The λ calculus was developed by Alonzo Church in the 1930s. It’s a simple and powerful way of notating and handling functions and computations (it might be familiar from programming). This handout is a brief non-comprehensive reference.

1 What is a λ(-term)?

- a variable \((x, y, f, P)\)
- a combination of a λ-term \(τ\) and a variable \(x\): \(λx[τ]\) or \(λx.τ\)
- a combination of two λ-terms where one is interpreted as a function \(τ\) and the other is an input \(σ\) to the function: \(τ(σ)\)
- nothing else is a λ-term

2 How to interpret a λ-term

\[λx.f(x)\]

"Give me an \(x\) and get back \(f\) applied to \(x\)"

\[λx.f(x)(y) \rightarrow f(y)\]

"Apply the function \(λx.f(x)\) to the input \(y\)”  reduces to  “\(f\) applied to the input \(y\)”
3 Some correspondences

Example 3.1. Two ways of writing the function that always returns 2:
(a) \( f(x) = 2 \)  
(b) \( \lambda y.2 \)

Example 3.2. Two ways of writing the squaring function:
(a) \( f(x) = x^2 \)  
\[ f(2) = 2^2 = 4 \]
(b) \( \lambda y.y^2 \)  
\[ \lambda y.y^2(2) = 2^2 = 4 \]

Example 3.3. Two ways of writing the \([\text{even}]\) function:
(a)  
\[ [\text{even}](x) = \begin{cases} 
T & \text{if } x \% 2 = 0 \\
F & \text{else} 
\end{cases} \]
(b) \( \lambda y.(T \text{ if } x \% 2 = 0, \text{ else } F) \)

**NB:** "\( x \% 2 \)" gives you the remainder when you divide \( x \) by 2

Example 3.4. Three ways of writing the \([\text{daughter}]\) function on the Simpson children:

\[ U = \left\{ \begin{array}{c} 
\begin{array}{c}
\end{array} \\
\begin{array}{c}
\end{array} \\
\begin{array}{c}
\end{array} \\
\end{array} \right\} \]

(a)  
\[ [\text{daughter}] = \begin{bmatrix} 
\Rightarrow T \\
\Rightarrow F \\
\Rightarrow T 
\end{bmatrix} \]

(b)  
\[ [\text{daughter}](x) = \begin{cases} 
T & \text{if } x \in \left\{ \begin{array}{c} 
\begin{array}{c}
\end{array} \\
\begin{array}{c}
\end{array} \\
\end{array} \right\} \\
F & \text{else} 
\end{cases} \]

(c) \( \lambda x. \left( T \text{ if } x \in \left\{ \begin{array}{c} 
\begin{array}{c}
\end{array} \\
\begin{array}{c}
\end{array} \\
\end{array} \right\}, \text{ else } F \right) \)