Interactive Session 10: Kinesthetic haptic devices: Stability

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Questions from prerecorded video?
effect of time delay
review stability in the context of the s-plane

common second-order system: \( m\ddot{x} + b\dot{x} + kx = f \)

take the Laplace transform of both sides:

\[
\mathcal{L}[m\ddot{x} + b\dot{x} + kx] = \mathcal{L}[f] \\
m s^2 X(s) + b s X(s) + k X(s) = F(s) \\
(ms^2 + bs + k)X(s) = F(s)
\]

transfer function/characteristic equation:

\[
\frac{F(s)}{X(s)} = ms^2 + bs + k
\]
Time Delay

\[ \frac{1}{ms^2 + bs} \]
Padé approximation

\[ Y(s) = R(s) + K_P + K_D s \]

\[ x_{wall} + \frac{1}{ms^2 + bs} \]

\[ delay = e^{-sT} \]

\[ e^{-sT} \approx \frac{1 - \left(\frac{sT}{2}\right)}{1 + \left(\frac{sT}{2}\right)} \]

This adds a left half plane pole and a right half plane zero!

In breakout groups: Discuss/calculate how time delay affects the maximum stable stiffness of a virtual wall.
\[ x = \frac{ke^{-st}}{ms^2+bs} \left( x_{\text{wall}} - e^{-st}x \right) \]

\[ x(1 + e^{-st} \frac{ke^{-st}}{ms^2+bs}) = \frac{ke^{-st}}{ms^2+bs} x_{\text{wall}} \]

\[ \frac{x}{x_{\text{wall}}} = \frac{ke^{-st} \left( \frac{1}{ms^2+bs} \right)}{1 + e^{-st} \frac{ke^{-st}}{ms^2+bs} e^{-2st}} \]

Chap. eqn.
\[ 1 + \frac{k}{m s^2 + 6s} \cdot \frac{1 - Ts}{1 + Ts} \frac{1}{(e^{-2sT})} = 0 \]

Diagram:
- No delay
- Delay

Note: Corrected from lecture. (I was using the higher-order approx. which gave complex delays instead of the real version used in the past.) It seems from the notes the delay given above...
HAPKITS!!

Fill out address form TODAY
(see announcement on Canvas)
Reminders:

Assignment 4 due today
Look for Assignment 5 posted later today

Quiz will be discussed later — there are still a couple students who need to take it

Office Hours/Q&A with Allison until 10 am.
Question queue (see tab with today’s date): [https://tinyurl.com/HapticsAllison](https://tinyurl.com/HapticsAllison)