Some formal analyses of determiners

Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2024

This handout provides some model answers for the technical questions on Assignment 4. We hope also that it helps to further illuminate the determiner properties of intersectivity, conservativity, and monotonicity.

1 Non-intersectivity of *all but two*

Here is a possible (though not necessarily empirically correct) definition of the phrasal determiner *all but two* as in *all but two students passed*:

$$[\![\textit{all but two}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X - Y| = 2, \text{ else } \mathsf{F}\Big)\Big)$$

A determiner D is intersective iff $D(A)(B) = D(B)(A)$ for all A, B. This determiner is not intersective.

Consider $A = \{a, b, c\}$ and $B = \{a\}$. Then

$$[\![\textit{all but two}]\!](A)(B) = \mathsf{T} \text{ if } |A - B| = 2, \text{ else } \mathsf{F}$$

resolves to T, since $A - B = \{b, c\}$. However,

$$[\![\textit{all but two}]\!](B)(A) = \mathsf{T} \text{ if } |B - A| = 2, \text{ else } \mathsf{F}$$

resolves to F, since $B - A = \emptyset$, which has cardinality 0. For intersectivity, all it takes is one failure of entailment in one direction to establish that the determiner is not intersective, so our job is done.

2 Monotonicity of the first argument to *few*

Here is a possible (though not necessarily empirically correct) definition of the determiner $[\![\textit{few}]\!]$:

$$[\![\textit{few}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| < j, \text{ else } \mathsf{F}\Big)\Big)$$

where $j > 0$ is a pragmatic free variable (presumably set to a very small integer, though the size might depend on the nature of the first argument).

A determiner D is downward monotone on its first argument iff $D(A)(B)$ entails $D(X)(B)$ for all A, B, X where $X \subseteq A$. We can show that the first argument slot for $[\![\textit{few}]\!]$ is downward monotone.

Assume $[\![\textit{few}]\!](A)(B) = \mathsf{T}$ for arbitrary A and B, with j also set to some value. Then $|A \cap B| < j$ holds. Moving to a subset X of A can only make $|X \cap B| \leqslant |A \cap B|$, so truth is preserved no matter how j is set, and hence $[\![\textit{few}]\!](X)(B) = \mathsf{T}$.

3 Non-monotonicity of the first argument to *between 2 and 4*

Here's a proposed meaning for the phrasal determiner *between 2 and 4*:

$$[\![\textit{between 2 and 4}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } 2 \leqslant |X \cap Y| \leqslant 4,\ \text{else } \mathsf{F}\big)\Big)$$

This determiner is nonmonotone on its first argument. Let $A = \{a, b, c, d, e\}$ and $B = \{b, c, d, e, f\}$. Then

$$[\![\textit{between 2 and 4}]\!](A)(B) = \mathsf{T} \text{ if } 2 \leqslant |A \cap B| \leqslant 4,\ \text{else } \mathsf{F}$$

resolves to T because $A \cap B = \{b, c, d, e\}$, which has cardinality 4.

Now suppose we take $X = \{a, b, c, d, e, f\}$. This is a superset of A, but $X \cap B = \{b, c, d, e, f\}$, which has cardinality 5. This shows that the determiner is not upward monotone on the first argument.

Now suppose we set $X = \{b\}$. This is a subset of A, but $X \cap B = \{b\}$, which has cardinality 1. This shows that the determiner is not downward on the first argument.

Since $[\![\textit{between 2 and 4}]\!]$ is neither upward nor downward monotone on its first argument, we conclude that it is nonmonotone on its first argument.

4 Conservativity of *not every*

Here is a proposed meaning for the phrasal determiner *not every*;

$$[\![\textit{not every}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } X \nsubseteq Y,\ \text{else } \mathsf{F}\big)\Big)$$

A determiner D is conservative iff $D(A)(B) = D(A)(A \cap B)$ for all A, B. This determiner is conservative.

To see this, first assume $[\![\textit{not every}]\!](A)(B) = \mathsf{T}$ for arbitrary sets A and B. Then we have that $A \nsubseteq B$. This means there is at least one x such that $x \in A$ but $x \notin B$. Any such x is also not in $A \cap B$ (because that would require $x \in B$), so $A \nsubseteq (A \cap B)$ holds, and thus $[\![\textit{not every}]\!](A)(A \cap B) = \mathsf{T}$.

