6 Lab: Motor Constants

Motors are useful for actuating and controlling systems. Understanding how they work is helpful for designing and building real systems. Two quantities that characterize a motor’s function are the motor torque constant $k_m$ and the motor voltage constant $k_v$ that appear in the following formulas

$$T_m = k_m i_m \quad v_m = k_v \omega_m$$

where $T_m$ is the motor torque, $i_m$ is the current passing through the motor, $v_m$ is motor back-EMF voltage, and $\omega_m$ is the motor’s angular speed.

The values of these constants are usually found on a motor specification sheet that is experimentally determined by the motor’s manufacturer. The motors and encoders used in this lab were purchased second-hand. Since the motor constants are not known, we will run an experiment to determine their values.

6.1 PreLab

Soon, you will do your own MIPSI lab. You will choose a physical system, and use the MIPSI technique, i.e., Model, Identifiers, Physics, Simplify and Solve, and Interpret your results. To prepare for the future MIPSI lab, brainstorm two physical systems that have an interesting question to be answered and are not a Ph.D. dissertation.

Short system description and question to be answered

Rough system schematic

Short system description and question to be answered

Rough system schematic
6.2 Experimental

A schematic of an ideal DC motor is shown to the right. This motor model includes the voltage supplied to the motor ($v_i$), the motor’s coil resistance ($R_m$), the motor’s inductance ($L_m$), and the motor’s back-EMF ($v_m$). The ODE that relates the motor’s angular speed $\omega_m$ to the input voltage $v_i$ is:

$$L_m \ddot{\omega}_m + (L_m b + R_m J) \dot{\omega}_m + (R_m b + k_m v_v) \omega_m = k_m v_i - R_m T_{Load} - L_m \dot{T}_{Load}$$

Shown to the right is the linear relationship between the motor’s steady-state angular speed $\omega_m$ and a constant value of $T_{Load}$ (when $v_i$ is constant).

Notice that increasing the input voltage $v_i$ increases the line’s offset.

Note: The slope of this line is called the motor’s speed-torque gradient constant, $k_{gradient}$.

6.2.1 Stall torque

Write the ODE governing $T_{Load}$ when both $v_i$ and $\omega_m$ are constant and determine the steady-state part of $T_{Load}$ as a function of $\omega_m$ and $v_i$ when both $v_i$ and $\omega_m$ are constant.

Result:

$$L_m \dot{T}_{Load} + R_m T_{Load} = k_m v_i - (R_m b + k_m v_v) \omega_m$$

$$T_{Load} = \begin{cases} \omega_m + \frac{k_m}{R_m} v_i & \text{when } v_i \text{ is constant} \\
\text{constant} & \text{when } \omega_m \text{ is constant} \end{cases}$$

Solve for $k_m$ in terms of the steady-state value of $T_{Load}$ when $v_i$ is constant and the motor is stalled, i.e., $\omega_m(t) = 0$.

$$k_m = \frac{T_{load}}{\omega_m}$$

While the motor is stalled, use the multi-meter to measure the resistance of the motor coil at several angular positions.

Average Resistance $R_m \approx \underline{\text{Ohms}}$

Using the motor attached to the cart via the wire, measure the stall torque for an appropriate range of $v_i$ from 3 to 5 volts. Assume the spring is linear. Note the length of the moment arm. **Do not stall the motor for long** or it will overheat and burn out.
### 6.2.2 No-load angular speed

Solve for $k_v$ in terms of the steady-state value of $\omega_m$ when $v_i$ is constant and there is no load on the motor, i.e., $T_{\text{Load}}(t) = 0$.

\[ k_v = \] 

### 6.2.3 Estimation with an Encoder

We use several pieces of equipment to measure and record the motor’s angular speed,

- **Encoder:**
  
  Our optical quadrature encoder determines our motor’s rotational speed by detecting alternating light and dark patterns on a disk. For example, the encoder on the right shows 8 transitions (from light to dark or vice-versa). A quadrature encoder has the ability to detect both angular speed and direction. Our encoder has 1000 transitions (500 black sections and 500 white sections) and counts 1000 tics per rev.

- **Oscilloscope:**
  
  The oscilloscope receives a digital signal from the encoder (i.e., bits of one and zero) and displays them to the screen. For example, the oscilloscope pattern to the right shows bits of one (“high bits”) and bits of zero (“low bits”). The value $\tau_{\text{period}}$ is the time interval from a transition from low to high to the next transition from low to high, e.g., measured in microseconds ($10^{-6}$ seconds).

Using a motor without load, measure the steady-state value of $\omega_m$ (use the oscilloscope and a single output channel of the encoder) for an appropriate range of $v_i$ from 0 to 6 volts.

\[
\omega_m = \frac{1 \text{ interval}}{\tau_{\text{period}} \mu\text{sec}} \times \frac{1 \text{ rev}}{500 \text{ intervals}} \times \frac{10^6 \mu\text{sec}}{1 \text{ sec}} \times \frac{2 \pi \text{ rad}}{1 \text{ rev}} \approx \frac{12566 \text{ rad}}{\tau_{\text{period}} \text{ sec}}
\]

Complete the following table assuming $b \approx 0$.

<table>
<thead>
<tr>
<th>Approx. $v_i$</th>
<th>Measured $v_i$</th>
<th>Period X</th>
<th>Angular speed $\omega_m \left( \frac{\text{rad}}{s} \right)$</th>
<th>$k_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Average $k_v$
†In lab 1 we found that these motors are dominated by Coulomb friction. Based on the data just taken, find a better estimate for $k_v$ by including a friction loading term, $T_f$. Also find the magnitude of the frictional loading $T_f$ in N m.

6.2.4 Comparison and verification

Determine the percent error\textsuperscript{11} in the difference in your estimates for $k_m$ and $k_v$.

$$\frac{k_m - k_v}{k_v} \times 100 = \%$$

• If $k_m$ and $k_v$ differ more than 10%, why do you think that this is the case?

• Which estimate do you think is more accurate and why?

• What measurement would you perform to improve your accuracy?

\textsuperscript{11}The definition of “volt” unit leads to (for an ideal DC motor) $k_m = k_v$ when SI units are used.