Quantifiers
Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2018
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Note  This is a companion to Keenan 1996. It is only partially complete in the sense that it’s waiting for you to fill in various blanks, flesh out ideas, and insert new examples.

1 Summary of semantic claims so far

(1) All meanings are defined in terms of a universe of entities $U$. For this handout, I’ll often rely on this Simpsons universe:

\[ U = \{ \text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns} \} \]

(2) Proper names directly refer to members of the domain:

\begin{align*}
\text{[Maggie]} &= \text{Maggie} \\
\text{[Lisa]} &= \text{Lisa} \\
\text{[Bart]} &= \text{Bart} \\
\text{[Marge]} &= \text{Marge} \\
\text{[Homer]} &= \text{Homer} \\
\text{[Burns]} &= \text{Burns}
\end{align*}

Simple model, useful for calculations — but it can be misleading

(3) Common nouns denote sets of entities, subsets of the universe $U$. Examples:

\begin{align*}
\text{[Simpson]} &= \{ \text{Maggie}, \text{Lisa}, \text{Bart}, \text{Marge}, \text{Homer}, \text{Burns} \} \\
\text{[boss]} &= \{ \text{Homer} \}
\end{align*}

(4) Intransitive verbs also denote sets of entities, subsets of the universe $U$. Examples:

\begin{align*}
\text{[skateboards]} &= \{ \text{Maggie}, \text{Bart}, \text{Burns} \} \\
\text{[studies]} &= \{ \text{Lisa} \} \\
\text{[worries]} &= \{ \text{Bart}, \text{Burns} \}
\end{align*}

(5) Adjectives denote functions from sets into sets. (For the special case of intersective adjectives, we can treat them as sets.) We won’t need to deal directly with adjectives for this unit, but they’ll become important again when we work with the full semantic grammar.

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2 Quantificational determiner meanings

(6) “So we think of a Det$_1$ as combining with an N to make an NP, the latter combining with $P_1$s to make Ss.” (Keenan 1996: 42)

(7) $\text{[every]} = \{\langle A, B \rangle : A \subseteq B \}$

(8) $\text{[a(n)]} = \{\langle A, B \rangle : A \cap B \neq \emptyset \}$

(9) $\text{[no]} = \{\langle A, B \rangle : A \cap B = \emptyset \} \cup \{\langle A, B \rangle : |A \cap B| = 0 \}$

(10) $\text{[at least three]} = \{\langle A, B \rangle : |A \cap B| \geq 3 \}$

(11) $\text{[at most three]} = \{\langle A, B \rangle : |A \cap B| \leq 3 \}$

(12) $\text{[exactly three]} = \{\langle A, B \rangle : |A \cap B| = 3 \}$
Quantifiers

(13) \([\text{most}] = \left\{ (A, B) : \frac{|A \cap B|}{|A|} > \frac{1}{2} \right\} = \left\{ (A, B) : |A \cap B| > |A - B| \right\}\]

(14) \([\text{more than four}] = \left\{ (A, B) : |A \cap B| > 4 \right\}\]

(15) \([\text{between five and ten}] = \left\{ (A, B) : 5 \leq |A \cap B| \leq 10 \right\}\]

(16) \([\text{every child}] = \{ B : [\text{child}] \subseteq B \}\]

(17) \([\text{every child skateboards}] = [\text{child}] \subseteq [\text{skateboards}]\]

(18) \([\text{most children}] = \left\{ B : \frac{|\text{child} \cap \text{skateboards}|}{|\text{child}|} > \frac{1}{2} \right\} \right\} = \left\{ B : |\text{child} \cap \text{skateboards}| > |\text{child} - \text{skateboards}| \right\} \]
2.1 A closer look at *most*

2.1.1 Mark Liberman’s survey

Mark Liberman noticed (19) and wrote: “I (think I) always took most to mean exactly “more than half”, so Irving’s “I wouldn’t say ‘most’ but I’d say ‘more than half’” took me aback.”

From: http://languagelog.ldc.upenn.edu/nll/?p=2510

(19) Kurt Andersen: I- I read somewhere that you said that now m- most of your audience, you believe, reads you not in English. They are not only overseas but people not in the United Kingdom or Australia. It's- it's people reading in-

John Irving: I wouldn't say- I wouldn't say “most” but I’d say “more than half”. Sure, more than half, definitely. I mean I- I sell more books in Germany than I do in the U.S. Uh I s- sell almost as many uh books in- in the Netherlands as I do in the- in the U.S.

Lots of readers left comments on Liberman’s post articulating their assumptions about what *most* means, and he collected them in a follow-up:

http://languagelog.ldc.upenn.edu/nll/?p=2511

(20) I think ‘most’ licenses a default generalization, relative to a bunch of pragmatic factors, …

(21) I think ‘most’ has a normative or qualitative sense in addition to a quantitative sense.

(22) For me too, “most” has a defeasible implicature of “much more than a majority”.

(23) I would be with John Irving - 51% of a population isn't “most” but around 60-75% would be. (90% or more would be “almost all”; well, until it hit “all” at 100%; and 75-90% would be “a very large majority”).

(24) “Most X are Y”, to me, means a substantial majority of X are Y—certainly more than 50%-plus-1. Even two-thirds feels borderline.

(25) Most has always meant “more than half (but less than all)” to me. If there are 100 of us and I say “Most of us stayed behind” I mean between 51 and 99.