For the other direction: assume $[\![\textit{not every}]\!](A)(A \cap B) = \mathsf{T}$. Then $A \nsubseteq (A \cap B)$ holds. This means there is at least one x such that $x \in A$ but $x \notin (A \cap B)$. Since we know $x \in A$, it must be that $x \notin B$, and thus we have $A \nsubseteq B$, which means $[\![\textit{not every}]\!](A)(B) = \mathsf{T}$.

5 A (non-existent) non-conservative determiner

Consider the hypothetical determiner $[\![\textit{somenon}]\!]$:

$$[\![\textit{somenon}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } ((U - X) \cap Y) \neq \emptyset,\ \text{else } \mathsf{F}\big)\Big)$$

This hypothetical determiner is not conservative. To see this, we can just note that

$$[\![somenon]\!](A)(A\cap B) = \mathsf{T} \text{ if } ((U - A)\cap(A\cap B)) \neq \emptyset, \text{ else } \mathsf{F}$$

always resolve to F, since $(U - A)\cap A = \emptyset$ and this is preserved under intersection (of either side). Thus, any situation in which $[\![somenon]\!](A)(B)$ is true will work as a counterexample to conservativity. For example, suppose the universe $U = \{a, b\}$, $A = \{a\}$, and $B = \{b\}$. Then

$$[\![somenon]\!](A)(B) = \mathsf{T} \text{ if } \big(\{b\}\cap\{b\}\big) \neq \emptyset, \text{ else } \mathsf{F}$$

which resolves to T, but

$$[\![somenon]\!](A)(A\cap B) = \mathsf{T} \text{ if } \big(\{b\}\cap\{a\}\cap\{b\}\big) \neq \emptyset, \text{ else } \mathsf{F}$$

which resolves to F.

6 Where *ever* can appear

The English adverbial particle *ever* has a highly restricted distribution. On the basis of the following examples (where * marks ungrammatical cases, as usual), formulate a generalization in terms of the monotonicity properties of determiners about where *ever* can appear:

(7)
a. No [$_{NP}$ students who have ever taken semantics] [$_{VP}$ have been to Peru]
b. No [$_{NP}$ students] [$_{VP}$ have ever been to Peru]
c. *Some [$_{NP}$ students who have ever taken semantics] [$_{VP}$ have been to Peru]
d. *Some [$_{NP}$ students] [$_{VP}$ have ever been to Peru]
e. At most three [$_{NP}$ students who have ever taken semantics] [$_{VP}$ have been to Peru]
f. At most three [$_{NP}$ students] [$_{VP}$ have ever been to Peru]
g. Exactly three [$_{NP}$ students who have ever taken semantics] [$_{VP}$ have been to Peru]
h. Exactly three [$_{NP}$ students] [$_{VP}$ have ever been to Peru]
i. Every [$_{NP}$ student who has ever taken semantics] [$_{VP}$ has been to Peru]
j. *Every [$_{NP}$ student] [$_{VP}$ has ever been to Peru]

Please restrict your attention to this set of examples when formulating your generalization, and accept the grammaticality judgments as given (even if you disagree with them).

Note: I've used square bracketing to indicate the basic syntactic structure of these cases. In all cases, the string inside [$_{NP}$ ...] corresponds to the restriction of the determiner semantically, and the string inside [$_{VP}$ ...] corresponds to the scope of the determiner semantically.

Model answer

The correct generalization is that *ever* can appear only in environments that are **not upward monotone**.

The *no* and *at most three* cases both involve downward monotone environments, and they are good. The *some* cases involve upward monotone environments, and they are bad. The *exactly three* cases are neither upward nor downward, i.e., non-monotone, and they are good. Finally, *every*'s restriction is downward and licenses *ever*, whereas its scope is upward and does not license *ever*. Thus, **not upward monotone** seems to be the correct generalization for this dataset.

7 Other questions

(1) Prove that *exactly three* is intersective.

(2) Prove that *exactly three* is conservative.

(3) Prove that *not every* is upward monotone on its first argument.