Math 20, Fall 2016 — Schaeffer
Stanford University

Homework assignment 2, due Friday, April 21st

Complete all assigned book problems as well as supplemental problems D and E.

**Book Problems**

- **Section 6.2:** 18, 22, 38, 42, 48, 50, 58, 60
- **Section 7.1:** 2ab, 8, 10, 12, 16, 20, 24, 32, 36, 42

**Supplemental Problem D**

a. Explain how to use supp. problems A and B from HW1 to evaluate the integrals

\[ \int \sqrt{1 - x^2} \, dx \quad \text{and} \quad \int \frac{dx}{x^2 - 1} \]

b. Remember that \( y = \sqrt{1 - x^2} \) graphs the upper semicircular arc of radius 1 centered at \((0, 0)\). Using this, we computed that

\[ \int_{-1}^{1} \sqrt{1 - x^2} \, dx = \text{area of a semicircle of radius 1} = \frac{\pi}{2} \]

Graph \( y = \sqrt{1 - x^2} \) and shade the region corresponding to the definite integral

\[ \int_{0}^{1/2} \sqrt{1 - x^2} \, dx \]

and then evaluate that definite integral (using a formula from part (a)).

**Supplemental Problem E**

Here are two integral formulas: If \( a > 0 \) is a constant,

\[ \int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \left( \frac{u}{a} \right) + C \quad \text{and} \quad \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C \]

a. Are these formulas true if \( a = 0 \)? Write down a formula for the integral \( \int \frac{du}{u^2} \).

b. Evaluate the integral \( \int \frac{dx}{4x^2 - 1} \)

(\text{More next page, plus a hint.})
c. Evaluate the integral \( \int \frac{dx}{x^2 + 3} \)

d. Evaluate the integral \( \int \frac{dx}{x^2 - 2x + 5} \) (Hint: complete the square)

e. Evaluate the integral \( \int \frac{dx}{4x^2 - 4x - 1} \) (Hint: complete the square)

f. Evaluate the integral \( \int \frac{dx}{9x^2 + 6x + 1} \) (Hint: complete the square)

Example: Suppose I want to integrate \( \int \frac{dx}{4x^2 + 4x + 17} \).

Completing the square gives me \( 4x^2 + 4x + 17 = 4(x + \frac{1}{2})^2 + 16 \). There are two ways to proceed from here: (1) Factor out the 4 and sub \( u = x + \frac{1}{2} \) (so \( du = dx \)) and \( a = 2 \):

\[
\int \frac{dx}{4x^2 + 4x + 17} = \int \frac{dx}{4(x + \frac{1}{2})^2 + 16} = \frac{1}{4} \int \frac{dx}{(x + \frac{1}{2})^2 + 4}
\]

\[
= \frac{1}{4} \left( \frac{1}{2} \text{arctan} \left( \frac{x + \frac{1}{2}}{2} \right) \right) + C = \frac{1}{8} \text{arctan} \left( \frac{2x + 1}{4} \right) + C
\]

Or (2) Use \( 4(x + \frac{1}{2})^2 = (2x + 1)^2 \) then sub \( u = 2x + 1 \) (so \( du = 2dx \)) and \( a = 4 \).