LAST TIME

Given \( B_n(z) \), finding \( B(z) \)

TODAY

Designing \( B_n(z) \)

Equal Ripple
Least Squares

Alternate Phase Profiles
Linear Phase
Minimum Phase
Non-Linear Phase
**Basic Problem**

RF Pulse Design with SIR consists of:

1) Designing $B_n(z)$
2) Choosing a compatible, minimum power $P_n(z)$.
3) Performing the back recursion

All determined once $B_n(z)$ has been designed.

**How Do We Design $B_n(z)$?**

**Several Issues:**

1) Goal is specified in terms of $w_{xy}$, or $w_{xy}$.

$B_n(z)$ is $w_{xy}(x) / z$.

The goal is a non-linear function of the input $B_n(z)$.

We need to figure out what to ask for to get what we want.
2) The parameters we care about are:

- Passband Error ($\delta_1$)
- Stopband Error ($\delta_2$)
- Slice Width in Frequency ($B_\text{w}$)
- Pulse Length ($T$)

Given these, we would like a minimum transition width ($F_s - F_p$) = $\Delta F$

What filter design programs want is:

- Passband Edge ($F_p$)
- Stopband Edge ($F_s$)
- Passband to Stopband Error Ratio ($\delta_1 / \delta_2$)

We need to redefine what we want to what we need to specify.
Basic IDEA

Equil-Ripple Filters (Parics - M. Cleeman)
Are Determined by Band Edges and Ripple Amplitudes

All we have to do is figure out what the effective ripple produced in the slice profile or interest is.

$\delta_1$ - Passband Maximum Ripple

$\delta_2$ - Stopband Maximum Ripple

This will depend on the profile. Once we have these relations, we can invert them to determine what $(\delta_1, \delta_2)$ to specify.
Example: Inversion Pulses

Inversion Pulse

\[ M_z^+(x) = (1 - z |B(x)|^2) M_0 \]
\[ = (1 - z |B(x)|^2) M_0 \left| \text{if } x \in \text{substrate} \right. \]

This case we can actually solve for explicitly by differentiating \( M_z^+(x) \), and factoring it. We will return to this.

Out-of-Slice Ripple

An input ripple of \( \delta_z \) produces

\[ \delta_z^e = 2 \delta_z \]

So

\[ \delta_z = \sqrt{\delta_z^e / 2} \]

In-Slice Ripple

Ripple is scaled to be less than 1.

\[ \text{Diagram showing ripple amplitude} \]

(4)
Maximum ripple occurs in $m_2$ when $m_0(z)$ is minimum

$$m_2 = (1 - 2(1-2\delta_1)^2) m_0$$

$$= (1 - 2(1 - 4\delta_1 + 4\delta_1^2)) m_0$$

$$= (1 + 8\delta_1) m_0$$

So

$$\delta_1^e = 8 \delta_1$$

$$\delta_1 = \frac{1}{8} \delta_1^e$$

For example, if we want an inversion profile with $\delta_1^e = 0.01$ and $\delta_2^e = 0.01$, we need to design $m_0(z)$ with

$$\delta_2 = \sqrt{\delta_1^e / 2} = \sqrt{0.01 / 2} = 0.07$$

much lower!

$$\delta_1 = \frac{1}{8} \delta_1^e = \frac{0.01}{8} = 0.0013$$

much smaller!

More importantly, the ratio

$$\frac{\delta_2}{\delta_1} = 53$$

far from the unity ratio $0.0 = m_2$. 
Similar relations can be derived for other types of pulses.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small TIP</td>
<td>$\delta_1^e$</td>
<td>$\delta_2^e$</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>$\sqrt{\delta_1^e}$</td>
<td>$\delta_2^e/\sqrt{2}$</td>
</tr>
<tr>
<td>Inversion</td>
<td>$\delta_1^e/2$</td>
<td>$\sqrt{\delta_2^e/2}$</td>
</tr>
<tr>
<td>Spin Echo</td>
<td>$\delta_1^e/4$</td>
<td>$\sqrt{\delta_2^e}$</td>
</tr>
<tr>
<td>Saturation</td>
<td>$\delta_1^e/2$</td>
<td>$\sqrt{\delta_2^e}$</td>
</tr>
</tbody>
</table>

WE KNOW RIPPLE AMPLITUDES $S_1, S_2$

How do we find passband edges?

FROM DIGITAL FILTER DESIGN

\[
T(\Omega F) = \frac{D_0(L, S_1, S_2)}{\text{degree function in Hz}} \quad \text{constant, for } S_1, S_2
\]

OR

\[
T(\beta_0) \left( \frac{\Omega F}{\beta_0} \right) = D_0(L, S_1, S_2)
\]

Do has been determined empirically for EQUI-RIPPLE FILTERS

Intuitively we expect

\[
\Omega F \sim \frac{1}{T}
\]

for a sinc, and

\[
\Delta F \sim \frac{2}{T}
\]

for a windowed sinc
So $D_0$ should be on the order of 1 to 2. This is a good estimate!

