The concept of a set

Chapter 1

Basic Concepts of Set

Theory
1.2 Specification of sets

The notation that numbers could be written as the union of sets, e.g., \( A \cup B \cup C \), is the first president of the United States and the number three. The president of the United States and the number three are the names of sets. The names of sets are written in a line, separated by commas: \( \{ A, B, C \} \). The set containing the members of a set is written in parentheses: \( (A, B, C) \).

The notation \( A \cup B \cup C \) or \( (A, B, C) \) describes each method separately.

There are three distinct ways to specify a set: (1) listing all its members, (2) describing a set by stating a property which all of its members have to qualify as a member, and (3) describing a set by defining a set with rules which determine its members. We

The combination of numbers are written with lower case letters. Sometimes the same letter stands for different numbers, e.g., \( a, b, c \). According to the set notation, a member of set \( A \) is an element of set \( B \). A subset is a set that is a member of a larger set. We denote the empty set by \( \emptyset \). The union of sets is the set of all elements that are in at least one of the sets: \( A \cup B \). The intersection of sets is the set of all elements that are in both sets: \( A \cap B \). The difference of sets is the set of all elements that are in one set but not the other: \( A \setminus B \).

We denote the union of sets by \( A \cup B \), the intersection of sets by \( A \cap B \), and the difference of sets by \( A \setminus B \).

For sets \( A \) and \( B \), \( A \subseteq B \) if every element of \( A \) is also an element of \( B \). If \( A \subseteq B \) and \( B \subseteq A \), then \( A = B \). If \( A \subseteq B \), then \( B \supseteq A \). If \( A \subseteq B \) and \( B \subseteq C \), then \( A \subseteq C \).

Mathematical statements of theorems often begin with the phrase "Let \( A \) be a set." The set of all integers is denoted by \( \mathbb{Z} \). The set of all real numbers is denoted by \( \mathbb{R} \). The set of all complex numbers is denoted by \( \mathbb{C} \).

The notation \( \mathbb{Z} \) or \( \mathbb{R} \) or \( \mathbb{C} \) denotes a set of numbers. For example, \( \mathbb{Z} \) denotes the set of all integers. The set of all integers is denoted by \( \mathbb{Z} \). The set of all real numbers is denoted by \( \mathbb{R} \). The set of all complex numbers is denoted by \( \mathbb{C} \).

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It is a question of the type of set.

Let us consider a set of numbers. We may define the property of being a subset of a given set. The property of being a subset of a given set is not a property of the set itself. However, it is a property of the members of the set.

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Se-theroretic Identities and Cardinality

1.3 Se-theroretic Identities and Cardinality

Both methods are commonly used. Both methods of specialization work on set theory itself, which is not a natural number. We will take up the subject of infinite sets of the same cardinality. The set of all natural numbers is the same cardinality as the set of all integers, and since the set of all rational numbers is the same cardinality as the integers, the set of all real numbers is the same cardinality as the integers.

In Chapter 2, we will consider the set of all real numbers. The set of all real numbers is the same cardinality as the integers. The set of all real numbers is the same cardinality as the integers. The set of all real numbers is the same cardinality as the integers. The set of all real numbers is the same cardinality as the integers.

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We now introduce two operations which take a pair of sets and produce

\[ \text{1.6 Union and Intersection} \]

\[ \text{Power sets} \]

\[ \text{Sets and Subsets} \]
The intersection of \( A \) and \( B \), written \( A \cap B \). In predicate notation this

members are just the members of both \( A \) and \( B \). This operation is called

The second operation on arbitrary sets \( A \) and \( B \) produces a set whose


sets \( A \) and \( B \)


\[
T = \{p, q\} = \emptyset \cap T
\]

\[
Y = \{q, r\} = \emptyset \cap Y
\]

\[
\{p, q\} = (W \cap T) \cap Y = W \cap (T \cap Y)
\]

\[
\{q, r\} = W \cap T
\]

\[
\{p, q\} = W \cap Y
\]

\[
\{p, q\} = T \cap Y
\]

Chapter 6, Section 2. For example,

Note that the disjunction, \( \{ x \mid x \in A \lor x \in B \} \) for any object to be a member

are just the objects which are members of \( A \) or \( B \) or of both. In the

The union of two sets \( A \) and \( B \), written \( A \cup B \), is the set whose members


Some more examples:

Figure 1.2: The Venn diagram for the set-theoretic difference \( A - B \) is shown in the text.

The Venn diagram for the set-theoretic difference \( A - B \) is shown in the text.

The set \( A - B \) is defined as the set of elements that are in \( A \) but not in \( B \). It is the complement of \( B \) relative to \( A \).

\[ \{ x \in A : x \notin B \} \]

The Venn diagram for \( A - B \) shows how \( A \) and \( B \) are related.

Another binary operation on arbitrary sets \( A \) and \( B \) is the difference, written \( A - B \).

Figure 1.3: Set-theoretic intersection \( A \cap B \)

The intersection of two sets \( A \) and \( B \) is the set of elements that are in both \( A \) and \( B \). It is the set-theoretic intersection of \( A \) and \( B \).

\[ \{ x \in A : x \in B \} \]

The Venn diagram for \( A \cap B \) shows how \( A \) and \( B \) overlap.

