This homework assignment continues our exploration of differential equations and practices separation of variables.

Note: For the first two problems, you might need to first read Section 11.2, “Slope Fields.” Slope fields are a useful tool in trying to understand what differential equations are telling us, as well as the shape of solutions to differential equations, but rather difficult to lecture about. So, if they weren’t sufficiently covered in your class, the book can help you do these problems.

**Problem 1:** Match each of the slope field segments in (I)-(VI) with one or more of the differential equations in (a)-(f). Be sure to briefly justify your choices! This can be something like, “the slope should be positive when $x$ is [blah].” It doesn’t have to be a long paragraph but we do want to know how you thought about it.

(a) $y' = e^{-x^2}$  
(b) $y' = \cos y$  
(c) $y' = \cos(4 - y)$  
(d) $y' = y(4 - y)$  
(e) $y' = y(3 - y)$  
(f) $y' = x(3 - x)$
**Problem 2:** Match the slope fields below to the corresponding differential equations. *Be sure to briefly justify your choices! This can be something like, “the slope should be positive when $x$ is [blah].” It doesn’t have to be a long paragraph but we do want to know how you thought about it.*

(a) $y' = xe^{-x}$  
(b) $y' = \sin x$  
(c) $y' = \cos x$  
(d) $y' = x^2 e^{-x}$  
(e) $y' = e^{-x^2}$  
(f) $y' = e^{-x}$

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**Problem 3:** Is the following statement true or false: The solutions of the differential equation $\frac{dy}{dx} = x^2 + y^2 + 1$ are increasing at every point.

*Note: Be sure to give an explanation for your answer. The correct answer without justification will get no credit.*
Problems 4-9: Use separation of variables to find the solutions to the differential equations subject to the given initial conditions.

Problem 4: \( \frac{dz}{dy} = zy, \ z = 1 \ \text{when} \ y = 0 \)

Problem 5: \( \frac{dy}{dx} = 2y - 4, \ \text{through} \ (2, 5) \)

Problem 6: \( \frac{dB}{dt} + 2B = 50, \ B(1) = 100 \)

Problem 7: \( \frac{dz}{dt} = t e^z, \ \text{through the origin} \)

Problem 8: \( \frac{dw}{d\theta} = \theta w^2 \sin(\theta^2), \ w(0) = 1 \)

Problem 9: \( x(x + 1)\frac{du}{dx} = u^2, \ u(1) = 1 \)

Problem 10: A circular oil spill grows at a rate given by the differential equation \( \frac{dr}{dt} = k \), where \( k \) is some constant, \( r \) represents the radius of the spill in feet, and time is measured in hours. If the radius of the spill is 400 feet 16 hours after the spill begins, what is the value of \( k \)? Include units in your answer.

Problem 11: Solve the differential equation \( \frac{dR}{dx} = a(R^2 + 1) \), assuming \( a \) is some non-zero constant. 
*Note: In this context, “solve” means “find the most general solution.”*

Problem 12: Solve the differential equation \( t \frac{dx}{dt} = (1 + 2 \ln t) \tan x \).
*Note: You may assume that \( x > 0 \) and \( t > 0 \).*

There is a table of integrals on the next page, if you need them.
Table of potentially useful integrals

Note: If you use one of these formulas, please cite it by number in your solution.

1. \[\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \left( \frac{x}{a} \right) + C, \ a \neq 0\]

2. \[\int \frac{bx + c}{x^2 + a^2} \, dx = \frac{b}{2} \ln |x^2 + a^2| + \frac{c}{a} \arctan \left( \frac{x}{a} \right) + C, \ a \neq 0\]

3. \[\int \frac{1}{(x - a)(x - b)} \, dx = \frac{1}{a - b} \left( \ln |x - a| - \ln |x - b| \right) + C, \ a \neq b\]

4. \[\int \frac{cx + d}{(x - a)(x - b)} \, dx = \frac{1}{a - b} \left[ (ac + d) \ln |x - a| - (bc + d) \ln |x - b| \right] + C, \ a \neq b\]

5. \[\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \left( \frac{x}{a} \right) + C, \ a > 0\]

6. \[\int \frac{1}{\sqrt{x^2 \pm a^2}} \, dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C, \ a > 0\]

7. \[\int \sqrt{a^2 \pm x^2} \, dx = \frac{1}{2} \left( x \sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} \, dx \right) + C, \ a > 0\]

8. \[\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2} \left( x \sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} \, dx \right) + C, \ a > 0\]