Better filter designs have lower $D_0$ limit to how much can be gained.
\[ D_\infty(\delta_1, \delta_2) = (a_1 L_1^2 + a_2 L_1 + a_3) L_2 + (a_4 L_1^2 + a_5 L_1 + a_6) \]

Where

\[ L_1 = \log_{10} \delta_1 \quad \text{and} \quad L_2 = \log_{10} \delta_2 \]

and

\[ a_1 = 5.309 \times 10^{-3} \]
\[ a_2 = 7.114 \times 10^{-2} \]
\[ a_3 = -4.761 \times 10^{-1} \]
\[ a_4 = -2.66 \times 10^{-3} \]
\[ a_5 = -5.941 \times 10^{-1} \]
\[ a_6 = -4.278 \times 10^{-1} \]
**Design Example**

**Exponential Pulse**

\[ T = 4 \text{ ms} \]
\[ BW = 2 \text{ kHz} \]
\[ T(\beta W) = 8 \]
\[ \delta_1 = 0.01 \]
\[ \delta_2 = 0.01 \]

From previous example
\[ \delta_1 = \frac{\delta^e}{8} = 0.00125 \]
\[ \delta_2 = \sqrt{\frac{\delta^e}{12}} = 0.0707 \]

Then
\[ D_{\infty} (0.00125, 0.0707) = 2 \]

The transition width is then

\[ \Delta f = \frac{D_{\infty}(\delta_1, \delta_2)}{T} = \frac{2}{4 \text{ ms}} = 500 \text{ Hz} \]

\[ 1 - 0.25 = 0.75 \text{ kHz} \]
\[ 1 + 0.25 = 1.25 \text{ kHz} \]
In MATLAB, design this filter with

\[ \text{firpm} \]

(\text{was \ rein.m})

**Inputs**

\[ f = \left[ 0 \ 750 \ 1250 \ 3200 \right] / 32000; \]
\[ m = \left[ 1 \ 1 \ 0 \ 0 \ 0 \right] \]
\[ \omega = \left[ 1 \ 8 \pi / 2 \right] = \left[ 1 \ 0.0177 \right] \]

Then

\[ b = \text{firpm}(255, f, m, \omega) \]

This is BETA. Scale to \( \sin(\theta/2) \), then apply inverse SLP.
PM Inversion

Beta

RF

Inversion Profile

Inversion Profile

Time, ms

Time, ms

Frequency, kHz

Frequency, kHz
Dynamics of Run Designs

1) Large spikes common at first/cast samples (convolutions)
2) Spikes get larger as N increases
3) Integrated absolute value or stopband (10-90%) for example can be large.

Another alternative is weighted least squares

\[ b = \text{finds}(n, t, m, w) \]

Same inputs as Remie.

DoG (S_1, S_2) will be different for finds, but not known.

Fortunately the DoG (S_1, S_2) for pure filters is reasonably close.

Finds designs are recommended unless there are other important factors.
PM vs LS Filter Design

Time-Bandwidth = 8, 1% pass/stopband ripples
Same band edges
Weighted Least Squared Inversion

Beta

Time, ms

0.04
0.03
0.02
0.01
0
-0.01
0
1
2
3
4

RF

Time, ms

0.2
0.15
0.1
0.05
0
-0.05
0
1
2
3
4

Inversion Profile

Frequency, kHz

1
0.5
0
-0.5
-1
-4
-2
0
2
4

Inversion Profile

Frequency, kHz

-0.95
-0.96
-0.97
-0.98
-0.99
-1
-1
-0.5
0
0.5
1
Comparison Between PM and LS Inversions

![Graphs showing comparison between PM and LS Inversions.](image)
Comparison Between PM and LS Inversions
**Minimum/Maximum Phase Pulses**

**Excitation as late as possible**

![Waveform diagram](image)

**Benefits**
- Sharper profile
- Less refocusing
- Shorter echo time

**Uses**
- Slab select pulse
- Saturation pulses
- Short echo time excitations
- Inversion pulses
Minimum Phase CAUSAL Signal/Filter

Causal Minimum Phase Signal
Signal concentrated at beginning
Passband zeroes inside unit circle

Minimum Phase RF Pulse
Signal concentrated at end (origin)
Passband zeroes outside unit circle

Causal design much more familiar
Design causal filters, reverse for RF pulse.
Any signal has a minimum phase signal with same magnitude.

These both have same magnitude profile! No gain in selectivity.

Only really want single zeros inside unit circle.

Basic idea: Design a special linear phase pulse factor into minimum phase component.
Linear Phase Filter is a convolution of a minimum and a maximum phase filter.

\[ \text{Linear Phase} \quad = \quad \text{Min Phase} \quad + \quad \text{Max Phase} \]

We want to design linear phase filter to be easy to factor.