The general case of intersection of arbitrary sets \( A \) and \( B \) is represented by the Venn diagram of Figure 1.3.

Figure 1.4: The intersection of \( A \cap B \) and \( C \)

The intersection of three sets \( A \), \( B \), and \( C \) is the set of elements that are in all three sets. It is the set-theoretic intersection of \( A \), \( B \), and \( C \).

\[ \{ x \in A : x \in B : x \in C \} \]

The Venn diagram for \( A \cap B \cap C \) shows how \( A \), \( B \), and \( C \) overlap.

Figure 1.5: The intersection of \( A \cap B \) and \( C \)

The intersection of two sets \( A \) and \( B \) is the set of elements that are in both \( A \) and \( B \). It is the set-theoretic intersection of \( A \) and \( B \).

\[ \{ x \in A : x \in B \} \]

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Figure 1.6: The intersection of \( A \cap B \) and \( C \)

The intersection of three sets \( A \), \( B \), and \( C \) is the set of elements that are in all three sets. It is the set-theoretic intersection of \( A \), \( B \), and \( C \).

\[ \{ x \in A : x \in B : x \in C \} \]

The Venn diagram for \( A \cap B \cap C \) shows how \( A \), \( B \), and \( C \) overlap.
I.8 Set-theoretic equalities

\[
\forall x \in \mathbb{N}, \exists y \in \mathbb{N} \mid x + y = 10
\]

Figure 1-6: The set-theoretic complement \( A' \)

\[
\text{Figure 1-5: Set-theoretic difference } A - B
\]

Chapter 1
\( A = A \cap X \) (representing \( A \subseteq X \))

We think of \( A \) as a subset of \( X \), for which the entire set \( X \) is the entire universe, and the union of both \( A \) and \( X \) is in \( X \).

The Consistency Principle is so called because it is concerned with the properties of the universe of sets.

\[ X = A \cup X \cap A \subseteq X \quad (q) \]
\[ X = \neg \neg X \cap A \subseteq X \quad (a) \]
\[ \neg \neg X \cap A \subseteq X \quad (a) \]

De Morgan's Laws

\[ A \cap X = (A \cup X) \quad (q) \]
\[ A \cup X = (A \cap X) \quad (q) \]

Commutative Laws

\[ A \cap X = X \cap A \quad (q) \]
\[ A \cup X = X \cup A \quad (q) \]

Distributive Laws

\[ (A \cup X) \cap X = (A \cap X) \cap X \quad (q) \]
\[ (A \cap X) \cup X = (A \cup X) \cup X \quad (q) \]

The relationship between these laws is that the order in which the set \( X \) is combined with the other sets is the same as the order in which the other sets are combined with \( X \).

\[ X \cap X = X \quad (q) \]
\[ X \cup X = X \quad (q) \]

Chapter 1

Set-Theoretic Equivalences
example, in set theory, we may replace \((B \cap C)(B \cup C)\), as the operation of intersection followed by union, with the operation of union followed by intersection, \((B \cup C)(B \cap C)\). The result will then be an equivalent expression. The idea is that in any set-theoretic expression, the order in which the operations are performed may be irrelevant, as long as the order of the operands is maintained.

For the moment our concern is not with the structure of the algebra.

For the moment, our concern is not with the structure of the algebra.

What we have seen is that there is an algebra of sorts which is

\[(x \cdot x) + (k \cdot x) = (x + k) \cdot x\]

for all numbers \(x, k, h, z\).

However, there is a distributive law relating + and \(\ast\), as follows:

\[(x \cdot (k + l)) = (x \cdot k) + (x \cdot l)\]

for all numbers \(x, k, l, h, z\).

and an associative law:

\[(k \cdot (x \cdot l)) = (k \cdot x) \cdot l\]

for all numbers \(x, k, l, h, z\).

Now a commutative law:

\[(k \cdot x) = (x \cdot k)\]

for all numbers \(x, k, l, h, z\).

(apply a commutative law)

Note from the above, the operations of addition and multiplication are not associative or commutative.

Members of the algebra are \(\{x \in X | \phi(x)\}\) and \(\{x \in X | \psi(x)\}\), where \(\phi(x)\) and \(\psi(x)\) are members of the algebra. The corresponding elements in the diagrams are labeled according to their membership in the algebra. A member of the algebra \(X\) may be empty or have elements, and \(\phi(x)\) and \(\psi(x)\) are the corresponding elements. The union of two members of the algebra \(X\) is the set of elements that are in either or both members.

