

# Introduction to the Rational Speech Acts model

Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2019

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## 1 Overview

This handout is meant to be a technical companion to our core RSA reading, Goodman & Frank 2016,<sup>1</sup> which provides relevant background, conceptual motivation, and model details. It's best to study that paper carefully before working with this handout. There's also a screencast available from the course website that provides a bit more conceptual background and walks through the calculations reviewed here.

## 2 Reference games

It's very useful to ground RSA calculations in specific reference games:

- (1) A reference game is a structure  $(R, M, \llbracket \cdot \rrbracket, P, C)$ , where
  - a.  $R$  is a set of states (worlds, referents, propositions, etc.).
  - b.  $M$  is a set of messages.
  - c.  $\llbracket \cdot \rrbracket : M \mapsto R \mapsto \{0, 1\}$  is a semantic interpretation function.
  - d.  $P : R \mapsto [0, 1]$  is a prior probability distribution over states.
  - e.  $C : M \mapsto \mathbb{R}_{\leq 0}$  is a cost function on messages.

The intuitive idea here is that we have a speaker and a listener in a shared context/game. The speaker is privately assigned a *target* referent  $r^* \in R$ , and the speaker's goal is choose a message from  $M$  that will lead the listener to pick  $r^*$  as the target.

Implicitly, RSA assumes that the speaker and listener would both prefer for the listener to correctly identify  $r^*$ . This is a version of the cooperativity assumptions from Grice's work.

The guiding idea behind RSA is that these agents can do better at games like this by reasoning about each other rather than just about the truth conditions built into  $\llbracket \cdot \rrbracket$ . This connects very deeply with the definition of conversational implicature, which also centers around this back-and-forth reasoning.

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<sup>1</sup>Goodman, Noah D. and Frank, Michael C. 2016. Pragmatic language interpretation as probabilistic inference. *Trends in Cognitive Sciences* 20(11): 818–829.

### 3 RSA and Grice

#### 3.1 The RSA model

(2)

$$P_{\text{Lit}}(r | m) = \frac{\llbracket m \rrbracket(r) \cdot P(r)}{\sum_{r' \in R} \llbracket m \rrbracket(r') \cdot P(r')}$$

(3)

$$P_S(m | r) = \frac{\exp(\alpha \cdot (\log P_{\text{Lit}}(r | m) + C(m)))}{\sum_{m' \in M} \exp(\alpha \cdot (\log P_{\text{Lit}}(r | m') + C(m')))}$$

(4)

$$P_L(r | m) = \frac{P_S(m | r) \cdot P(r)}{\sum_{r' \in R} P_S(m | r') \cdot P(r')}$$

#### 3.2 Simplification where priors are flat, costs are 0, and $\alpha = 1$

(5)

$$P_{\text{Lit}}(r | m) = \frac{\llbracket m \rrbracket(r)}{\sum_{r' \in R} \llbracket m \rrbracket(r')}$$

(6)

$$P_S(m | r) = \frac{P_{\text{Lit}}(r | m)}{\sum_{m' \in M} P_{\text{Lit}}(r | m')}$$

(7)

$$P_L(r | m) = \frac{P_S(m | r)}{\sum_{r' \in R} P_S(m | r')}$$

#### 3.3 Connections to Grice

We don't *need* to reconstruct Grice's theory, but it's reassuring that we can make connections.

Grice	RSA
Quality	All agents assign 0 probability to false utterances.
Quantity	The speaker favors informative utterances.
Manner	The cost function $C$ .
Relevance	Basic RSA doesn't engage this directly, though the referent prior helps.

The recursive nature of RSA aligns with the definition of conversational implicature (the speaker believes that the listener believes ...).