Equal-ripple ( Parks-McClellan) filter
START WITH A PM FILTER

ADD A BIAS OF \( \theta_{2,0} \)

THIS MAGNITUDE PROFILCE IS THE SAME AS

PERFECT SQUARE!
TAKE SQUARE ROOT OF PROFILE

\[ \sqrt{V_1 + \delta_1, e} \]
\[ \sqrt{V_2 e} \]

USE HILBERT TRANSFORM RELATIONSHIP TO FIND PHASE

\[ \sqrt{V_1 + \delta_1, e} \pm \delta_1, e \]
\[ \pm \sqrt{V_2 e} \]

EQUAL RIPPLE, MINIMUM PHASE PULSE

SHARPEST TRANSITION

HOW DO WE DESIGN THE ORIGINAL LINEAR PHASE FILTER TO GIVE A SPECIFIED MINIMUM PHASE PROFILE?
**Passband Ripple**

\[
\sqrt{1 + \delta_{z,1}^2} \approx \frac{\delta_{z,1}}{\delta_{z,1}} = 1 + \delta_{z,1} + \frac{1}{2}\delta_{z,1}^2
\]

\[
\delta_{z,1m} = \frac{1}{2} \delta_{z,1}
\]

\[
\delta_{z,1} = 2 \delta_{z,1m}
\]

**Stopband Ripple**

\[
\delta_{z,1m} = \sqrt{2} \delta_{z,1}
\]

\[
\delta_{z,1} = \delta_{z,1m} \sqrt{2}
\]

**Design Relation for Linear Phase Filter**

\[
(2T)(\Delta F) = D_{oo}(\delta_{z,1}, \delta_{z,1})
\]

\[
(2T)(\Delta F) = D_{oo}(2\delta_{z,1m}, \delta_{z,1m} \sqrt{2})
\]

**Where**

- \( T \) - Length of minimum phase filter

**Then**

\[
T \Delta F = \frac{1}{2} D_{oo}(2\delta_{z,1m}, \delta_{z,1m} \sqrt{2})
\]

\[
= D_{oo,1m}(\delta_{z,1}, \delta_{z,1})
\]
WHERE

\[ D_{0,1,m}(s_1,s_2) = \frac{1}{2} D_{0}(2s_1,2s_2^2/2) \]

RECALL

\[ \Delta f = \frac{D_{0,1,m}(s_1,s_2)}{T} \]

SO FOR A GIVEN T, A MINIMUM PHASE FILTER CAN HAVE HALF THE TRANSITION WIDTH OF LINEAR PHASE FILTER.

IN PRACTICE, THIS IS LESS.

TYPICAL NUMBERS ARE 70 - 90% INCREASES WITH T (13C0)
\[ D_\infty(\delta_1, \delta_2) \text{ vs } D_{\infty,m}(\delta_1, \delta_2) \]

\[ D_{\infty,m}(\delta_1, \delta_2) = \frac{1}{2} D_\infty \left( 2\delta_1, \frac{\delta_2}{2} \right) \]
Typical Tradeoffs

For any $\delta_2$, we can improve $\delta_1$ from 0.01 to 0.001

Factor of 10 in Passband Ripple

Similarly, fix $\delta_1$ and improve stopband ripple by factor of 10

Fix $\delta_1$ and $\delta_2$ and reduce transition width $1 = \delta_2 = 0.001$, $\delta_1 = 0.001$, $D_1 = 60\mu s$, from 2.6 to 2. W reduces to 75\%.
Linear vs Minimum Phase Inversion Pulses

\[ T(\beta \omega) = 4, \delta_1 = 0.01, \delta_2 = 0.0001 \]
OTHER PHASE PROFILES

Once we have a minimum phase design, there are many other phase profiles that have the same magnitude profile.

Minimum Phase

Non-linear Phase

Each passband zero may be flipped outside unit circle.

There are about \( T(BW) \) passband zeros:

\[ N_p \geq T(BW) \]

So there are

\[ Z^N_p \]

possible phase profiles.
IF profile phase is NOT a concern (sat pulses, inversion pulses), WE CAN CHOOSE PHASE TO OPTIMIZE SOME OTHER PARAMETER

5) PEAK RF AMPLITUDE

DESIGN PROCEDURE

1) DESIGN minimum PHASE RNC2

2) FACTOR (ROOT.m in MATLAB)

3) CHECK EACH COMBINATION OF ROOT FLIPS
   a) CALCULATE RNC2
   b) DESIGN RF PULSE

4) CHOOSE SOLUTION WITH MINIMUM PEAK R12; 16

CURRENTLY WORKS FOR 16 PASSBAND TERMS, IN TENS OF SECONDS OF CPU TIME.

OTHER APPROACHES FOR HIGHER ORDERS.
Non-Linear Phase Inversion Pulses

Minimum Phase Inversion

Optimized Phase Inversion

Peak Amplitude reduced by a factor of 2,
Peak power by a factor of 4