Figure 1-8: Venn diagram for

\[(Z \subseteq X) \land (\lambda \subseteq X)\]

\[(Z \cup X) \cap (\lambda \cup X)\]

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Exercises

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1. Given the sets A, B, and C as in Exercise 1, list the members of each of the following sets by listing the members:

   (i) A ∪ C
   (ii) A ∩ C
   (iii) (A ∪ C) ∩ (A ∩ C)
   (iv) (A ∪ C) ∩ (A ∩ C)

2. Given the sets A, B, and C as in Exercise 1, list the members of each of the following sets by listing the members:

   (i) A ∪ B
   (ii) A ∩ B
   (iii) (A ∪ B) ∩ (A ∩ B)
   (iv) (A ∪ B) ∩ (A ∩ B)

3. Write a specification by rules and one by predicates for each of the following sets:

   (i) {x | x ∈ A and x ∈ B}
   (ii) {x | x ∈ A or x ∈ B}
   (iii) {x | x ∈ A and x ∈ C}
   (iv) {x | x ∈ A or x ∈ C}

4. Consider the following sets:

   (i) {x | x ∈ A and x ∈ B}
   (ii) {x | x ∈ A or x ∈ B}
   (iii) {x | x ∈ A and x ∈ C}
   (iv) {x | x ∈ A or x ∈ C}

5. Specify each of the following sets by listing the members:

   (i) A ∪ B
   (ii) A ∩ B
   (iii) (A ∪ B) ∩ (A ∩ B)
   (iv) (A ∪ B) ∩ (A ∩ B)

6. Show by using the set-theoretic equations in Figure 1.7 for any sets A, B, and C:

   (a) A ∪ B = (A ∩ C) ∪ (B ∩ C)
   (b) A ∩ B = (A ∩ C) ∩ (B ∩ C)
   (c) A ∪ B = (A ∩ C) ∪ (B ∩ C)
   (d) A ∩ B = (A ∩ C) ∩ (B ∩ C)

7. Let A = {1, 2, 3, 4} and B = {3, 4, 5, 6}.

   (a) A ∪ B
   (b) A ∩ B
   (c) A ∪ B
   (d) A ∩ B

8. Let A = {1, 2, 3, 4}, and C = {1, 2, 3, 4}. What is the union of A and C?

   (a) A ∪ C
   (b) A ∩ C
   (c) A ∪ C
   (d) A ∩ C

9. Show by explicit construction that the set consisting of the first 100 positive integers is identical to the set of all even integers.

   (a) Which are members of 51?
   (b) Which are members of 59?
   (c) Which are members of 91?
   (d) Which are members of 99?

10. Show by explicit construction that the set consisting of the first 100 positive integers is identical to the set of all even integers.

   (a) Which are members of 51?
   (b) Which are members of 59?
   (c) Which are members of 91?
   (d) Which are members of 99?
Axiom of Comprehension, restricted to one type. It states that for any set $A$ and any property $P(x)$, the set of all elements $x$ of $A$ that have property $P$ is also a set. In symbols:

\[ \forall A \exists B \left( \forall x \left( x \in B \iff \exists y \left( y \in A \land P(y) \right) \right) \right) \]

We can denote this set as $x : P(x)$.

12. Call adjectives which are correctly predicated of themselves autologic.

\[ B \subseteq A \iff (A \cup \neg A) \subseteq (A \cup \neg A) \]

Show that $A$ is a subset of the universe if and only if $A$ is the universe.

\[ A + A = (A \cup A) \]

Show that if $A$, then $A$ is a subset of the universe if and only if $A$ is the universe.

\[ \emptyset + A = (\emptyset \cup A) \]

Show that if the empty set is a subset of $A$, then $A$ is the universe.

\[ (A \cup A) + (A \cup A) = (A \cup A) \]

Show that the universe is a subset of itself.

\[ (A \cup A) \subseteq (A \cup A) \]

Show that the universe is a subset of the universe.

\[ (A \cup A) + (A \cup A) = (A \cup A) \]

Show that the universe is a subset of the universe.

Express each of the following in terms of union, intersection, and complementation, and simplify using the set-theoretic equations:

\[ A + B = \emptyset \cup A, A \cup B \]

Show that $A$ and $B$ are disjoint if and only if $A \cap B = \emptyset$.

For $A$ and $B$, $A \cup B$ is the set of all elements that are in $A$, $B$, or both.

\[ (A \cup B) \cap (A \cap B) = A \]

Show that the intersection of the union with the intersection is equal to the set $A$.

\[ (B \cup A) \cap (B \cap A) = B \]

Show that the intersection of the union with the intersection is equal to the set $B$.

\[ (A \cup B) \cap (A \cap B) = A \]

Show that the intersection of the union with the intersection is equal to the set $A$.

\[ (B \cup A) \cap (B \cap A) = B \]

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Show that the intersection of the union with the intersection is equal to the set $A$.

\[ (B \cup A) \cap (B \cap A) = B \]

Show that the intersection of the union with the intersection is equal to the set $B$. 

11. The symmetric difference of two sets $A$ and $B$, denoted $A \triangle B$, is the set of elements which are in $A$ or $B$ but not in both.

\[ (A \cap B) \cup (A \triangle B) \]

The symmetric difference of two sets $A$ and $B$ is the set of all elements which are in $A$ or $B$ but not in both.

10. Show that the distributive law (4) is true by constructing Venn-diagrams for $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

11. Show that the distributive law (4) is true by constructing Venn-diagrams for $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.