Midterm 2 Expectations

Prerequisites

Math is a naturally cumulative subject so there are several important things that you are expected to know well from previous classes. You will not be tested directly on this material, but you are assumed to be familiar with it (items in red are prerequisites that have been added since Midterm 1 expectations):

- You should be able to find the graph of a line given two points on that line, or the slope of the line and a single point.
- You should be able to factor and expand simple polynomials.
- Given a quadratic polynomial, you should be able to write it in completed-square form. That is, given \( ax^2 + bx + c \) for constants \( a, b, c \) (with \( a \neq 0 \)) find constants \( h \) and \( k \) such that \( ax^2 + bx + c = a(x + h)^2 + k \).
- You are expected to be familiar with basic rules for powers, radicals, exponents, and logarithms.
- You are expected to know how the various trig functions are defined (in terms of angles and right triangles) and their values at integer multiples of \( \pi/2 \). For Midterm 1 you do not have to know any trig identities (any necessary identities will be provided to you).
- You should also know the definitions of the arcsine and arctangent functions. These are also called the inverse sine and inverse tangent, but BE CAREFUL: \( \text{arcsin} \ x \) is not the same as \( \frac{1}{\sin x} \) and \( \text{arctan} \ x \) is not the same as \( \frac{1}{\tan x} \). Rather, they are the functions that reverse sine and tangent:
  - For a given input \( x \in [-1, 1] \), the output of \( \text{arcsin} \ x \) is equal to the unique \( y \in [-\frac{\pi}{2}, \frac{\pi}{2}] \) such that \( \sin y = x \).
  - For a given input \( x \in (-\infty, \infty) \), the output of \( \text{arctan} \ x \) is equal to the unique \( y \in (-\frac{\pi}{2}, \frac{\pi}{2}) \) such that \( \tan y = x \). Note that the limit of \( \text{arctan} \ x \) as \( x \to \infty \) is \( \frac{\pi}{2} \) and as \( x \to -\infty \) is \( -\frac{\pi}{2} \).
  - For example, \( \text{arcsin}(-1) = -\frac{\pi}{2} \) because \( \sin(-\frac{\pi}{2}) = -1 \) and \( \text{arctan}(1) = \frac{\pi}{4} \) since \( \tan(\frac{\pi}{4}) = 1 \).
- You should be able to graph lines and recognize the graphs of other simple functions, as you learned in precalculus.
• You should know that the equations you need to graph a circle of radius \( r \) and center \((a, b)\) are \( y = b \pm \sqrt{r^2 - (x - a)^2} \).

• You should know how to find the derivative of a function (including the derivatives of the less familiar functions \( \tan x, \sec x, \arcsin x \) and \( \arctan x \)), and you should know what the derivative means.

• For now, you do not need to know about the hyperbolic functions \( \sinh x, \cosh x, \tanh x \), or inverse trig functions other than arcsine and arctangent.

• You should be able to take the limits of basic functions as \( x \to \infty \).

**What you need to know about geometry for Midterm 2**

• How to find the area of a region bounded by given curves in the \( xy \)-plane. (5.4 and 8.1)

• How to find the volume of a solid that has known cross-sections taken perpendicular to the \( x \)-axis. (8.1)

• How to find the volume of a solid of revolution about any horizontal or vertical line. (8.2)

• The basic area formulas for squares, rectangles, triangles, circles, and semicircles.

• The basic volume formulas for prisms, pyramids, and spheres.

**What you need to know about differential equations for Midterm 2**

• How to solve an differential equation of the form \( n \)th derivative of \( y = f(x) \) for the general solution or for a particular solution, given initial/boundary conditions. (6.3)

• Specifically, how to solve physical problems that invoke the relationship between position, velocity, and acceleration: Finding position functions from velocity and an initial position, etc. (6.3)

• The difference between velocity and speed. (6.3)

• How to verify that a particular curve (or family of curves) satisfies a given differential equation. (11.1)

• How to solve a first order separable differential equation for the general solution, or for a particular solution, given initial/boundary conditions. (11.4)

• How to use a phase plane to study autonomous differential equations and make qualitative conclusions about them (specifically the values they take as \( x \to \infty \)). (11.2)
• Terms related to differential equations: order (11.1), separable (11.4), autonomous.

• How to find equilibria of an autonomous differential equation, distinguish between stable and unstable equilibria (11.5, 11.7), and use equilibria calculations to make qualitative statements about the asymptotic behavior of a system (by taking limits as $x \to \infty$).

• The explicit solution to the exponential differential equation $P' = rP$, what the constant $r$ means and how its sign (positive or negative) affects solutions. You should also know that the exponential differential equation has applications in modeling population growth, computing continuously compounded interest, and radioactive decay (half-life). (11.5)

• To model basic phenomena using differential equations (11.6).

More practice problems from the book

• 6.3: 23, 25
• 8.1: 19, 21
• 8.2: 37, 39
• 11.1: 1, 3, 5, 25
• 11.4: 11, 19, 23, 25
• 11.5: 3, 7, 9
• 11.6: 21, 25
• 11.7: 25a (Note: you don’t need to know how to compute explicit solutions to the logistic equation, but this problem only asks you to do a qualitative analysis using the differential equation itself.)