Groenendijk’s (1999) Logic of Interrogation (LOI) is a dynamic statement of the partition semantics for questions, with a pragmatic theory layered atop it. The formulation is game-theoretic in the intuitive, but not in the technical, sense.

1 The logic

Definition 1 (Contexts). Let \( W \) be the set of worlds. A context is an equivalence relation on \( W \).

Definition 2 (Context change potentials). Let \( C \) be an equivalence relation on \( W \times W \).

- \( C[\varphi!] = \{ (w, w') \in C \mid [\varphi]^g,w = [\varphi]^{g,w'} = T \} \) (declaratives)
- \( C[\varphi?] = \{ (w, w') \in C \mid [\varphi]^g,w = [\varphi]^{g,w'} \} \) (interrogatives)
- Let \( \tau = \varphi_1 \ldots \varphi_n \). Then \( C[\tau] = C[\varphi_1] \ldots C[\varphi_n] \).

(1) a. \( W = \{ w_1, w_2, w_3, w_4 \} \) \( [p] = \{ w_1, w_2 \} \) \( [q] = \{ w_2, w_3 \} \)

b. \[
\begin{align*}
&\{ (w_1, w_1), (w_1, w_2), (w_1, w_3), (w_1, w_4), \\
& (w_2, w_1), (w_2, w_2), (w_2, w_3), (w_2, w_4), \\
& (w_3, w_1), (w_3, w_2), (w_3, w_3), (w_3, w_4), \\
& (w_4, w_1), (w_4, w_2), (w_4, w_3), (w_4, w_4) \}
\end{align*}
\]

c. \[
(W \times W)[?p] = \{ (w_1, w_1), (w_1, w_2), (w_1, w_3), (w_1, w_4), \\
(w_2, w_1), (w_2, w_2), (w_2, w_3), (w_2, w_4), \\
(w_3, w_1), (w_3, w_2), (w_3, w_3), (w_3, w_4), \\
(w_4, w_1), (w_4, w_2), (w_4, w_3), (w_4, w_4) \}
\]

d. \[
(W \times W)[?q] = \{ (w_1, w_1), (w_2, w_2), (w_3, w_3), (w_4, w_4), \\
(w_2, w_1), (w_2, w_2), (w_2, w_3), (w_2, w_4), \\
(w_3, w_1), (w_3, w_2), (w_3, w_3), (w_3, w_4), \\
(w_4, w_1), (w_4, w_2), (w_4, w_3), (w_4, w_4) \}
\]

e. \[
(W \times W)[?(p \land q)] = \{ (w_1, w_1), (w_1, w_3), (w_1, w_4), \\
(w_2, w_1), (w_2, w_3), (w_2, w_4), \\
(w_3, w_1), (w_3, w_3), (w_3, w_4), \\
(w_4, w_1), (w_4, w_3), (w_4, w_4) \}
\]
2 Discourse game

In Groenendijk’s (1999) simple game of interrogation, there are two players: interrogator and witness. The interrogator asks questions and the witness answers them. The paper is not explicit about the informational asymmetry between witness and interrogator, but the following seems in keeping with the descriptions and examples:

(2) a. The witness and interrogator each have their own contexts, $C_w$ and $C_i$
    b. The witness’s context does not change. The interrogator’s does: he asks questions, the witness answers them, and the interrogator faithfully updates $C_i$ with the answer.
    c. The game finishes when $C_w = C_i$.

Definition 3 (“Groengrice maxims”). Groenendijk’s (1999) definition of pertinence: $\varphi$ is pertinent after $\tau$ iff $\varphi$ consistent, informative, and licensed wrt $\tau$:

i. $\varphi$ is consistent with $\tau$ iff there is a $C$ such that $C[\tau][\varphi] \neq \emptyset$ (quality)

ii. $\varphi$ is informative with respect to $\tau$ there is a $C$ such that $C[\tau] \neq C[\tau][\varphi]$ (quantity)

iii. $\tau$ licenses $\varphi$ iff $\forall C, w, w’$ if $\langle w, w’ \rangle \in C[\tau]$ and $\langle w, w \rangle \notin C[\tau][\varphi]$, then $\langle w’, w’ \rangle \notin C[\tau][\varphi]$ (relevance)

The notions of consistency and entailment are standard logical notions. New is at most that they indiscriminately apply to statements and questions, and that we focus on the use of these notions in the formulation of Quality and Quantity requirements for the cooperative exchange of information, instead of as criteria for the soundness and validity of reasoning.

(Groenendijk 1999:115)

Play the Game of Interrogation online!
http://christopherpotts.net/ling/implementations/loi/
3 Examples

Unlicensed partial answer

(3)  $$\text{[\text{red}(a)]^g = \{w_2, w_3, w_4\}; \text{[red}(b)\text{]}^g = \{w_1\}$$

$$\begin{align*}
(W \times W)[?\text{red}(a)] &= \begin{cases}
\langle w_1, w_1 \rangle, & \langle w_2, w_2 \rangle, & \langle w_2, w_3 \rangle, & \langle w_2, w_4 \rangle, \\
\langle w_3, w_2 \rangle, & \langle w_3, w_3 \rangle, & \langle w_3, w_4 \rangle \\
\langle w_4, w_2 \rangle, & \langle w_4, w_3 \rangle, & \langle w_4, w_4 \rangle \\
\langle w_5, w_1 \rangle, & \langle w_5, w_2 \rangle, & \langle w_5, w_3 \rangle, & \langle w_5, w_4 \rangle
\end{cases}
\end{align*}$$

$$\begin{align*}
[(\text{red}(a) \lor \text{red}(b))] &= \begin{cases}
\langle w_1, w_1 \rangle, & \langle w_1, w_2 \rangle, & \langle w_2, w_2 \rangle, & \langle w_2, w_3 \rangle, & \langle w_2, w_4 \rangle, \\
\langle w_3, w_2 \rangle, & \langle w_3, w_3 \rangle, & \langle w_3, w_4 \rangle \\
\langle w_4, w_2 \rangle, & \langle w_4, w_3 \rangle, & \langle w_4, w_4 \rangle \\
\langle w_5, w_1 \rangle, & \langle w_5, w_2 \rangle, & \langle w_5, w_3 \rangle, & \langle w_5, w_4 \rangle
\end{cases}
\end{align*}$$

Licensed partial answer  Groenendijk (1999) observes that this is a retreat from the requirement of strongly exhaustive answers, though not one that demands a change to the semantics.