Liberman looked at some dictionaries:

(26) *OED*: modifying a plural count noun: the greatest number of; the majority of

(27) *Merriam-Webster*: the majority of

(28) *American Heritage*: in the greatest quantity, amount, measure, degree, or number: to win the most votes
2.1.2 Theories

\[ \left\lceil \frac{\text{most}}{\text{}} \right\rceil = \left\{ \langle A, B \rangle : \frac{|A \cap B|}{|A|} > \frac{1}{2} \right\} \]  

\[ = \left\{ \langle A, B \rangle : |A \cap B| > |A - B| \right\} \]  

\[ \left\lceil \frac{\text{most}}{\text{}} \right\rceil = \left\{ \langle A, B \rangle : \frac{|A \cap B|}{|A|} > f \right\} \]  

\[ \text{(where } f \gg \frac{1}{2} \text{) } \]

\[ \left\lceil \frac{\text{most}}{\text{}} \right\rceil = \left\{ \langle A, B \rangle : |A \cap B| > |C \cap B| \text{ for all relevant contrasts groups } C \right\} \]  

(29) most homes (39 percent) have a separate room where the pc is

(30) found that most of them (42 percent) focus on what he dubs

(31) most of the country (42 percent) will

Liberman did an additional post giving lots of citations and abstracts for psycholinguistic and theoretical work on most:

http://languagelog.ldc.upenn.edu/nll/?p=2516
3 Quantifier properties

3.1 Intersectivity

(32) A determiner $D$ is intersective iff $D(A)(B) = D(B)(A)$ for all $A$ and $B$.

- **some?** A Simpson skateboards. $\iff$ A skateboarder is a Simpson

- **every?** Every Simpson skateboards $\iff$ Every skateboarder is a Simpson.

- **no?**

- **at most four?**

3.2 Conservativity

(33) A determiner $D$ is conservative iff $D(A)(B) = D(A)(A \cap B)$ for all $A$ and $B$.

- **some?** Some Simpson skateboards $\iff$ Some Simpson is a Simpson who skateboards.

- **every?** Every Simpson skateboards $\iff$ Every Simpson is a Simpson who skateboards.

- **no?**

- **few?**

- **most?**

Proposed universal $^2$ Every lexical determiner in every language is conservative.

Keenan (1996: 55)

“With at most a few exceptions $^1$ English Dets denote conservative functions.”

From Keenan’s footnote 1:

“All putative counterexamples to Conservativity in the literature are ones in which a sentence of the form Det A’s are B’s is interpreted as $D(B)(A)$, where $D$ is conservative. So the problem is not that Det fails to be conservative, rather it lies with matching the Noun and Predicate properties with the arguments of the Det denotation.”

3.2.1 Potential counterexample #1: only

(34) Only dogs bark.

But! The evidence strongly suggests that only is not a determiner.

i. It can modifier a wide range of constituents, not just nominals.

ii. It precedes determiner elements (e.g., only some books).

3.2.2 Potential counterexample #2: many

(35) Many Scandinavians have won the Nobel Prize.

\[
[\text{many}] = \left\{ (A, B) : \frac{|A \cap B|}{|B|} \geq k \right\}
\]

Expected semantics\(^3\)

\[
\frac{|[\text{scandinavian}] \cap [\text{nobelist}]|}{|[\text{scandinavian}]|} \geq k
\]

- Suppose that there are 20 million Scandinavians.
- Suppose that there are 1000 Nobel Prize winners, 600 of whom are Scandinavian.
- Then the expected semantics is wildly false.

The flipped semantics (expected for Many Nobel Prize winners are Scandinavian).

\[
\frac{|\{x : \text{scandinavian}(x) \land \text{nobelist}(x)\}|}{|\{x : \text{nobelist}(x)\}|} \geq k
\]

- Suppose that there are 1000 Nobel Prize winners, 600 of whom are Scandinavian.
- Then the flipped semantics is going to be true as long as \(k\) is \(\frac{3}{5}\) (60%) or greater.

Alternative reading for many

(37) \([\text{many}_{\text{cardinal}}] = \{ (A, B) : |A \cap B| > n \}\) where \(n\) is some contextually supplied value.

(38) a. There are many cookies left.
    b. There are many books in the library.
    c. There are many cars in the U. S.

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3.3 Monotonicity

(39) A function $D$ is increasing (upward monotone) iff wherever $A \subseteq B$, $D(A) \subseteq D(B)$.

(40) A function $D$ is decreasing (downward monotone) iff wherever $A \subseteq B$, $D(B) \subseteq D(A)$.

(41) A function $D$ is nonmonotone iff $D$ is neither increasing nor decreasing.

(42) a. some $(\uparrow)(\uparrow)$
    b. no $(\downarrow)(\downarrow)$
    c. every $(\downarrow)(\uparrow)$
    d. at most ten $(\downarrow)(\downarrow)$
    e. few $(\quad)(\quad)$
    f. exactly three $(\quad)(\quad)$
    g. most $(\quad)(\quad)$

Polarity sensitivity

(43) a. Sam didn’t ever take notes.
    b. *Sam ever took notes.

(44) a. At no time did Sam ever take notes.
    b. *At some times, Sam ever took notes.

(45) a. Sam took the class without ever taking notes.
    b. *Sam took the class while ever taking notes.

(46) a. No student ever took notes.
    b. No student who ever studied semantics took notes.

(47) a. *A student ever took notes.
    b. *A student who ever studied semantics took notes.

(48) a. Every student who ever studied semantics took notes.
    b. *Every student ever took notes.