Linguist 130a/230a: section, week 2

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1 Basic set-theoretic concepts

1.1 Definitions

- A set is a collection of objects.

  A set is notated by $\{\}$.

  For instance, {Richard Montague} is the set that contains Richard Montague.

  Alternatively, we can say that Richard Montague is a member or an element of the set {Richard Montague}.

  {{Richard Montague}} is the set that contains the set that contains Richard Montague.

- Sets can be empty.

  The empty set is notated by $\varnothing$.

  While $\varnothing$ is the empty set, $\{\varnothing\}$ is not the empty set.

  $\{\varnothing\}$ is the set that contains the empty set.

- A difference in the order of elements in a set doesn't change set identity.

  $\{1, 2, 3\} = \{2, 1, 3\}$

- As a set is a collection of objects, multiple occurrences of an element don't change set identity either.

  $\{1, 2\} = \{2, 2, 2, 1, 2\}$

- There are different ways of defining a given set.

We can define a set by enumeration notation, i.e., by enumerating all of its members: S = $\{2\}$

We can also define a set by predicate notation, i.e., by specifying a property for its members in defining the set:

S′ = $\{x$: x is an even prime$\}$

The above can be read as *S′ is the set that contains all x such that x is an even prime*.

Is S = S′?

- To be able to talk about set sizes and comparing them, we introduce the notion of cardinality.

  A = $\{1, 2, 3, 4\}$ contains more things than B = $\{1, 2\}$. In other words, A is larger than B.

  More specifically, A contains 4 things and B contains 2 things.

  Cardinality of a set is notated by $|\ \ |$.

  In other words, $|\{1, 2, 3, 4\}|$ tells you the set size of $\{1, 2, 3, 4\}$, which is four.

  Where A = $\{1, 2, 3, 4\}$, $|$A$|$ = 4.

  Where A = $\{1, 2, 3, 4\}$ and C = $\{100, 1000\}$, $|$A$| > |$C$|$.

1.2 $\in$, $\subset$, and $\subseteq$ relations

- $\in$ notates the membership relation.

  If x is a member of a set A, then it is true that $x \in$ A.

  For the set $\{1, 2\}$, it is true that $1 \in \{1, 2\}$.

  $\notin$ notates that $\in$ doesn't hold.

  For the set $\{1, 2\}$, it is true that $3 \notin \{1, 2\}$.

- $\subseteq$ notates the subset relation.

  For two sets A and B, A $\subseteq$ B iff for all x, if $x \in$ A, then $x \in$ B.

  For the set $\{1, 2\}$, it is false that $1 \subseteq \{1, 2\}$, as 1 is not a set.

  However, for sets $\{1\}$ and $\{1, 2\}$, it is true that $\{1\} \subseteq \{1, 2\}$

  $\subseteq$ is a reflexive relation. What does this mean? For any set A, it is true for A that A $\subseteq$ A.

$\nsubseteq$ notates that $\subseteq$ doesn't hold.

- $\subset$ notates the proper subset relation.

  For sets A and B, if A $\subseteq$ B and B $\nsubseteq$ A, then A $\subset$ B.

  Take $\{1, 2\}$ and $\{1\}$. $\{1\} \subseteq \{1, 2\}$, but $\{1, 2\} \nsubseteq \{1\}$. Therefore, $\{1\} \subset \{1, 2\}$.

  Is $\subset$ a reflexive relation?

  $\not\subset$ notates that $\subset$ doesn't hold.

1.3 Operations on sets

- $\mathcal{P}$(A) notates the power set of A.

  $\mathcal{P}$(A) is the set that contains all the subsets of A.

  If A = $\{1, 2\}$, then $\mathcal{P}$(A) = $\{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$

- A $\cap$ B is the intersection of A and B.

  A $\cap$ B = $\{x$: $x \in$ A and $x \in$ B$\}$

  Where A = $\{1, 2, 3\}$ and B = $\{1, 2\}$, A $\cap$ B = $\{1, 2\}$.

- A $\cup$ B is the union of A and B.

  A $\cup$ B = $\{x$: $x \in$ A or $x \in$ B$\}$

  Where A = $\{1, 2, 3\}$ and B = $\{3, 4, 5\}$, A $\cup$ B = $\{1, 2, 3, 4, 5\}$.

- The complement of a set X is the set of all those things that are not in X.

  Let the universe be such that it contains the entities, dax, wif, lug, and zup, and nothing else. Let A = {dax, wif}.

  Then the complement of A contains lug and zup, notated as $A^{\complement}$ = {lug, zup}.

- We can also talk about complements of a set in another set.

  Where A = $\{1, 2, 3, 4\}$ and B = $\{2, 3\}$, the complement of B in A, notated by A $-$ B, is the set of all things in A that are not in B.

  A $-$ B = $\{1, 4\}$.

2 Exercises

(1) The following sets are represented in the predicate notation. Convert them to the enumeration notation.

a. {3v: v is an even prime number}

b. {z: z is an integer and z $>$ 0 and z $<$ 10 and z^2 is a prime number}

c. {z: z is an integer and z $>$ 0 and z $<$ 5 and Barack Obama is a former US president}

(2) The following sets are represented in the enumeration notation. Convert them to the predicate notation.

a. $\{1, 2, 3, 4\}$

b. $\{2, 3, 5, 7\}$

c. {Barack Obama, Donald Trump, Joe Biden}

(3) Calculate the cardinality of each of the following sets:

a. $\{1, 2, 3, 4\}$

b. $\{1, 2, \{3, 4\}, 0\}$

c. $\varnothing$

d. $\{\varnothing\}$

e. $\{\{1, 2\}\}$

f. $\{\{1, 2, \{3, 4\}, 5\}, \{6, 7\}, 8\}$

g. {3v: v is an even prime number}

(4) Let A be an arbitrary set. Are the following always true?

a. $\varnothing \subset A$

b. $\varnothing \subseteq A$

c. $\varnothing \in A$

d. $A = \{y : y \in A\}$

e. $A = \{y : y \subseteq A\}$

(5) Write out the power set of each of the following sets:

a. $\{1, 2\}$

b. $\{1, 2, \{3, 4\}, 0\}$

c. $\varnothing$

d. $\{\varnothing\}$

(6) Calculate the following, writing out the answers in enumeration notation:

a. $\{1, 0, 3\} \cup \{1, 2, 3\}$
b. $\{1, 0, 3\} \cap \{2, 4\}$
c. $\{0, 1, 5\} - \{0, 1\}$
d. $\{0, 1, 5\} - \{2, 5, 8\}$
e. $\{y\colon y \text{ is not } 3\}^{C}$

(7) Suppose A - B = $\varnothing$. Are the following statements true?
a. $A \cup B = B$
b. $A \cap B = A$