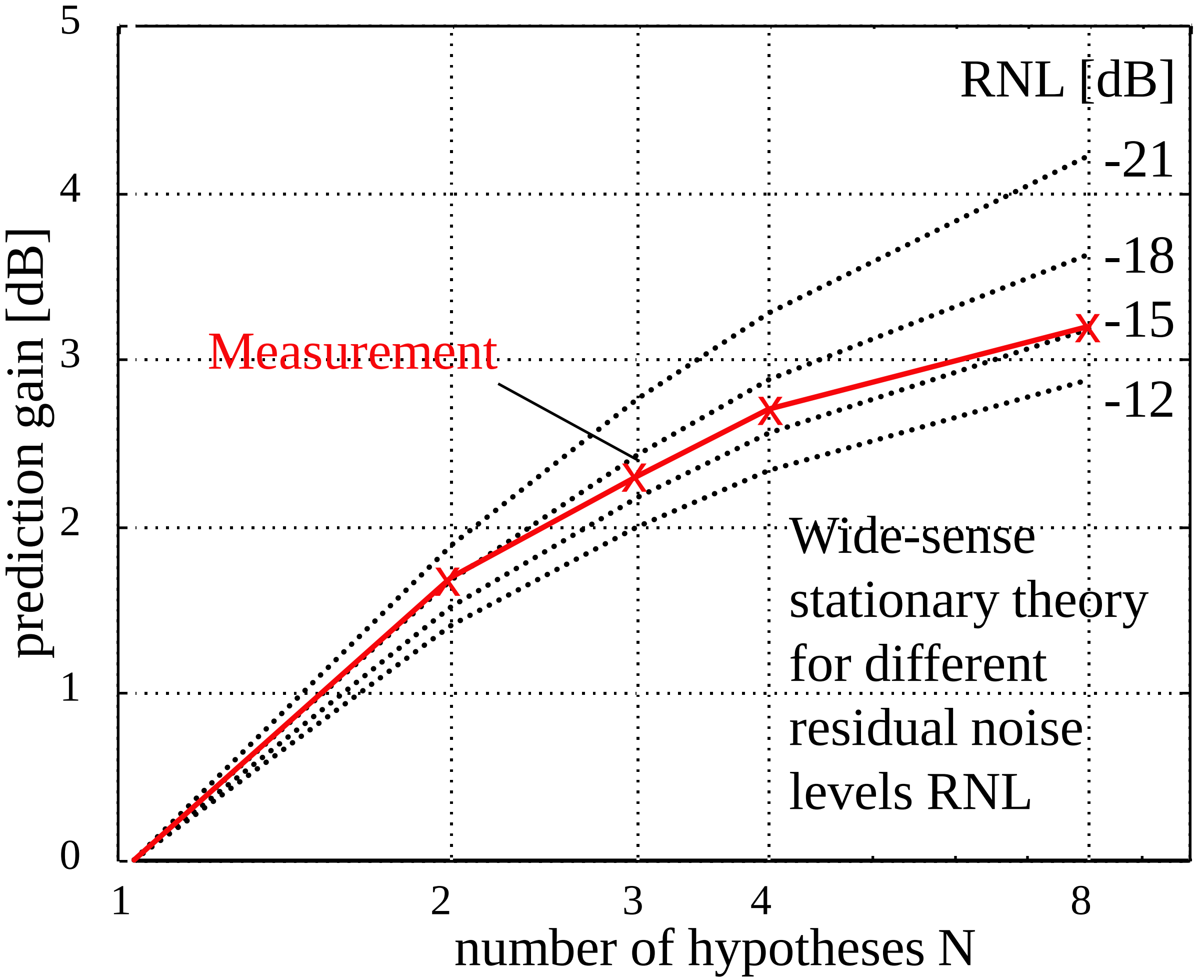
Theoretical rate reduction vs. intraframe coding for multihypothesis mc prediction



Wiener filter, noise -24 dB:

- Diminishing returns beyond critical accuracy
- Accurate motion compensation less important with larger N
- Diminishing returns with increasing N

Prediction gain by multihypothesis motion-compensated prediction



<u>Measurement</u>
Foreman, 16x16 blocks,
integer-pel accuracy
M = 10 previous frames

Reading

- B. Girod, “Motion-compensating prediction with fractional-pel accuracy,” IEEE Trans. Communications, vol. 41, no. 4, pp. 604-612, April 1993.
- B. Girod, “Efficiency Analysis of Multi-Hypothesis Motion-Compensated Prediction for Video Coding,” IEEE Trans. Image Processing, vol. 9, no. 2, pp. 173-183, February 2000.