Lecture 7:
Kinesthetic haptic devices: multi-DOF design

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kinematics

(Hapkit reminder)
transmission

Capstan drive

Friction drive
Hapkit kinematics

\[ r_{\text{pulley}} \theta_{\text{pulley}} = r_{\text{sector}} \theta_{\text{sector}} \]

\[ x_{\text{handle}} = r_{\text{handle}} \theta_{\text{sector}} \]

\[ x_{\text{handle}} = \frac{r_{\text{handle}} r_{\text{pulley}}}{r_{\text{sector}}} \theta_{\text{pulley}} \]
Hapkit force/torque relationships

The relationship between force and torque:

\[ \tau = Fr \]

\[ \frac{\tau_{\text{pulley}}}{r_{\text{pulley}}} = \frac{\tau_{\text{sector}}}{r_{\text{sector}}} \]

\[ F_{\text{handle}} = \frac{\tau_{\text{sector}}}{r_{\text{handle}}r_{\text{pulley}}} \tau_{\text{pulley}} \]
kinematics
(a more complete introduction)
suggested references

• Introduction to robotics: mechanics and control
  John J. Craig

• Robot modeling and control
  Mark W. Spong, Seth Hutchinson, M. Vidyasagar

• A mathematical introduction to robotic manipulation
  Richard M. Murray, Zexiang Li, S. Shankar Sastry

• Springer handbook of robotics
  B. Siciliano, Oussama Khatib (eds.)
kinematics

• The study of movement

• The branch of classical mechanics that describes the motion of objects without consideration of the forces that cause it

• Why do you need it?
  – Determine endpoint position and/or joint positions
  – Calculate mechanism velocities, accelerations, etc.
  – Calculate force-torque relationships
degrees of freedom

• Number of independent position variables needed to in order to locate all parts of a mechanism
• DOF of motion
• DOF of sensing
• DOF of actuation
• The DOF of a mechanism does not always correspond to number of joints
it will help to prototype!

round head paper fasteners

[Images of round head paper fasteners]

www.rogersconnection.com/triangles

[Images of triangular structures]
joints

• Think of a manipulator/interface as a set of bodies connected by a chain of joints

• **Revolute** is the most common joint for robots

From Craig, p. 69
forward kinematics for higher degrees of freedom

For mechanical trackers that use joint angle sensors, you need a map between \textit{joint space} and \textit{Cartesian space}.

\textbf{fwd kinematics: from joint angles, calculate endpoint position}
joint variables

Be careful how you define joint positions

Absolute

Relative
absolute forward kinematics

\[ x = L_1 \cos(\theta_1) + L_2 \cos(\theta_2) \]
\[ y = L_1 \sin(\theta_1) + L_2 \sin(\theta_2) \]

(Often done this way for haptic devices)
relative forward kinematics

\[ x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \]
\[ y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \]

(Often done this way for robots)
Inverse Kinematics

• Using the end-effector position, calculate the joint angles necessary to achieve that position

• Not used often for haptics
  – But could be useful for planning/design

• There can be:
  – No solution (workspace issue)
  – One solution
  – More than one solution
Two possible solutions

Two approaches:
  – algebraic method (using transformation matrices)
  – geometric method

Your devices should be simple enough that you can just use geometry
computing end-effector velocity

• forward kinematics tells us the endpoint position based on joint positions
• how do we calculate endpoint velocity from joint velocities?
• use a matrix called the *Jacobian*

\[ \dot{x} = J \dot{q} \]
formulating the Jacobian

multidimensional form of the chain rule:

\[
\begin{align*}
\dot{x} &= \frac{\partial x}{\partial q_1} \dot{q}_1 + \frac{\partial x}{\partial q_2} \dot{q}_2 + \cdots \\
\dot{y} &= \frac{\partial y}{\partial q_1} \dot{q}_1 + \frac{\partial y}{\partial q_2} \dot{q}_2 + \cdots \\
&\vdots
\end{align*}
\]

assemble in matrix form:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} \\
\frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2}
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
\]

\[
\dot{x} = J \dot{q}
\]
Singularities

• Many devices will have configurations at which the Jacobian is singular

• This means that the device has lost one or more degrees of freedom in Cartesian Space

• Two kinds:
  – Workspace boundary
  – Workspace interior
Singularity Math

- If the matrix is invertible, then it is non-singular.

\[
\dot{\theta} = J^{-1} \dot{x}
\]

- Can check invertibility of \(J\) by taking the determinant of \(J\). If the determinant is equal to 0, then \(J\) is singular.

- Can use this method to check which values of \(\theta\) will cause singularities.
Calculating Singularities

Simplify text: \( \sin(\theta_1 + \theta_2) = s_{12} \)

\[
\begin{vmatrix}
-L_1 s_1 - L_2 s_{12} & -L_2 s_{12} \\
L_1 c_1 + L_2 c_{12} & L_2 c_{12}
\end{vmatrix}
\]

\[
= (-L_1 s_1 - L_2 s_{12})L_2 c_{12} + (L_1 c_1 + L_2 c_{12})L_2 s_{12}
\]

For what values of \( \theta_1 \) and \( \theta_2 \) does this equal zero?
compute the necessary joint torques

the Jacobian can also be used to relate joint torques to end-effector forces:

$$\tau = J^T f$$

this is a key equation for multi-degree-of-freedom haptic devices
how do you get this equation?

the **Principle of virtual work**

states that changing the coordinate frame does not change the total work of a system

\[ f \cdot \delta x = \tau \cdot \delta q \]
\[ f^T \delta x = \tau^T \delta q \]
\[ f^T J \delta q = \tau^T \delta q \]
\[ f^T J = \tau^T \]
\[ J^T f = \tau \]
force generation signals

desired force (in computer)

