Unconventional RF Pulses and Pulse Pairs

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Typical Spin Echo Pulse Sequence

- Pulses designed independently
- Echo time limited
- Large dynamic range, demanding for RF amp
Design the spin echo instead of the pulses!

- Matched pulse pairs produce a linear phase echo
- Non-linear phase pulses have lower peak power
- Much more selective, less demanding for the RF amp
Short Echo Times

- The pulses can overlap, allowing short spin echo times
- Arbitrary flip angles
- Phase cycles

Slice Selective Excitation

Choose $B_1(t)$ with a limited range of frequencies

$\omega(x) = \gamma G_x x$

Slice Profile
Small-Tip-Angle Approximation

\[ \Delta M_{xy} = \gamma B_1(t) dt \]

\[ k(t) = -\gamma \int_t^T G(s) ds \]

\[ M_{xy}(x) = \int_0^T \gamma B_1(t)e^{ik(t)x} dt \]
Designing Small-Tip-Angle Pulses

Key Parameters:
- Duration, $T$
- Number of zeros, $N$ (here 8)

Windowed Sinc
RF Pulse:

N zeros

$T$

Slice Profile:

$F$

Width $N/T$

Transition Band $2/T$

$N$ is the Time–Bandwidth Product of the pulse
Large-Flip-Angle Pulses

Fourier-based designs work well to 90°

\[ M_{xy} \]

30° Excitation

\[ M_{xy} \]

90° Excitation

\[ M_{xy} \]

180° Refocusing

Non-linear problem beyond 90°
Spin Domain

Solve for rotations instead of magnetization

Rotation represented by 2x2 unitary matrix

\[
Q = \begin{pmatrix}
\alpha & -\beta^* \\
\beta & \alpha^*
\end{pmatrix}
\]

α and β are

\[
\alpha = \cos(\varphi/2) - in_z\sin(\varphi/2)
\]

\[
\beta = -i(n_x + in_y)\sin(\varphi/2)
\]

Rotation is by angle \(\varphi\) about axis \((n_x, n_y, n_z)^T\)

Magnitude constraint: \(\alpha\alpha^* + \beta\beta^* = 1\)
Magnetization from Spin Domain

Simple to compute slice profiles from spin domain:

\[ M_{xy} = 2a^{*}\beta M_0 \]  \hspace{1cm} \text{Excitation}

\[ M_z = (1-2\beta\beta^{*}) M_0 \]  \hspace{1cm} \text{Inversion/Saturation}

\[ M_{xy}^{*} = -\beta^2(M_{xy})^* \]  \hspace{1cm} \text{Refocusing}
Hard Pulse Approximation

- Represent RF as impulses separated by free precession intervals (discrete time approximation)
- Spinors are two polynomials $A_N(z)$ and $B_N(z)$ in $z = e^{i\gamma G x \Delta t}$ where $\Delta t$ is sampling time
- This is invertible, given $A_N(z)$ and $B_N(z)$ we can find $B_1(t)$
Shinnar-Le Roux Algorithm

Design $B_N(z)$ to approximate the desired flip angle profile

$$\beta(x) = -i(n_x + i n_y) \sin(\varphi(x)/2)$$

using a Fourier design (filter design).

Find a consistent $A_N(z)$ using magnitude constraint, minimum power (min phase)

Solve for $B_1(t)$

$$B_1(t) = \text{SLR}^{-1}(A_N(z), B_N(z))$$
Example: Spin Echo Pulse

Desired $\beta$ profile

$\sin\left(\frac{\varphi(x)}{2}\right)$

$\text{FT}$

$\text{SLR}^{-1}$

Bloch Eq

$B_N(z)$ coefficients

$M_x$

$M_y$

$X$

$N$

$B_N(z)$
Minimum phase pulse can have half the transition width.
Other Phase Profiles

- **Minimum Phase**: High peak power
- **Non-linear Phase**: Optimized non-linear phase reduces peak power by 31%
- Same total power (SAR)
- Identical magnitude profile
Designing Spin Echoes

\( M_{xy} \) at the echo is

\[
M_{xy} = (2a_{90}\beta^{*}_{90})(-\beta^{2}_{180}) M_0
\]

Choose

\[
\beta_{90} = \left(\frac{1}{\sqrt{2}}\right)(i\beta^{2}_{180})
\]

