Quantifiers
Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2019
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Note  This is a companion to Keenan 1996. It is only partially complete in the sense that it’s waiting for you to fill in various blanks, flesh out ideas, and insert new examples.

1 Summary of semantic claims so far

(1) All meanings are defined in terms of a universe of entities $U$. For this handout, I’ll often rely on this Simpsons universe:

$$U = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns}\}$$

(2) Proper names directly refer to members of the domain:

$$[\text{Maggie}] = \text{Maggie} \quad [\text{Lisa}] = \text{Lisa} \quad [\text{Bart}] = \text{Bart}$$

$$[\text{Marge}] = \text{Marge} \quad [\text{Homer}] = \text{Homer} \quad [\text{Burns}] = \text{Burns}$$

(3) Common nouns denote sets of entities, subsets of the universe $U$. Examples:

$$[\text{Simpson}] = \{\text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}\} \quad [\text{boss}] = \{\text{Homer}\}$$

(4) Intransitive verbs also denote sets of entities, subsets of the universe $U$. Examples:

$$[\text{skateboards}] = \{\text{Maggie}, \text{Lisa}\} \quad [\text{studies}] = \{\text{Marge}\} \quad [\text{worries}] = \{\text{Homer}, \text{Burns}\}$$

(5) Adjectives denote functions from sets into sets. (For the special case of intersective adjectives, we can treat them as sets.) We won’t need to deal directly with adjectives for this unit, but they’ll become important again when we work with the full semantic grammar.

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2 Quantificational determiner meanings

(6) “So we think of a Det₁ as combining with an N to make an NP, the latter combining with P₁'s to make Ss.” (Keenan 1996: 42)

(7) \[ \text{every} = \{ \langle A, B \rangle : A \subseteq B \} \]

(8) \[ \text{a(n)} = \{ \langle A, B \rangle : A \cap B \neq \emptyset \} \]

(9) \[ \text{no} = \{ \langle A, B \rangle : A \cap B = \emptyset \} \]

(10) \[ \text{at least three} = \{ \langle A, B \rangle : |A \cap B| \geq 3 \} \]

(11) \[ \text{at most three} = \{ \langle A, B \rangle : |A \cup B| \leq 3 \} \]

(12) \[ \text{exactly three} = \{ \langle A, B \rangle : |A \cap B| = 3 \} \]
(13) \[\text{[most]} \equiv \left\{ (A, B) : \frac{|A \cap B|}{|A|} > \frac{1}{2} \right\} = \left\{ (A, B) : |A \cap B| > |A - B| \right\}\]

As before, As that are more

(14) \[\text{[more than four]} = \left\{ (A, B) : |A \cap B| > 4 \right\}\]

(15) \[\text{[between five and ten]} = \left\{ (A, B) : 5 \leq |A \cap B| \leq 10 \right\}\]

(16) \[\text{[every child]} = \{B : [\text{child}] \subseteq B\}\]

(17) \[\text{[every child skateboards]} = [\text{child}] \subseteq [\text{skateboards}]\]

(18) \[\text{[most children]} = \left\{ B : \frac{|\bigcap \text{child} \cap B|}{|\bigcap \text{child}|} > \frac{1}{2} \right\}\]
2.1 A closer look at *most*

2.1.1 Mark Liberman's survey

Mark Liberman noticed (19) and wrote: “I (think I) always took most to mean exactly “more than half”, so Irving’s “I wouldn't say ‘most' but I'd say ‘more than half’” took me aback.”

From: http://languagelog.ldc.upenn.edu/nll/?p=2510

(19) Kurt Andersen: I- I read somewhere that you said that now m- most of your audience, you believe, reads you not in English. They are not only overseas but people not in the United Kingdom or Australia. It’s- it’s people reading in-

John Irving: I wouldn't say- I wouldn't say “most” but I’d say “more than half”. Sure, more than half, definitely. I mean I- I sell more books in Germany than I do in the U.S. Uh I s- sell almost as many uh books in- in the Netherlands as I do in the- in the U.S.

Lots of readers left comments on Liberman’s post articulating their assumptions about what *most* means, and he collected them in a follow-up:

http://languagelog.ldc.upenn.edu/nll/?p=2511

(20) I think ‘most' licenses a default generalization, relative to a bunch of pragmatic factors, …

(21) I think ‘most' has a normative or qualitative sense in addition to a quantitative sense.

(22) For me too, “most” has a defeasible implicature of “much more than a majority”.

(23) I would be with John Irving - 51% of a population isn't “most” but around 60-75% would be. (90% or more would be “almost all”; well, until it hit “all” at 100%; and 75-90% would be “a very large majority”).

(24) “Most X are Y”, to me, means a substantial majority of X are Y—certainly more than 50%-plus-1. Even two-thirds feels borderline.

(25) Most has always meant “more than half (but less than all)” to me. If there are 100 of us and I say “Most of us stayed behind” I mean between 51 and 99.

Liberman looked at some dictionaries:

(26) *OED*: modifying a plural count noun: the greatest number of; the majority of

(27) *Merriam-Webster*: the majority of

(28) *American Heritage*: in the greatest quantity, amount, measure, degree, or number: to win the most votes
2.1.2 Theories

(H) \[ \text{most} = \left\{ (A, B) : \frac{|A \cap B|}{|A|} > \frac{1}{2} \right\} \] ('more than half'; repeated from (13))

\[ = \left\{ (A, B) : |A \cap B| > |A - B| \right\} \]

(GH) \[ \text{most} = \left\{ (A, B) : \frac{|A \cap B|}{|A|} > f \right\} \] (where \( f \gg \frac{1}{2} \))

(P) \[ \text{most} = \left\{ (A, B) : |A \cap B| > |C \cap B| \text{ for all relevant contrasts groups } C \right\} \] (plurality)

2.1.3 Corpus experiments

(a) Liberman's results for Googling "most * percent" and picking our the first 150 hits with numerical percentages.

