Failure to follow the instructions below is a breach of the Stanford Honor Code:

- You may not use or consult any book or notes during the exam.*
- You may not use a calculator or the calculator function on any electronic device during the exam.*
- You may not access any internet-capable electronic device during the exam,* including smartphones and smartwatches, for any reason. These devices must be switched to “airplane mode” and disconnected from all wireless networks (both cellular and wifi) during the exam*.
- You must sit in your assigned seat.
- You may not communicate with anyone other than the course staff during the exam,* or look at anyone else’s solutions.

*“During the exam” is defined as: After you start the exam, and before you turn in the exam and leave the testing site.

- You have **50 minutes** to complete this exam. If the course staff must ask you to stop writing or to turn in your exam more than once after time is called, you may receive a score of zero.

I understand and accept these instructions. All smart devices on my person are in airplane mode and disconnected from all wireless networks.

Signature: _______________________________________________________

Remember to show your work and justify your answer if required (additional tips are on the next page). Present all solutions in as organized a manner as possible. GOOD LUCK!
Here are some tips:

- If you have time, it’s always a good idea to check your work.

- If you get the wrong answer for an integral but show your work, chances are good that we can award you partial credit.

- DO NOT attempt to estimate any of your answers as decimals. For example, $1 - \frac{1}{\pi}$ is a much better answer than 0.682, because it is exact.

- The boxes at the end of each topic are for grading purposes only. Do not touch or look at these boxes. Pretend they are not there.

- The last page of the exam is blank, and can be used for extra work. If you think it would help for us to look at this work, you should indicate that CLEARLY on the problem’s page.

Integration table entries you might need:

I. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$

II. $\int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \left( \frac{u}{a} \right) + C$

IIIa. $\int \sqrt{u^2 + a^2} \, du = \frac{1}{2} u \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln \left| u + \sqrt{u^2 + a^2} \right| + C$

IIIb. $\int \sqrt{u^2 - a^2} \, du = \frac{1}{2} u \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$

IV. $\int \sqrt{a^2 - u^2} \, du = \frac{1}{2} u \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsin \left( \frac{u}{a} \right) + C$

Trigonometric identities you might need:

$\sin^2 x + \cos^2 x = 1 \quad \sin^2 x = 1 - \cos^2 x \quad \cos^2 x = 1 - \sin^2 x$

$\sin^2 x = \frac{1 - \cos(2x)}{2} \quad \cos^2 x = \frac{1 + \cos(2x)}{2}$

Above, $\sin^n x$ means $(\sin x)^n$. 
Unless otherwise mentioned, you do not need to justify your answers in 1–4.

1. a. Which of the following is/are true statements about the definite integral? Circle all true statements.
   
   i. If \( f(x) \) and \( g(x) \) are continuous, then \( \int_a^b [f(x) + g(x)] \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \).
   
   ii. If \( f(x) \) is a differentiable function, then \( \int_a^b f'(x) \, dx = f(b) - f(a) \).
   
   iii. If \( f(x) \) is a differentiable function, then \( \int_a^b f'(x) \, dx = f(x) + C \).
   
   iv. If \( f(x) \) is a continuous function, then \( \int_a^b [f(x)]^2 \, dx = [\int_a^b f(x) \, dx]^2 \).
   
   v. If \( a < b < c \) then \( \int_a^c f(x) \, dx = \int_a^b f(x) \, dx + \int_b^c f(x) \, dx \).
   
   vi. If \( n \) is an odd positive integer, then \( \int_{-1}^1 x^n \, dx = 0 \).
   
   vii. None of the above.

b. Which of i.–vi. in part (a) is the (First) Fundamental Theorem of Calculus?

2. a. Which of the following is an antiderivative of \( 2^x \)? Circle the correct answer.
   
   \[
   2^x \quad (\ln 2) \cdot 2^x \quad \frac{2^{x+1}}{x+1} \quad \text{None}
   \]

b. Which of the following is an antiderivative of \( e^{x^3} \)? Circle the correct answer.
   
   \[
   \frac{e^{x^3}}{3x^2} \quad 3x^2 e^{x^3} \quad e^{x^3} \quad \text{None}
   \]

3. Integration by parts is an integration technique that is based on which derivative rule?
4. Suppose a tank of water is filling up at a rate of $R(t)$ cubic meters per hour. You may assume that $R(t)$ is always positive and that $t$ is measured in hours. Express the following quantity as a definite integral: The amount of water gained by the tank over the three hours beginning at $t = 2$.

5. Below is graphed $y = f(t)$.

![Graph of $f(t)$](image)

Let $F(x) = \int_{5}^{x} f(t) \, dt$. Caution: The graph above is of $y = f(t)$, not of $y = F(x)$.

a. Is $F(0)$ positive, negative, or zero?

b. Evaluate $F'(10)$.
The problems on this page refer to the functions \( f(t) \) and \( F(x) \) from the previous page.

c. At which point(s) \((x\text{-value(s)})\) in the interval \(0 \leq x \leq 25\) is \( F'(x) = 0\)?

d. At what point \((x\text{-value})\) in the interval \(0 \leq x \leq 25\) is \( F(x) \) maximized?

e. Now, let \( G(x) = \int_0^x f(t) \, dt \). Briefly explain why \( F(x) - G(x) \) must be a constant.
In problems 6–10, evaluate the indefinite integral, showing your work. If you use an integration technique or a table entry, be sure to indicate that clearly. Note that there are integration table entries and trigonometric identities on the second page of the exam. These may prove helpful!

For each of 6–10, draw a box around your final answers.

6. \[ \int [\cos(3x) - 2\sin(5x)] \, dx \]

7. \[ \int x^5 \ln x \, dx \]
8.  \int \sqrt{x - 9} \, dx

9.  \int x^2 e^{4x^3 + 1} \, dx
10. $\int \sqrt{4x^2 - 4x + 5} \, dx$ — Hint: $(2x - 1)^2 = ??$
DO NOT DETACH THIS (OR ANY OTHER) PAGE.

The remaining space is provided for any extra work. If you think this work is important to one of your solutions, please indicate that on the page of the relevant problem (otherwise we won’t know to look!).