D/A

amplifiers

volts

counts

actuator

force/torque

voltage or current

transmission & kinematics

endpoint

force/torque
Phantom Omni
kinematics
Phantom Omni

![Image of Phantom Omni device with a hand holding the handle and a diagram showing the mechanical components with labels for distances and angles.]
phantom omni

link lengths
phantom omni

singular configurations
If $\theta_3$ is fixed, we can regard this as a two-link manipulator. When $\theta_3 = 0$, on the $y - z$ plane,

\[
\begin{bmatrix}
  y \\
  z
\end{bmatrix} =
\begin{bmatrix}
  l_1 \sin \theta_1 + l_2 \sin \theta_2 \\
  l_1 \cos \theta_1 + l_2 \cos \theta_2
\end{bmatrix}
\]

Now, let us consider $\theta_3$. Note that $\theta_3$ affects $x$ and $z$ position, but not $y$ position. So, we first calculate a link length, $L$, on the $z - x$ plane:

\[L = l_1 \cos \theta_1 + l_2 \cos \theta_2\] (1)

Then, rotating the link length $L$ around the $y$-axis gives you the forward kinematics of an Omni:

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} =
\begin{bmatrix}
  L \sin \theta_3 \\
  l_1 \sin \theta_1 + l_2 \sin \theta_2 \\
  L \cos \theta_3
\end{bmatrix} =
\begin{bmatrix}
  (l_1 \cos \theta_1 + l_2 \cos \theta_2) \sin \theta_3 \\
  l_1 \sin \theta_1 + l_2 \sin \theta_2 \\
  (l_1 \cos \theta_1 + l_2 \cos \theta_2) \cos \theta_3
\end{bmatrix}
\]
phantom omni

By definition, Jacobian matrix for the Omni is

$$J = \begin{bmatrix}
\frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\
\frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\
\frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3}
\end{bmatrix}
$$

$$= \begin{bmatrix}
-l_1 \sin \theta_1 \sin \theta_3 & -l_2 \sin \theta_2 \sin \theta_3 & (l_1 \cos \theta_1 + l_2 \cos \theta_2) \cos \theta_3 \\
l_1 \cos \theta_1 & l_2 \cos \theta_2 & 0 \\
-l_1 \sin \theta_1 \cos \theta_3 & -l_2 \sin \theta_2 \cos \theta_3 & -(l_1 \cos \theta_1 + l_2 \cos \theta_2) \sin \theta_3
\end{bmatrix}$$
pantograph mechanism
pantograph

**Definition 1:** a mechanical linkage connected in a manner based on parallelograms so that the movement of one pen, in tracing an image, produces identical movements in a second pen.

**Definition 2:** a kind of structure that can compress or extend like an accordion.
pantograph haptic device

Xiyang Yeh, ME 327 2012
http://charm.stanford.edu/ME327/Xiyang
pantograph haptic device

Sam Schorr and Jared Muirhead, ME 327 2012
http://charm.stanford.edu/ME327/JaredAndSam
The Pantograph Mk-II: A Haptic Instrument

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Abstract
— We describe the redesign and the performance evaluation of a high-performance haptic device system called the Pantograph. The device is based on a two degree-of-freedom parallel mechanism which was designed for optimized dynamic performance, but which also is well kinematically conditioned. The results show that the system is capable of producing accurate tactile signals in the DC–400 Hz range and can resolve displacements of the order of 10 µm. Future improvements are discussed.


I. INTRODUCTION
The scientific study of touch, the design of computational methods to synthesize tactile signals, studies in the control of haptic interfaces, the development of force reflecting virtual environments, and other activities, all require the availability of devices that can produce reliable haptic interaction signals. In some cases, it is needed to produce well controlled stimuli. In other cases, it is important to have the knowledge of the structural dynamics of a device, but in all cases, these activities entail having devices which are well characterized. Following SensAble's Phantom and Immersion's Impulse Engine, several new commercially-available general-purpose haptic devices have been recently introduced: MPB’s Freedom-6S, Force Dimension's Omega, Haption's Virtuose; plus other application-specific devices. In addition, interesting, low-complexity, high-performance devices have also become available, either from research institutions or from commercial sources [9], [10], [15], [21]. We felt, nonetheless, that a general-purpose laboratory system having high performance features, would be a valuable tool.

With this in mind, we set out to redesign the 'Pantograph' haptic device, first demonstrated at the 1994 ACM SIGCHI conference in Boston, MA [22]. Our first goal was the creation of an open architecture system which could be easily replicated from blueprints and from a list of off-the-shelf components. The second goal was to obtain a system which would have superior and known performance characteristics so that it could be used as a scientific instrument. Our intention is to make the system available in open-source, hardware and software.

An important aspect of the Pantograph, a planar parallel mechanism (Fig. 1d), is the nature of its interface: a non-slip plate on which the fingerpad rests (Fig. 1e). Judiciously programmed tangential interaction forces at the interface (Fig. 1e) have the effect of causing fingertip deformations and tactile sensations that resemble exploring real surfaces.

Fig. 1. Pantograph Mk II electromechanical hardware. 

a) Side view showing the main electromechanical components.

b) Front view.

c) Photograph.

d) Top view of the five-bar mechanism and plate constrained to 2-DOF.

e) The interaction force has two components: fN is measured by the load cell and fT results from coupling the finger tip to the actuators via linkages.
graphkit

Design by Tara Gholami and Joey Campion
http://hapkit.stanford.edu/twoDOF.html
example

Find the forward kinematics
Find the Jacobian