(b) My experiment using regular expressions to search the Gigaword corpus, a 1 billion word corpus of English newswire text. Link to my examples.

Liberman: “it’s pretty clear that the whole range from 50.1 to 99.9 is getting some action.”

My three surprising cases below 50% seem to involve implicit additional restrictions:

(29) most homes (39 percent) have a separate room where the pc is

(30) found that most of them (42 percent) focus on what he dubs

(31) most of the country (42 percent) will

Liberman did an additional post giving lots of citations and abstracts for psycholinguistic and theoretical work on most:

http://languagelog.ldc.upenn.edu/nll/?p=2516
3 Quantifier properties

3.1 Intersectivity

(32) A determiner $D$ is intersective iff $D(A)(B) = D(B)(A)$ for all $A$ and $B$.

(33) $\text{some?}$

Some student is a dancer

Some dancer is a student

It's a law of set theory that $A \cap B = B \cap A$ for all sets $A$ and $B$, and this suffices to show that $\{\{A, B\} : A \cap B \neq \emptyset\}$ is intersective.

(34) $\text{every?}$

Every student is a dancer

Every dancer is a student

Consider $A = \{3\}$ and $B = \{1, 2\}$. This pair is in $\{\{A, B\} : A \subseteq B\}$, but $\{3, 1\}$ is not, which establishes that $\{\{A, B\} : A \cap B \neq \emptyset\}$ is intersective.

(35) $\text{no?}$

No student is a dancer

No dancer is a student

$\{\{A, B\} : A \cap B = \emptyset\} = \{\{A, B\} : |A \cap B| = 0\}$

Since these receive 0 depend only on the cardinality of the intersection, they are guaranteed to be intersective.

(36) $\text{at most four?}$

At most four students are dancers

At most four dancers are students

$\{\{A, B\} : |A \cap B| \leq 4\}$

Equivalent by the law that $A \cap B = B \cap A$ for all $A$.

3.2 Conservativity

(37) A determiner $D$ is conservative iff $D(A)(B) = D(A)(A \cap B)$ for all $A$ and $B$.

(38) $\text{some?}$

Some student is a dancer

Some student is a student and a dancer

 Equivalent by the law that $A \cap B = B \cap A$ for all $A$.

(39) $\text{every?}$

Every student is a dancer

Every student is a student and a dancer

(40) $\text{no?}$

Consider the hypothetical determiner $D$ such that $D(A)(B) = \{\{A, B\} : |A \cap B| \neq \emptyset\}$. To show this is not conservative, consider $A = \{3\}$ and $B = \{1, 2\}$. This pair $\{A, B\}$ is in $\{\{A, B\} : |A \cap B| \neq \emptyset\}$, however, $\{A, A \cap B\}$ is not, because $|A| = 1$ but $|A \cap B| = 0$. This pair is thus a counterexample, which is all it take to prove non-conservative. More generally, any pair of disjoint, non-empty sets will be counterexamples.

(41) $\text{few?}$

(42) $\text{most?}$
**Proposed universal**  Every lexical determiner in every language is conservative.\(^2\) See also Keenan’s footnote 1.

**Potential counterexample: only**

(43) Only dogs bark.

But!  The evidence strongly suggests that *only* is not a determiner.

i.  It can modify a wide range of constituents, not just nominals.

ii.  It precedes determiner elements (e.g., *only some books*).

### 3.3 Monotonicity

(44) Upward monotonicity (increasing)

\[ (a) \quad \text{A determiner } D \text{ is upward monotone on its first argument iff } D(A)(B) \Rightarrow D(X)(B) \text{ for all } A, B, X \text{ where } A \subseteq X. \]

\[ (b) \quad \text{A determiner } D \text{ is upward monotone on its second argument iff } D(A)(B) \Rightarrow D(A)(X) \text{ for all } A, B, X \text{ where } B \subseteq X. \]

(45) Downward monotonicity (decreasing)

\[ (a) \quad \text{A determiner } D \text{ is downward monotone on its first argument iff } D(A)(B) \Rightarrow D(X)(B) \text{ for all } A, B, X \text{ where } X \subseteq A. \]

\[ (b) \quad \text{A determiner } D \text{ is downward monotone on its second argument iff } D(A)(B) \Rightarrow D(A)(X) \text{ for all } A, B, X \text{ where } X \subseteq B. \]

(46) A determiner *D* is nonmonotone on an argument iff *D* is neither upward nor downward monotone on that argument.

(47) some $(\uparrow)(\uparrow)$

(48) no $(\downarrow)(\downarrow)$

(49) every $(\bigcup)(\uparrow)$

(50) at most ten $(\bigcup)(\bigcup)$

(51) few $(\ )$( )

(52) exactly three $(\bigcap)(\bigcup)$

\[ \{ \langle A, B \rangle : |A \cap B| = 3 \} \]

Consider $A = \{1,2,3,4\}$ and $B = \{2,3,4,5,6\}$.

$\langle A, B \rangle \notin \text{exactly three}$.

However $X = \{1,3\}$ is a subset of $A$, and $\langle X, B \rangle \in \text{exactly three}$.

Since $|X \cap B| = 1$. Similarly, $Y = \{1,2,3,4,5\}$ is a superset of $A$, and $\langle X, B \rangle \notin \text{exactly three}$.

Since $|Y \cap B| = 3$. This shows that \text{exactly three} is nonmonotone on its first argument. The same sort of argument works for the second argument.

(53) most $(\ )$( )