(1) Find the volume of the solid whose base is the region in the first quadrant bounded by \( y = \sqrt{1 - x^2} \) and the \( x \)-axis, shown below, and whose cross-sections perpendicular to the \( x \)-axis are squares.

![Diagram of the region](image)

(2) Start by sketching the region described. Then setup but do not evaluate integrals that give the volume of the following solids:

(a) The solid obtained by rotating the region between \( y = \sin x \) and the \( x \)-axis from \( x = 0 \) to \( x = \pi \) about the \( x \)-axis.

(b) The solid obtained by rotating the region between \( y = \sin x \) and the \( x \)-axis from \( x = 0 \) to \( x = \pi \) about the line \( y = 2 \).

(c) The solid obtained by rotating the region between \( x = 2\sqrt{y} \) and the \( y \)-axis from \( y = 4 \) to \( y = 9 \) about the \( y \)-axis.

(d) The solid obtained by rotating the region between \( x = y^2 \) and \( x = 1 - y^2 \) about the line \( x = 3 \).

(e) The solid obtained by rotating the region enclosed by \( y = x^3 \), \( y = 1 \) and \( x = 2 \) about the line \( y = -3 \).

*Solutions to be posted on the Schedule and Homework pages of the website by the end of the day*
(3) A shape that shows up a lot in certain areas of mathematics is called a *torus*, though you might more readily recognize it as a bagel or a donut. One way to construct a torus would be to rotate the circle of radius $1/2$ centered at $(1, 0)$ about the $y$-axis.

![Diagram of a torus with the equation $(x-1)^2 + y^2 = \frac{1}{4}$](image)

The following questions will help us determine the volume of this torus.

(a) Do you want to integrate with respect to $x$ or $y$?

(b) The cross-sections will be washers. Write an expression for the inner radius $r_{in}$ and $r_{out}$ in terms of the variable you chose in part (a).

(c) Setup and evaluate the integral to find the volume of this torus.

   *Note:* your final answer should be $\frac{\pi^2}{2}$

The following table entries might be useful for this worksheet:

(a) $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \left( \frac{x}{a} \right) + C, \ a > 0$

(b) $\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left| x + \sqrt{x^2 + a^2} \right| + C, \ a > 0$

(c) $\int \sqrt{a^2 + x^2} \, dx = \frac{1}{2} \left( x \sqrt{a^2 + x^2} + a^2 \int \frac{1}{\sqrt{a^2 + x^2}} \, dx \right) + C, \ a > 0$

(d) $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left( x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C, \ a > 0$