## 4 Simple scalar implicature

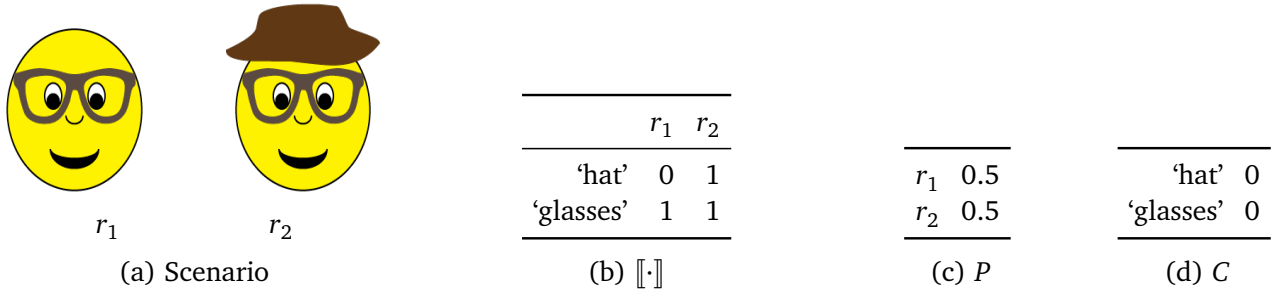


Figure 1: A communication game supporting a scalar implicature. For the calculations,  $\alpha = 1$ .

(8) a.

$P_{\text{Lit}}$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.5	0.5

$$P_{\text{Lit}}(r | m) = \frac{[[m]](r)}{\sum_{r' \in R} [[m]](r')}$$

b.

	'hat'	'glasses'
$r_1$	0	1
$r_2$	0.67	0.33

$$P_S(m | r) = \frac{P_{\text{Lit}}(r | m)}{\sum_{m' \in M} P_{\text{Lit}}(r | m')}$$

c.

	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.75	0.25

$$P_L(r | m) = \frac{P_S(m | r)}{\sum_{r' \in R} P_S(m | r')}$$

**Remarks** This captures the scalar implicature pattern that a general term will tend to exclude any more specific salient terms. It's not clear whether this is an explanation based on quantity (informativity) or ambiguity avoidance (manner), but perhaps the distinction doesn't matter here!

Here are the calculations above in more detail:

(9) Start with the lexicon:

	$r_1$	$r_2$
'hat'	0	1
'glasses'	1	1

(10) Normalize the rows (divide each value by the sum of the values in its row):

$P_{Lit}$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.5	0.5

(11) Transpose the matrix so that the states are along the rows:

	'hat'	'glasses'
$r_1$	0	0.5
$r_2$	1	0.5

(12) Normalize the rows:

$P_S$	'hat'	'glasses'
$r_1$	0	1
$r_2$	0.67	0.33

(13) Transpose:

	$r_1$	$r_2$
'hat'	0	0.67
'glasses'	1	0.33

(14) Normalize the rows:

