Failure to follow the instructions below will constitute a breach of the Stanford Honor Code:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone or any other electronics during the exam for any reason.
- You must take the exam during the class period in which you are officially enrolled, and you must sit in your assigned seat.
- You may not communicate with anyone other than the course staff during the exam, or look at anyone else’s solutions.
- Additionally, you may not discuss or communicate directly or indirectly the contents of this exam with ANYONE other than the course staff until tomorrow, October 22nd.

I will not discuss this exam with ANYONE other than the course staff until: ____________________

I understand and accept these instructions.

Signature: ____________________________________________________________

Remember to show your work and justify your answer if required (additional tips are on the next page). Present all solutions in as organized a manner as possible.

GOOD LUCK!

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Here are some tips:

- If you have time, it's always a good idea to check your work.
- If you get the wrong answer for an integral but show your work, chances are good that we can award you partial credit.
- DO NOT attempt to estimate any of your answers as decimals. For example, $1 - \frac{1}{\pi}$ is a much better answer than 0.682, because it is exact.
- The boxes at the end of each topic are for grading purposes only. Do not touch or look at these boxes. Pretend they are not there.
- The last page of the exam is blank, and can be used for extra work. If you think it would help for us to look at this work, you should indicate that CLEARLY on the problem's page.

Integration table entries you might need:

I. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$

II. $\int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \left( \frac{u}{a} \right) + C$

III. $\int \sqrt{u^2 \pm a^2} \, du = \frac{1}{2} u \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| u \pm \sqrt{u^2 \pm a^2} \right| + C$

IV. $\int \sqrt{a^2 - u^2} \, du = \frac{1}{2} u \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsin \left( \frac{u}{a} \right) + C$

Trigonometric identities you might need:

$\sin^2 x + \cos^2 x = 1$ \quad $\sin^2 x = 1 - \cos^2 x$ \quad $\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = \frac{1 - \cos(2x)}{2}$ \quad $\cos^2 x = \frac{1 + \cos(2x)}{2}$

REMEMBER that $\sin^n x$ means $(\sin x)^n$. 

Unless otherwise specified, no justification or explanation is necessary for problems 1–6.

1. Briefly explain in your own words why indefinite integral formulas always end with $+ C$.

2. What is the name of the formal statement that asserts an inverse relationship between integration and differentiation? Just write down the name.

3. Complete the integral formula below:

$$\int \sin^2(3\theta) + C \ d\theta$$

4. Below are three functions defined in terms of integrals.
   a. Which of the integrals below can NOT be evaluated symbolically using the methods of integration we have learned?
      Circle the undoable integral.
      i. $F_1(x) = \int_0^x te^t \ dt$
      ii. $F_2(x) = \int_0^x te^{t^2} \ dt$
      iii. $F_3(x) = \int_0^x e^{t^2} \ dt$
   b. Which of the three functions above is decreasing on the interval $(0, 1)$?
      Circle all true statements.
      i. $F_1(x)$ is decreasing.
      ii. $F_2(x)$ is decreasing.
      iii. $F_3(x)$ is decreasing.
      iv. None of them is decreasing.
      v. It cannot be determined from the given information.
5. The graph below represents the flow in and out of the Sunset Reservoir during the year 2007.

Let \( I(t) \) = inflow in gallons per day at time \( t \), \( O(t) \) = the outflow in gallons per day at time \( t \), and \( W(t) \) = the amount of water in the reservoir in gallons at time \( t \), where \( t \) is measured in days from midnight on January 1st.

a. Which of the following statements about \( I, O, W \) is/are true? 
   
   *Circle all true statements.*
   
   i. \( W(t) = I(t) + O(t) \)
   
   ii. \( W'(t) = I(t) - O(t) \)
   
   iii. The amount of water in the reservoir at the end of January 1st is equal to 
   
   \[ W(0) + \int_0^1 [I(t) - O(t)] \, dt \]
   
   iv. None of the statements above are true.

b. By inspection of the graph, would you say there was more water in the reservoir at the beginning of the year or at the end of the year? *Circle the correct answer.*

   i. There was more water in the reservoir at the *beginning* of the year.
   
   ii. There was more water in the reservoir at the *end* of the year.
   
   iii. The reservoir held the same amount of water at the beginning and end of the year.
   
   iv. It cannot be determined from the information given.

   v. At some point between April and October.

c. When in 2007 did the reservoir hold the least water? *Circle the correct answer.*

   i. At the beginning of the year.
   
   ii. At some point between January and April.
   
   iii. At some point between April and October.
   
   iv. At some point between October and the end of the year.
   
   v. At the end of the year.
6. Partario is attempting to evaluate the indefinite integral \( \int \sec^3 \theta \, d\theta \).

Like a good Math 20 student, he shows his work!

a. Here’s the first step in his evaluation:

\[
\int \sec^3 \theta \, d\theta = \sec \theta \tan \theta - \int \sec \theta \tan^2 \theta \, d\theta
\]

Which method of integration do you think Partario used in the step above?
Integration by substitution or integration by parts?

*Just write down your answer.*

b. Two steps later, Partario obtains

\[
\int \sec^3 \theta \, d\theta = \sec \theta \tan \theta - \ln |\sec \theta + \tan \theta| - \int \sec^3 \theta \, d\theta
\]

Finish the job! Continuing from where Partario left off, evaluate the integral \( \int \sec^3 \theta \, d\theta \).

*Again, you do not need to provide any additional justification.*

*Draw a box around your answer.*
For problems 7–10 you should show your work and clearly indicate the method you used, when appropriate. Any table entries you may need are on the second page of the exam.

7. Evaluate $\int \frac{1 + 3x^5}{x^3} \, dx$. Draw a box around your answer.

8. Evaluate $\int \frac{x}{x^2 + 1} \, dx$. Draw a box around your answer.
9. Evaluate $\int x \sin x \, dx$. Draw a box around your answer.

10. Evaluate $\int \sqrt{4x^2 + 12x + 5} \, dx$. Hint: $(2x + 3)^2 = (??)$

   Draw a box around your answer.