Quantifiers
Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2020
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Note  This is a companion to Keenan 1996. It is only partially complete in the sense that it's waiting for you to fill in various blanks, flesh out ideas, and insert new examples.

1 Summary of semantic claims so far

(1)  All meanings are defined in terms of a universe of entities $U$. For this handout, I'll often rely on this Simpsons universe:

$$U = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns}\}$$

(2)  Proper names directly refer to members of the domain:

$$[\text{Maggie}] = \text{Maggie}$$  $$[\text{Lisa}] = \text{Lisa}$$  $$[\text{Bart}] = \text{Bart}$$

$$[\text{Marge}] = \text{Marge}$$  $$[\text{Homer}] = \text{Homer}$$  $$[\text{Burns}] = \text{Burns}$$

(3)  Common nouns denote sets of entities, subsets of the universe $U$. Examples:

$$[\text{Simpson}] = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns}\}$$  $$[\text{boss}] = \{\text{Homer}\}$$

(4)  Intransitive verbs also denote sets of entities, subsets of the universe $U$. Examples:

$$[\text{skateboards}] = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns}\}$$
$$[\text{studies}] = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns}\}$$
$$[\text{worries}] = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns}\}$$

(5)  Adjectives denote functions from sets into sets. (For the special case of intersective adjectives, we can treat them as sets.) We won't need to deal directly with adjectives for this unit, but they'll become important again when we work with the full semantic grammar.

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2 Quantificational determiner meanings

(6) “So we think of a $\text{Det}_1$ as combining with an $\text{N}$ to make an $\text{NP}$, the latter combining with $\text{P}_1$s to make $\text{S}$s.” (Keenan 1996: 42)

\[
\begin{array}{c}
S \\
\text{NP} \\
\text{Det}_1 \quad \text{N} \\
\text{every} \quad \text{child} \\
\end{array}
\]

\[
\begin{array}{c}
\text{P}_1 \\
\text{studies} \\
\text{scope} \\
\end{array}
\]

(7) \[[\text{every}] = \{\langle A, B \rangle : A \subseteq B \}\]

(8) \[[\text{a(n)}] = \{\langle A, B \rangle : A \cap B \neq \emptyset \}\]

(9) \[[\text{no}] =

(10) \[[\text{at least three}] = \{\langle A, B \rangle : |A \cap B| \geq 3 \}\]

(11) \[[\text{at most three}] =

(12) \[[\text{exactly three}] =
(13) \[ \text{[most]} = \left\{ (A, B) : \frac{|A \cap B|}{|A|} > \frac{1}{2} \right\} \\
= \left\{ (A, B) : |A \cap B| > |A - B| \right\} \]

(14) \[ \text{[more than four]} = \]

(15) \[ \text{[between five and ten]} = \]

(16) \[ \text{[every child]} = \left\{ B : [\text{child}] \subseteq B \right\} \]

\[ \left\{ B : [\text{child}] \subseteq B \right\} \]

\[ \left\{ (A, B) : A \subseteq B \right\} \]

\[ [\text{child}] \]

(17) \[ \text{[every child skateboards]} = [\text{child}] \subseteq [\text{skateboards}] \]

\[ [\text{child}] \subseteq [\text{skateboards}] \]

\[ \left\{ B : [\text{child}] \subseteq B \right\} \]

\[ [\text{skateboards}] \]

\[ \left\{ (A, B) : A \subseteq B \right\} \]

\[ [\text{child}] \]

(18) \[ \text{[most children]} = \]
2.1 A closer look at *most*

2.1.1 Mark Liberman’s survey

Mark Liberman noticed (19) and wrote: “I (think I) always took *most* to mean exactly “more than half”, so Irving’s “I wouldn’t say ‘most’ but I’d say ‘more than half’” took me aback.”

From: http://languagelog.ldc.upenn.edu/nll/?p=2510

(19) Kurt Andersen: I- I read somewhere that you said that now m- most of your audience, you believe, reads you not in English. They are not only overseas but people not in the United Kingdom or Australia. It’s- it’s people reading in-

John Irving: I wouldn’t say- I wouldn’t say “most” but I’d say “more than half”. Sure, more than half, definitely. I mean I- I sell more books in Germany than I do in the U.S. Uh I s- sell almost as many uh books in- in the Netherlands as I do in the- in the U.S.

Lots of readers left comments on Liberman’s post articulating their assumptions about what *most* means, and he collected them in a follow-up:

http://languagelog.ldc.upenn.edu/nll/?p=2511

(20) I think ‘most’ licenses a default generalization, relative to a bunch of pragmatic factors, …

(21) I think ‘most’ has a normative or qualititative sense in addition to a quantitative sense.