$P_L$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.75	0.25

## 5 The role of message costs

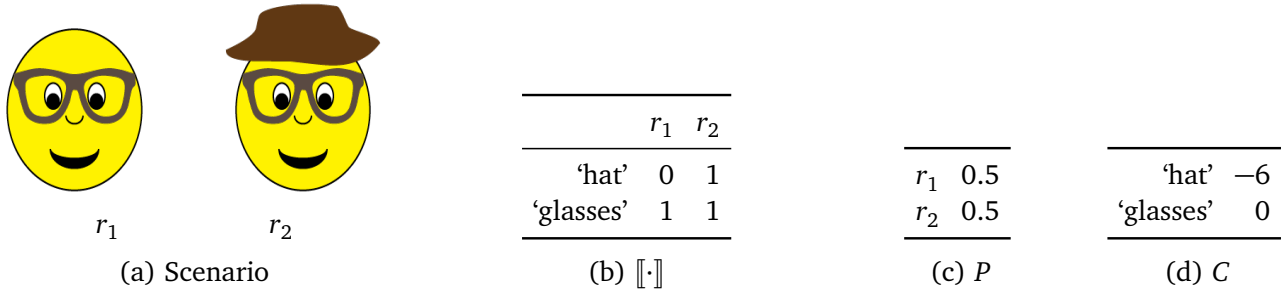
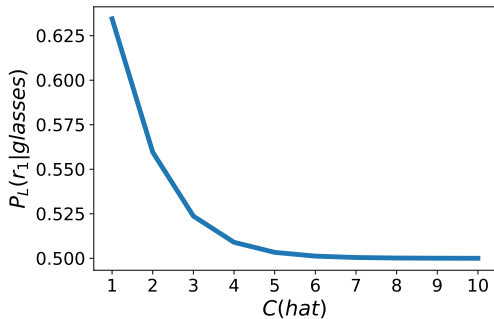
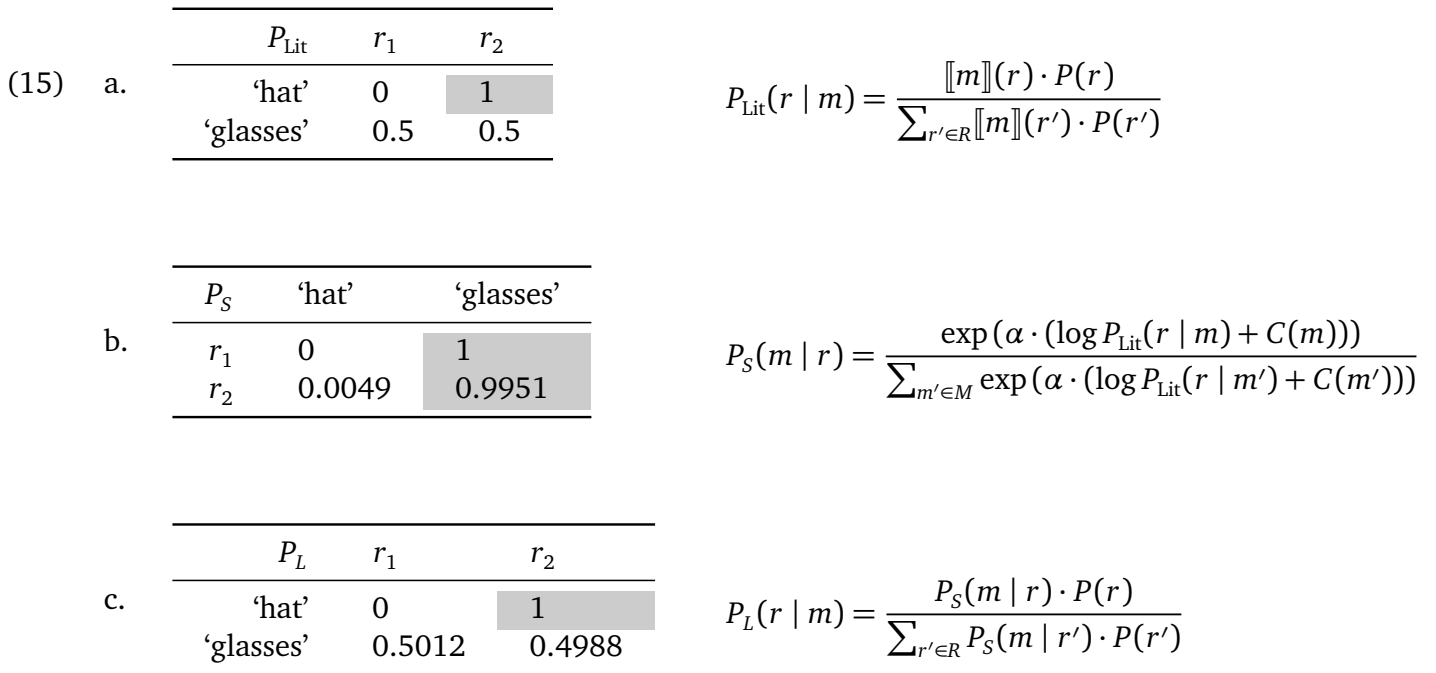


Figure 2: A communication game with very high costs on one message. For the calculations,  $\alpha = 1$ .



Increasing the cost of a term (here 'hat') lets us model situations where that term is regarded as marked – perhaps too marked to be used. In such situations, that term doesn't compete with the terms it entails, so the implicature disappears. In the figure, we see that, by  $C(\text{'hat'}) = -7$ ,  $P_L$  doesn't really treat 'glasses' as referring to  $r_1$ , because  $P_L$  expects 'glasses' to be the least marked way of referring to  $r_2$  as well.

Figure 3: Exploring the space of cost functions.

