Homework #7 (24 pts)
Solutions

1. *Zero Forcing Code (8 pts)*

   (a) No. User u is able to coordinate the reception of signals on L_y dimentions.

   (b) Yes, it can. For example, a zero forcing W_u can reduce the data rate by enhancing the noise.

   (c) No. In section 14.2, the optimality of MMSE-GDFE and ZF-GDFE are conditioned on that the receivers does not cooperate so that the W reduces to block diagonal matrix. But for user u alone, the condition does not hold.

   (d) $H = QR$. We know in BC MMSE-GDFE reduces to ZF-GDFE. Thus $R_f = H^*H = R^*R = G^*G$. Let $S_0 = \text{diag}(\text{diag}(R_f)$, we have $G = S_0^{-\frac{1}{2}}R = \text{diag}(\text{diag}(R))^{-1}R$. The feedforward matrix $W = S_0^{-1}G^{-*}H^* = S_0^{-\frac{1}{2}}Q^*$. Therefore the ZF GDFE decoding is to rotate the received signal by Q^* and scale it by $S_0^{-\frac{1}{2}}$. The dirty-paper encoding can be done at the tranceiver using $G = \text{diag}(\text{diag}(R))^{-1}R$, an upper diagonal matrix. This scheme correspondes to $\mathbf{v} = QS_0^{\frac{1}{2}}\mathbf{x}$ at the crypto precoder of the transmitter.

2. *Admission Problem (9 pts)*

   (a) See Fig. 1.

   (b) For this scalar BC channel, by the duality of BC and MAC, we only need to check if $\mathbf{b}$ is in the dual MAC region with a sum unit energy, i.e., to run minPMAC with a uniform weight vector and then check if the total energy contraint is satisfied or not.

   (c) We can use bisection to find $\mathbf{b}$. The code is given below.

```
%MATLAB Code for 14.5 (c)
% Hao Zou, Nov. 29, 2008
clear all;

k_vector  = [10 0.1 1];
b = zeros(2,1,3);
for i = 1:length(k_vector)
    k = k_vector(i); % b2 = k*b1;
    b1_high = 10;
    b1_low = 0;
    eps = 0.5e-4;
    H = zeros(1,2,1);
    H(1,1,1) = 0.9/sqrt(0.01);
    H(1,2,1) = 0.6/sqrt(0.01);
    while (b1_high - b1_low > eps)
```

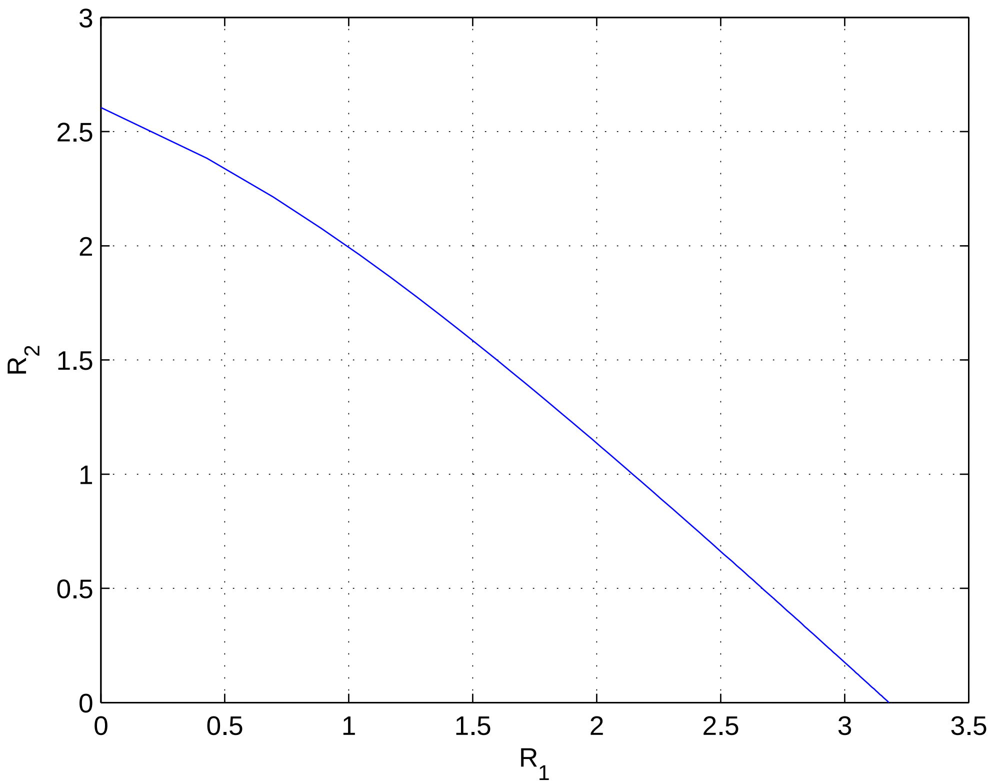

Figure 1: The rate region for 14. 5 (a)

```
        b1 = (b1_high + b1_low)/2;
        b_temp = (b1*[1 k])';
        [E, theta, B] = minPMAC(H, b_temp ,[1 1]');
        if (sum(E)>1)
            b1_high = b1;
        else
            b1_low = b1;
        end
    end
    b(:,1,i) = b1*[1 k]';
end

Results:

b(:,:,1) =

    0.2484
    2.4837


b(:,:,2) =

    2.8849
    0.2885


b(:,:,3) =

    1.5454
    1.5454
```

We can verify the results with Fig. 1.

(d) If the weight vector is not uniform, then minPMAC is not minimizing the sum energy of the dual MAC and thus the rate region of BC cannot be traced as we have done in 14.5 (c) - (d).

3. *Reciprocity (16 pts)*

(a) The wireless hub can measure the upstream channel H_{MAC} by pilot sequences and use the symmetry to the channel to obtain the downlink channel $H_{\text{BC}} = H^*_{\text{MAC}}$. Similarly, the downlink receivers u and measure $H_{\text{BC},u}$ and obtain $H_{\text{MAC},u} = H^*_{\text{BC},u}$ by symmery of the channel.

(b) When using the channel in a ping-pong manner between uplink and downlink transmission, the echo in the transmission needs to be perfectly cancelled. Under ideal echo cancellation, and perfect reciprocity, using the channel in a time-division access manner is optimal. However, since these ideal conditions are rarely met in practice, time-division may not be optimal.

(c) In practice: (1) the noise at the downlink locations are not neccessarily the same as the uplink receiver. (2) The delay of TDMA system is high and limited by the longest round-trip distance of the system. (3) The wireless channel can change fast. (4) The channel may not be symmetric for the wireless channel.