1. Meanings and Context

LECTURE 1
Set Theory

1. Set Theory Concepts

(1) Basic notions about sets
(a) A set is a group of objects. Any group objects \( a, b, c \) can form a set. The basic way to represent a set is \( \{a, b, c\} \).
(b) If an object \( x \) is a member of a set \( A \), it is denoted by \( x \in A \). If \( b \) is not a member of \( A \), it is denoted by \( x \notin A \).
(c) Sets can include other sets: \( \{a, b, \{c, d\}, \{e, f, g\}\}, \{\{a, b\}, \{a, b\}\} \).
Sets are determined only by their members, not their order.
(d) Sets which contain only one member are called singletons:
\( \{a\}, \{\{a, b\}\} \) are singletons.
(e) A set which contains no members is called the empty set or the null set and is denoted by \( \{\}, \emptyset \).

(2) Relations between sets
(a) If two sets \( A, B \) do not share any common members, they are called disjoint sets: \( \{a, b\} \) and \( \{c, d\} \) are disjoint.
(b) If every member of a set \( A \) is a member of a set \( B \) than \( A \) is a subset of \( B \) and \( B \) is a superset of \( A \). It is denoted by \( A \subseteq B \) and \( B \supseteq A \) respectively. Examples: \( \{a, b, c\} \) is a subset of \( \{a, b, c, d, e\} \). Every set is a subset and a superset of itself: for every set \( A \), \( A \subseteq A \) and \( A \supseteq A \). Every set is a superset of the empty set: for every set \( A \), \( \emptyset \subseteq A \).

(3) Operations in sets
(a) For two sets $A,B$, the smallest set that contains elements of both sets is called the **union** of $A,B$ and denoted by $A \cup B$. For example, \{a, b, c\} $\cup$ \{c, d, e\} = \{a, b, c, d, e\}

(b) For two sets $A,B$, the largest set that includes only common elements of $A,B$ is called the **intersection** of $A,B$ and denoted by $A \cap B$. For example, \{a, b, c, d\}$\cap$\{c, d, e, f\} = \{c, d\}

4. Ways to define sets
   (a) Use capital letters to refer to particular well-known sets: $N$ is the set of all natural numbers, $R$ is the set of rational numbers.
   (b) Abstraction notation: $\{x | \phi(x)\}$ is the set of all entities $x$ such that $\phi(x)$ is true. For example, $A = \{x | \phi(x)\}$, where $\phi(x) = x$ lives in London or $\phi(x) = x \geq 5$.

2. Practice

2.1. Write the following in set notation

   (1) $d$ is a member of $B$
   (2) the intersection of $B$ and $C$
   (3) $A$ is a proper subset of $B$
   (4) the set of all linguists

2.2. Translate the following into idiomatic English

   (1) $\{x | x$ is an Englishman and $x$ studies philosophy$\}$
   (2) $\{x | x$ is a singer and John invited $x$}$
   (3) $\{x | x$ is an American and $x$ is a linguist$\}$

2.3. Indicate whether the following statements are true or false

   (1) $a \in \{b, a, f\}$
   (2) $f \notin \{b, a, f\}$
   (3) \{a, b, c\} $\subseteq$ \{c, b, a\}
   (4) \{a, b, c\} $\subset$ \{a, b, c\}
   (5) \{a\} $\notin \emptyset$
   (6) \{e, f\} $\subset$ \{d, e, f\}

2.4. Given the following sets: $A = \{2, 4, 5\}; B = \{a, b, c\}; C = \{2, 3, f\}$, what are the following:

   (1) $A \cup C$
   (2) $A \cap B$
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3. Homework

3.1. Write the following using set notation:
(1) $f$ is not an element of the union of $A$ and $C$
(2) $A$ is a set which consists of the members 2, 4, 5
(3) $B$ is included in $D$
(4) The empty set is a subset of $A$
(5) The set of all the musicians in the Israel Philharmonic Orchestra

3.2. Indicate whether the following statements are true or false regarding the sets $A = \{k, m, 3, 7\}; B = \{f, g, 7, 3, e\}$
(1) $k$ is an element of $A \cap B$
(2) $k$ is an element of $A \cup B$
(3) $A \cap B$ has two elements
(4) $\{3, 7\} \subset A$
(5) $\{3, 7\} \subseteq A$
(6) $\{3, 7\} \subset (A \cap B)$
(7) $\{m, g\} \subset (A \cup B)$
(8) $\{f, g\} \subset (B - A)$

3.3. Given the following sets: $A = \{8, d, f\}; B = \{1, 7, f, d\}; C = \{a, b, 7, e\}$ what are the following?
(1) $A \cap B$
(2) $A \cup C$
(3) $B \cup \emptyset$
(4) $B - A$
(5) $C \cup (A \cap B)$