(4)  $$\text{[red}(a)\text{]}^g = \{w_1, w_2\}; \text{[red}(b)\text{]} = \text{[red}(c)\text{]}^g = \{w_4, w_5\}$$

$$\begin{align*}
(W \times W)[?x\text{red}(x)] &= \begin{cases}
\langle w_1, w_1 \rangle, & \langle w_2, w_1 \rangle, & \langle w_2, w_2 \rangle, & \langle w_3, w_3 \rangle \\
\langle w_2, w_1 \rangle, & \langle w_2, w_2 \rangle \\
\langle w_3, w_3 \rangle
\end{cases}
\end{align*}$$

$$\begin{align*}
[\exists x \text{red}(x)] &= \begin{cases}
\langle w_1, w_1 \rangle, & \langle w_1, w_2 \rangle \\
\langle w_2, w_1 \rangle, & \langle w_2, w_2 \rangle \\
\langle w_3, w_3 \rangle
\end{cases}
\end{align*}$$

$$\begin{align*}
\langle w_1, w_1 \rangle, & \langle w_2, w_2 \rangle, & \langle w_2, w_3 \rangle, & \langle w_2, w_4 \rangle, \\
\langle w_3, w_2 \rangle, & \langle w_3, w_3 \rangle, & \langle w_3, w_4 \rangle \\
\langle w_4, w_2 \rangle, & \langle w_4, w_3 \rangle, & \langle w_4, w_4 \rangle \\
\langle w_5, w_1 \rangle, & \langle w_5, w_2 \rangle, & \langle w_5, w_3 \rangle, & \langle w_5, w_4 \rangle
\end{cases}
\end{align*}$$

$$\begin{align*}
\langle w_1, w_1 \rangle, & \langle w_1, w_2 \rangle, & \langle w_2, w_2 \rangle, & \langle w_2, w_3 \rangle, & \langle w_2, w_4 \rangle, \\
\langle w_3, w_2 \rangle, & \langle w_3, w_3 \rangle, & \langle w_3, w_4 \rangle \\
\langle w_4, w_2 \rangle, & \langle w_4, w_3 \rangle, & \langle w_4, w_4 \rangle \\
\langle w_5, w_1 \rangle, & \langle w_5, w_2 \rangle, & \langle w_5, w_3 \rangle, & \langle w_5, w_4 \rangle
\end{cases}
\end{align*}$$
4 Questions about the Game

(5) Interrogatives never remove identity pairs. What are the linguistic reasons for this? (See Facts 1 and 2.)

(6) The classical update property (p. 114) says that each update takes us to a subset (not necessarily proper) of the input information state. What are the virtues and vices of this limitation?

(7) Compare Quality in the LOI (p. 116) with Grice's original conception, given here:

Contribute only what you know to be true. Do not say false things. Do not say things for which you lack evidence.

Which aspects of this definition does LOI-Quality capture? Which does it ignore?

(8) Compare Quantity in the LOI (p. 116) with Grice's original conception, given here:

Make your contribution as informative as is required. Do not say more than is required

Which aspects of this definition does LOI-Quality capture? Which does it ignore?

(9) Compare Relevance in the LOI (p. 116) with Grice's original conception, given here:

Be relevant.

The LOI gives us the notion of relevance to a question. This is a huge gain. How does it achieve this? What are the linguistic intuitions behind the idea?

(10) What is a contextual alternative in the LOI?

(11) What exactly is the connection between the Presupposition Test (Fact 5) and presuppositions? What does it mean to say that “an indicative sentence presupposes the corresponding yes/no question” (p. 117)?
5 A few observations

Declaratives and their associated questions  Fact 4 (The ‘Relatedness test’) says that a declarative is licensed iff its corresponding yes/no question is non-inquisitive. This is just to say that you can’t assert $\varphi$ unless the question of whether $\varphi$ is really an issue. Groenendijk (1999:117) calls this a particular kind of presupposition, but I think that we need not use that terminology or theoretical conception.

No hybrids  In the terms the logic of interrogation, there are no hybrids — no expressions that can simultaneously delink worlds and eliminate world pairs (cf. Groenendijk and Roelofsen 2009). An interrogative with nontrivial presuppositions would have to do that, though. And there may well be other motivations.

Exhaustivity is best  We’ve seen situations in which non-exhaustive answers are licensed. However, even there, an exhaustive answer is considered to be the best sort of answer. Can we make this ordering precise?

Constraining the interrogator  The felicity conditions on asking questions are weak. The requirement is simply that some world-pairs get disconnected. The article reads, “licensing only puts constraints on the statements of the witness, but reckons any statement from the interrogator to be relevant” (p. 116), and the associated footnote 9 says, “This is a feature particular to the present set-up. One could add requirements of relatedness for the questions of the interrogator as well.” Questions can clearly be relevant or irrelevant. Can we capture this?

References
