Last two classes:

- Convergence vs. divergence for infinite sums and improper integrals, through examples.

For now: can forget about infinite sums & power series except as motivation — we will return to these topics later in the quarter more carefully.

Our Focus now: How can we tell if an improper integral converges or diverges?

Examples:

- \( \int_1^\infty \frac{1}{x^2} \, dx = \lim_{b \to \infty} \int_1^b \frac{1}{x^2} \, dx = \lim_{b \to \infty} \left[ \ln(b) - \ln(1) \right] = \lim_{b \to \infty} \left( 1 - \frac{1}{b} \right) = 1 \)
  
  **Converges**

- \( \int_1^\infty \frac{1}{x} \, dx = \lim_{b \to \infty} \int_1^b \frac{1}{x} \, dx = \lim_{b \to \infty} \left[ \ln(b) - \ln(1) \right] = +\infty \)
  
  **Diverges**

- \( \int_1^\infty 1 \, dx = \lim_{b \to \infty} \int_1^b 1 \, dx = \lim_{b \to \infty} \left[ b - 1 \right] = +\infty \)
  
  **Diverges**

- \( \int_1^\infty x \, dx = \lim_{b \to \infty} \int_1^b x \, dx = \lim_{b \to \infty} \left[ \frac{b^2}{2} - 1 \right] = +\infty \)
  
  **Diverges**

\( \lim_{x \to \infty} \frac{1}{x^2} = 0 \quad \lim_{x \to 0^+} \frac{1}{x} = \infty \)

"Decay" "stable" "grows without bound"

Improper integral \( \int_1^\infty f(x) \, dx \) can converge or diverge, depending on "how fast" the decay is.

(We'll define decay, stable, grows w/o bound more carefully next time.)

Examples: (classified in class)

- Converge: \( \int_0^\infty e^{-x^2} \, dx \), \( \int_0^1 x \, dx \), \( \int_0^1 x^5 \, dx \)
- Diverge: \( \int_0^\infty \frac{2x}{x^2+1} \, dx \), \( \int_0^\infty \frac{1}{x} \, dx \), \( \int_0^\infty \frac{1}{x^{1/2}} \, dx \)

What do these have in common?