Here are the calculations above in more detail:

(16) Start with the lexicon:

	$r_1$	$r_2$
'hat'	0	1
'glasses'	1	1

(17) Normalize the rows:

$P_{\text{Lit}}$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.5	0.5

(18) Transpose:

	'hat'	'glasses'
$r_1$	0	0.5
$r_2$	1	0.5

(19) Take the log of the values, subtract the costs, and exponentiate:

	'hat'	'glasses'		'hat'	'glasses'	
$r_1$	$\exp(\log(0) - 6)$	$\exp(\log(0.5) - 0)$	$\Rightarrow$	$r_1$	0	0.5
$r_2$	$\exp(\log(1) - 6)$	$\exp(\log(0.5) - 0)$		$r_2$	0.0025	0.5

(20) Normalize the rows:

$P_S$	'hat'	'glasses'
$r_1$	0	1
$r_2$	0.0049	0.9951

(21) Transpose:

	$r_1$	$r_2$
'hat'	0	0.0049
'glasses'	1	0.9951

(22) Normalize the rows:

$P_L$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.5012	0.4988

## 6 The role of the the alpha parameter

The alpha parameter can be seen as controlling how much pragmatics we see. Larger  $\alpha$  results in stronger pragmatic inferences, and smaller  $\alpha$  corresponds to weaker pragmatic inferences. Only the speaker agent uses  $\alpha$  directly, but this means that the pragmatic listener is also affected by it.

Here’s a version of the speaker with  $\alpha$  but no cost terms, to simplify it a bit:

(23)

$$P_S(m | r) = \frac{\exp(\alpha \cdot (\log P_{Lit}(r | m)))}{\sum_{m' \in M} \exp(\alpha \cdot (\log P_{Lit}(r | m')))}$$

These two speaker matrices convey how large  $\alpha$  amplifies the pragmatics:

(24)

$P_S$	'hat'	'glasses'
$r_1$	0	1
$r_2$	0.67	0.33

$\alpha = 1$

(25)

$P_S$	'hat'	'glasses'
$r_1$	0	1
$r_2$	0.94	0.06

$\alpha = 4$



As  $\alpha$  gets bigger, the implicature gets stronger in the sense that  $P_L$  is more certain that ‘glasses’ must pick out  $r_1$  even though it is also true of  $r_2$ . Where message costs are all the same, this reduces to the effect of multiplying  $\alpha$  by the log of the  $P_{Lit}$  values, which affects the differences between values. For example,  $1 \log(0.75) - 1 \log(0.25)$  is five times smaller than  $5 \log(0.75) - 5 \log(0.25)$ .

Figure 4: The effect of the rationality parameter  $\alpha$  on the pragmatic listener.