(22) For me too, “most” has a defeasible implicature of “much more than a majority”.

(23) I would be with John Irving - 51% of a population isn’t “most” but around 60-75% would be. (90% or more would be “almost all”; well, until it hit “all” at 100%; and 75-90% would be “a very large majority”).

(24) “Most X are Y”, to me, means a substantial majority of X are Y—certainly more than 50%-plus-1. Even two-thirds feels borderline.

(25) Most has always meant “more than half (but less than all)” to me. If there are 100 of us and I say “Most of us stayed behind” I mean between 51 and 99.

Liberman looked at some dictionaries:

(26) *OED*: modifying a plural count noun: the greatest number of; the majority of

(27) *Merriam-Webster*: the majority of

(28) *American Heritage*: in the greatest quantity, amount, measure, degree, or number: to win the most votes
2.1.2 Theories

(H) \[ \text{\texttt{most}} = \left\{ (A, B) : \frac{|A \cap B|}{|A|} > \frac{1}{2} \right\} \] (more than half; repeated from (13))

\[ = \left\{ (A, B) : |A \cap B| > |A - B| \right\} \]

(GH) \[ \text{\texttt{most}} = \left\{ (A, B) : \frac{|A \cap B|}{|A|} > f \right\} \] (where \( f \gg \frac{1}{2} \))

(P) \[ \text{\texttt{most}} = \left\{ (A, B) : |A \cap B| > |C \cap B| \right\} \] for all relevant contrasts groups \( C \) (plurality)

2.1.3 Corpus experiments

(a) Liberman’s results for Googling "most * percent" and picking our the first 150 hits with numerical percentages.

Liberman: “it’s pretty clear that the whole range from 50.1 to 99.9 is getting some action.”

Very close to an attested 100% example (via James Collins)

(29) The rape case was the only offense for which the majority of participants supported registration (in fact 100% supported the registry in this condition), whereas …

My three surprising cases below 50% seem to involve implicit additional restrictions:

(30) most homes (39 percent) have a separate room where the pc is

(31) found that most of them (42 percent) focus on what he dubs

(32) most of the country (42 percent) will

Liberman did an additional post giving lots of citations and abstracts for psycholinguistic and theoretical work on \textit{most}: http://languagelog.ldc.upenn.edu/nll/?p=2516
3 Quantifier properties

3.1 Intersectivity

(33) A determiner $D$ is intersective iff $D(A)(B) = D(B)(A)$ for all $A$ and $B$.

(34) $\text{some?}$

(35) $\text{every?}$

(36) $\text{no?}$

(37) $\text{at most four?}$

3.2 Conservativity

(38) A determiner $D$ is conservative iff $D(A)(B) = D(A)(A \cap B)$ for all $A$ and $B$.

(39) $\text{some?}$

(40) $\text{every?}$

(41) $\text{no?}$

(42) $\text{few?}$

(43) $\text{most?}$
Proposed universal  Every lexical determiner in every language is conservative.\textsuperscript{2} See also Keenan’s footnote 1.

Potential counterexample: only

(44) Only dogs bark.

But!  The evidence strongly suggests that only is not a determiner.

i. It can modify a wide range of constituents, not just nominals.

ii. It precedes determiner elements (e.g., only some books).

3.3 Monotonicity

(45) Upward monotonicity (increasing)

a. A determiner $D$ is upward monotone on its \textbf{first} argument iff $D(A)(B) \Rightarrow D(X)(B)$ for all $A, B, X$ where $A \subseteq X$.

b. A determiner $D$ is upward monotone on its \textbf{second} argument iff $D(A)(B) \Rightarrow D(A)(X)$ for all $A, B, X$ where $B \subseteq X$.

(46) Downward monotonicity (decreasing)

a. A determiner $D$ is downward monotone on its \textbf{first} argument iff $D(A)(B) \Rightarrow D(X)(B)$ for all $A, B, X$ where $X \subseteq A$.

b. A determiner $D$ is downward monotone on its \textbf{second} argument iff $D(A)(B) \Rightarrow D(A)(X)$ for all $A, B, X$ where $X \subseteq B$.

(47) A determiner $D$ is nonmonotone on an argument iff $D$ is neither upward nor downward monotone on that argument.

(48) some ( $\uparrow$ ) ( $\uparrow$ )

(49) no ( $\downarrow$ ) ( $\downarrow$ )

(50) every ( ) ( )

(51) at most ten ( ) ( )

(52) exactly three ( ) ( )

(53) most ( ) ( )