Matches profile and phase

\[
M_{xy} \approx |\beta_{180}|^4 M_0
\]
Non-Linear Phase Example

- Phases exactly cancel
- Perfect linear phase spin echo
- 90 is twice as long as 180
Near-Contiguous Spin Echoes

Used for arterial spin labeling, where the sharp transition is important

Non-Linear Phase CPMG

Goal: Different absolute phase at each z position
Non-Linear Phase CPMG

Goal:
Different absolute phase at each z position
Choose

\[ \beta_{90} = \frac{1}{\sqrt{2}}(i\beta_{180}) \Rightarrow M_{xy,90} \approx i\beta_{180} M_0 \]

\[ M_{xy} \approx i\beta_{180} |\beta_{180}|^2 M_0 \]

Echo has same phase profile as initial \( M_{xy} \)
Non-Linear Phase CPMG

- Echoes all have the same profile and polarity
- Works for any phase profile, including frequency sweeps (SPEN)
Overlapping Pulses

The two pulses don’t need to be distinct

Give \((\alpha_{90}, \beta_{90})\) and \((\alpha_{180}, \beta_{180})\)

\[
\beta_{pp} = z^D \beta_{90} \alpha^*_{180} + \alpha_{90} \beta_{180}
\]

\[
\alpha_{pp} = z^{-D} \alpha_{90} \alpha_{180} - \beta_{90} \beta^*_{180}
\]

D is the echo delay (in samples)

Find \(B_1(t)\) via SLR back recursion

Pulse Sequence

- z-Gradient needs added area
- Same as if it was on constantly until echo time
- Conjugate phase suppresses crushed component from 180
Slice Profiles

- Single pulse has spin echo and crushed signal
- Crushed component subtracts out
Slice Selective $^{31}$P Spectra

- Pulse pairs with different echo delays
- Axial 3.3 cm slice of brain
- 1.5T, 250 ms A/D, 2 s TR

Non-Linear Phase $\beta$’s

Many ways to design non-linear phase $\beta$
Widely studied for saturation pulses

Many options:
- Add quadratic phase to a linear phase $\beta$
- Use complex Remez algorithm
- Design a minimum phase pulse, factor, and zero flip
- Use an adiabatic pulse as a prototype
**Quadratic Phase**

- Complex Remez algorithm will produce an optimal design for a specified phase and amplitude profile.
- Paper by Schulte et al. tells you what to ask for!

Zero Flipping

Minimum Phase

Non-linear Phase

\[ z_i \Rightarrow \frac{1}{z_i^*} \]

TBW = N has \( 2^N \) possible phase profiles!

Max peak amplitude reduced \( \sim 1/\sqrt{N} \)
Adiabatic Pulses

Slice selective inversion
Insensitive to amplitude

\[ B_1(t) = A_0 \text{sech}(\beta t)e^{-i\mu\beta \tanh(\beta t)t} \]
Pulse pair with adiabatic 180, and overlapping phase compensated 90

Compensates for B1 at 7T
True Self Refocusing Pulses

Another option: start with a minimum phase excitation, and add phase to $\alpha$

Original $M_{xy}$ is

$$M_{xy} = 2\alpha^* \beta M_0$$

Add phase compensation $p$ to $\alpha$ to delay echo

$$M_{xy} = 2(\alpha p)^* \beta M_0$$

One cycle of linear phase across slice delays echo by one main lobe width

Adds one 180 in power (expensive!)
True Self Refocusing Pulses

- Zero echo time
- No crushed or unrefocused signal
- Echo delayed by adding phase, and RF power
Multi-Dimensional Pulses

- Excitation and acquisition are duals
- Any acquisition technique can be used as an excitation pulse (spiral, EPI)
- EPI RF pulses are most useful for MRS, Spectral-Spatial pulses
- Do two things with one pulse!
Spectral-Spatial Pulses

Product Pulse Design:
- Sampled spectral pulse (large tip angle)
- Spatial subpulses (small tip angle)
- Also, full 2D design
Spectral-Spatial Profile

- Periodic in frequency
- \(\frac{N}{2}\) “Ghost” sidelobes
- Continuous in space
Positive Contrast SPIO Imaging

Spin echo image of frequency shifted signals

Positive contrast

Conclusions

Much to be gained by focusing on the MR signal instead of the RF pulses

Benefits include

- Sharper profiles
- Shorter echo time
- Better control of the signal