## 7 The role of the referent prior

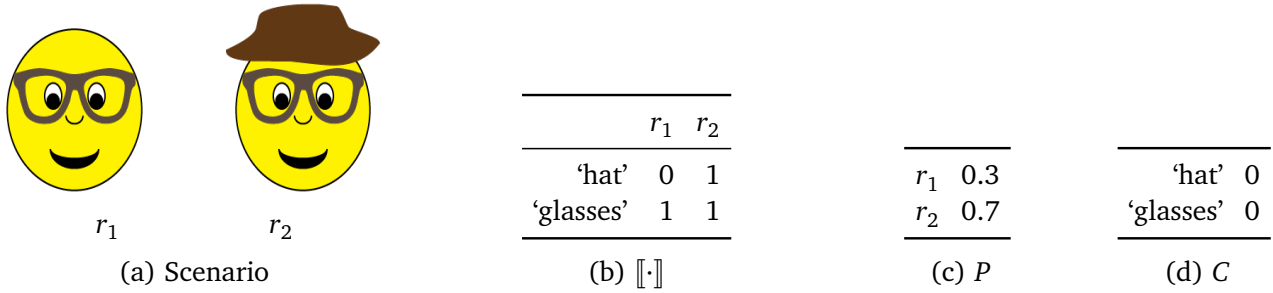
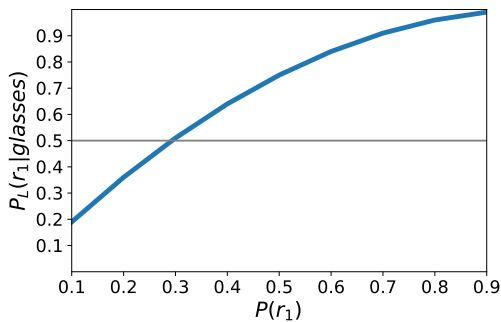
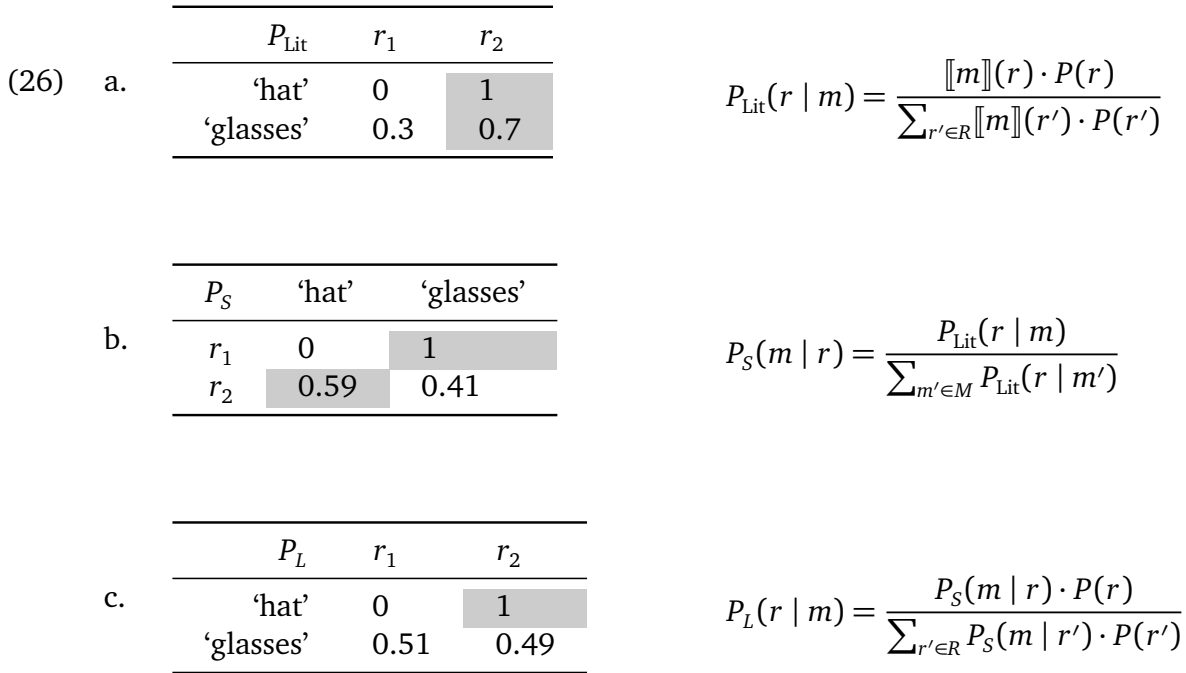


Figure 5: A communication game with skewed priors. For the calculations,  $\alpha = 1$ .



Decreasing the prior on one referent models a situation in which that referent is unlikely in the view of the discourse participants. As this value gets smaller, this referent becomes less relevant, and expected implicatures can disappear. In the figure, we see this happen: if  $P(r_1)$  is very low, hearing 'glasses' doesn't necessarily lead  $P_L$  to choose  $r_1$  as a referent, because  $r_1$  as a referent is so unlikely in general.

Figure 6: Exploring the space of referent priors.



Here are the calculations above in more detail:

(27) Start with the lexicon:

	$r_1$	$r_2$
'hat'	0	1
'glasses'	1	1

(28) Bring in the prior:

	$r_1$	$r_2$		$r_1$	$r_2$	
'hat'	$0 \cdot 0.3$	$1 \cdot 0.7$	$\Rightarrow$	'hat'	0	0.7
'glasses'	$1 \cdot 0.3$	$1 \cdot 0.7$		'glasses'	0.3	0.7

(29) Normalize the rows:

$P_{Lit}$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.3	0.7

(30) Transpose:

	'hat'	'glasses'
$r_1$	0	0.3
$r_2$	1	0.7

(31) Normalize the rows:

	'hat'	'glasses'
$r_1$	0	1
$r_2$	0.59	0.41

(32) Transpose:

$P_S$	$r_1$	$r_2$
'hat'	0	0.59
'glasses'	1	0.41

(33) Bring in the prior:

	$r_1$	$r_2$		$r_1$	$r_2$	
'hat'	$0 \cdot 0.3$	$0.59 \cdot 0.7$	$\Rightarrow$	'hat'	0	0.413
'glasses'	$1 \cdot 0.3$	$0.41 \cdot 0.7$		'glasses'	0.3	0.287

(34) Normalize the rows:

$P_L$	$r_1$	$r_2$
'hat'	0	1
'glasses'	0.51	0.49

## 8 Our in-class experiment

For a complete report on the experiment, see the experiment 3 packet:

<http://web.stanford.edu/class/linguist130a/surveys/ling130a-survey-2019-implicatures.zip>

### 8.1 Set-up

The materials were displayed on the classroom screen. Participants gave their responses on paper. The materials, response sheets, and raw response tables are available from the class website. 66 students participated.

