This homework assignment is all about the new integration techniques we’ve been learning. Remember that you can always check if your antiderivative is correct by taking the derivative.

**Important: you may NOT use a table for any of the integrals in Problems 1 - 8.** This is because the table entries that might help with these problems would not be provided on an exam. For this class, the only table to use is provided on the last page of this assignment. Be sure to show all your work and remember the constant of integration for indefinite integrals.

**Problem 1:** For each part, show that the two integrals are equal using a substitution. Do not evaluate either integral; just show that they are equal.

(a) \( \int_1^2 2 \ln(s^2 + 1) \, ds = \int_1^4 \frac{\ln(t + 1)}{\sqrt{t}} \, dt \)

(b) \( \int_0^\pi x \cos(\pi - x) \, dx = \int_0^\pi (\pi - t) \cos t \, dt \)

**Problem 2:** Evaluate the following integrals:

(a) \( \int \frac{1}{\sqrt{x}} \, dx \)

(b) \( \int \frac{1}{\sqrt{x + 1}} \, dx \)

(c) \( \int \frac{1}{\sqrt{x + 1}} \, dx \)

**For Problems 3 - 6, compute the indefinite integral.**

**Problem 3:** \( \int \theta^2 \cos(3\theta) \, d\theta \)

**Problem 4:** \( \int \ln(x^2) \, dx \)

**Problem 5:** \( \int \arcsin(w^2) \, dw \)

**Problem 6:** \( \int x(\ln x)^4 \, dx \)

**Problem 7:** Use the table with \( f(x) = F'(x) \) to find \( \int_0^5 x f'(x) \, dx \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>-5</td>
<td>-15</td>
<td>-1</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>( F(x) )</td>
<td>10</td>
<td>8</td>
<td>2</td>
<td>-5</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

**Problem 8:** Let \( F(a) \) be the area under the graph of \( y = x^2 e^{-x} \) between \( x = 0 \) and \( x = a \) for \( a > 0 \).

(a) Find a formula for \( F(a) \). *Note: if possible, your formula should not have an integral sign in it.*

(b) Is \( F \) an increasing or decreasing function?

(c) Is \( F \) concave up or concave down for \( 0 < a < 2 \)?

[More problems on next page]
For Problems 9 - 12, you may use the table on the next page as needed.

**Problem 9:** Evaluate the indefinite integrals.

(a) \( \int \frac{1}{\sqrt{9x^2 + 25}} \, dx \)  
(b) \( \int \frac{1}{y^2 + 4y + 4} \, dy \)

**Problem 10:** Evaluate the indefinite integral: \( \int \frac{1}{t^2 + 4t + 5} \, dt \)  
*Hint: you might want to start by completing the square.*

**Problem 11:** Evaluate the indefinite integral: \( \int w \sqrt{1 - w^4} \, dw \)

**Problem 12:** Compute the indefinite integrals.

(a) \( \int \frac{e^{2x}}{1 + e^x} \, dx \)  
(b) \( \int \frac{e^x}{1 + e^{2x}} \, dx \)
For problems where you need to use a table, please reference the following:

Note: If you use one of these formulas, please cite it by number in your solution.

1. \( \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left( \frac{x}{a} \right) + C, \ a \neq 0 \)

2. \( \int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + c a \arctan \left( \frac{x}{a} \right) + C, \ a \neq 0 \)

3. \( \int \frac{1}{(x - a)(x - b)} \, dx = \frac{1}{a - b} (\ln |x - a| - \ln |x - b|) + C, \ a \neq b \)

4. \( \int \frac{cx + d}{(x - a)(x - b)} \, dx = \frac{1}{a - b} [(ac + d) \ln |x - a| - (bc + d) \ln |x - b|] + C, \ a \neq b \)

5. \( \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \left( \frac{x}{a} \right) + C, \ a > 0 \)

6. \( \int \frac{1}{\sqrt{x^2 \pm a^2}} \, dx = \ln |x + \sqrt{x^2 \pm a^2}| + C, \ a > 0 \)

7. \( \int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left( x \sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C, \ a > 0 \)

8. \( \int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left( x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C, \ a > 0 \)