MATH 220/CME 303: PROBLEM SET 1
DUE AT 9AM, FRIDAY, OCTOBER 5, 2018

Problem 1. Classify the following PDEs by degree of non-linearity (linear, semi-linear, quasilinear, fully nonlinear):

(1) $(\cos x)u_x + u_y = u^2$.
(2) $uu_{tt} = u_{xx}$.
(3) $u_x - e^x u_y = \cos x$.
(4) $u_{tt} - u_{xx} + e^u u_x = 0$.

Problem 2.

(1) Solve
$$u_x + (\sin x)u_y = y,\ u(0,y) = 0.$$
(2) Sketch the projected characteristic curves for this PDE.

Problem 3.

(1) Solve
$$yu_x + xu_y = 0,\ u(0,y) = e^{-y^2}.$$
(2) In which region is u uniquely determined?

Problem 4.

(1) Solve $u_x + u_t = u^2,\ u(x,0) = e^{-x^2}$.
(2) Show that there is $T > 0$ such that u blows up at time T, i.e. u is continuously differentiable for $t \in [0,T)$, x arbitrary, but for some x_0, $|u(x_0,t)| \to \infty$ as $t \to T-$. What is T?

Problem 5. Solve
$$u_t + uu_x = 0,\ u(x,0) = -x^2$$
for $|t|$ small.

Problem 6. Consider the Euler-Lagrange functional
$$I(u) = \int_\Omega F(x,u,\partial u)\,dx$$
given by
$$F(x,z,p) = \frac{1}{2}c(x)^2\sum_{j=1}^n p_j^2 + \frac{1}{2}q(x)z^2 + fz,$$
where c, q, f are given functions (speed of waves, potential and forcing, respectively), and show that the corresponding Euler-Lagrange equation is
$$\nabla\cdot(c^2\nabla u) - qu = f,$$
which in the special case of constant c reduces to
$$c^2\Delta u - qu = f.$$