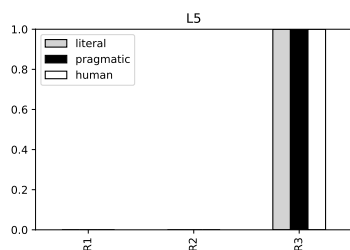
### 8.2 Design

The experiment itself involved 15 reference games. In phase 1 of the experiment, participants were in the Speaker role: presented with a scene of three potential referents, participants were asked to try to identify the target referent (boxed) from a highly restrictive vocabulary. The Speaker phase was short and intended mainly to get participants thinking about what it is like to be a speaker in this scenario. In phase 2, participants were in the Listener role. There were again three referents. Below them was a one-word message from the speaker. The task was to try to identify the speaker's intended referent based on that message.

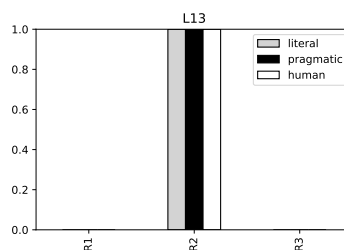
### 8.3 Rationale

The experiment allows us to do an informal assessment of the RSA model against data. In particular, in comparing the human responses with the predictions of a literal listener and a pragmatic one, we can support the claim that RSA-style pragmatic inference has value in explaining listener behavior.

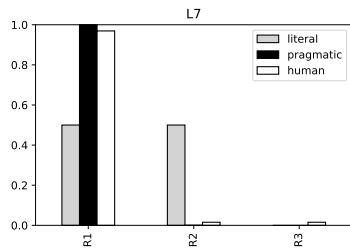
### 8.4 Selected items



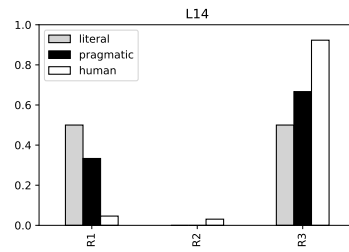
Note: purely truth-conditional



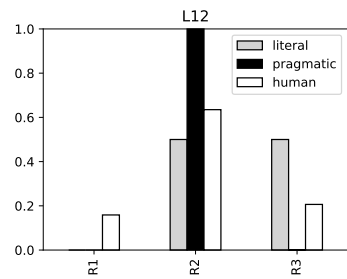
Note: purely truth-conditional



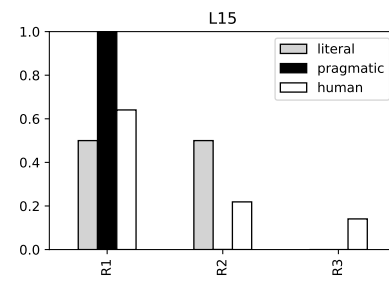
Note: basic scalar



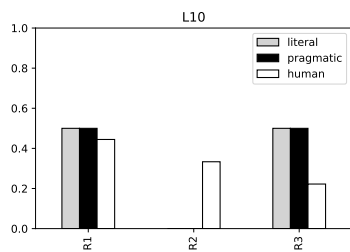
Note: basic scalar



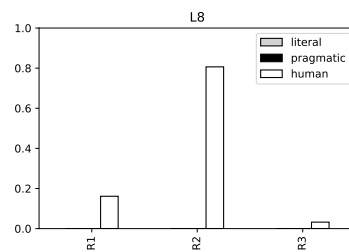
Note: complex scalar; expecting R2 because R1 is 'hat' and R3 is 'glass'



Note: complex; expecting R1 because R3 is 'mustache', creating scalar inference for R1 and R2



Note: unavoidable ambiguity



Note: impossible

## 8.5 Overall correlation

Over all items, the literal listener has a 0.68 correlation with the mean human responses, whereas the pragmatic listener has a 0.88 correlation with  $\alpha = 4$ .