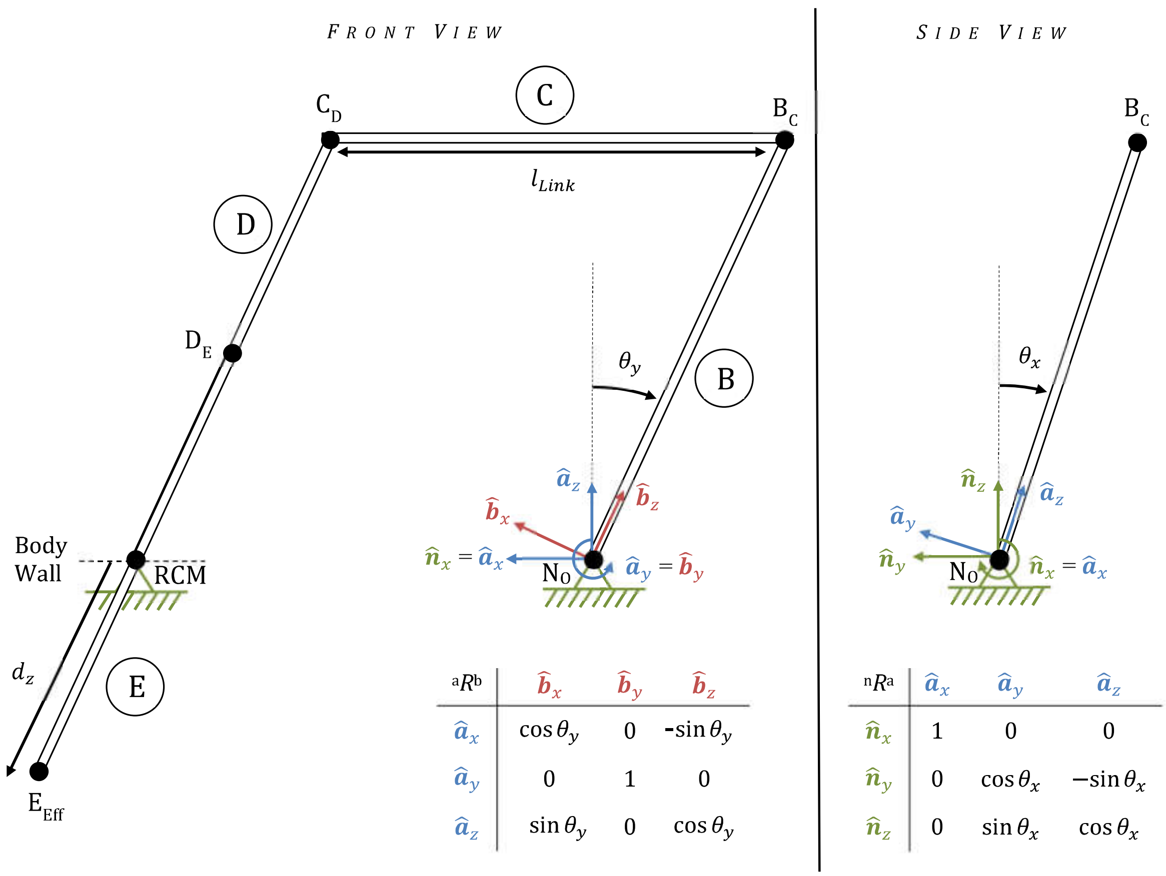
Appendix for Assignment 3, Problem 3

Details of Kinematics for RCM Robot



${}^{a}R^{b}$	$\hat{b}_x$	$\hat{b}_y$	$\hat{b}_z$
$\hat{a}_x$	$\cos\theta_y$	0	$-\sin\theta_y$
$\hat{a}_y$	0	1	0
$\hat{a}_z$	$\sin\theta_y$	0	$\cos\theta_y$

${}^{n}R^{a}$	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\hat{n}_x$	1	0	0
$\hat{n}_y$	0	$\cos\theta_x$	$-\sin\theta_x$
$\hat{n}_z$	0	$\sin\theta_x$	$\cos\theta_x$

Translational and Angular Velocity

Link 1 (Body B), Angular velocity in frame N, Translational velocity of mass center in frame N:

$${}^{N}\vec{\boldsymbol{\omega}}^{A} = \dot{\theta}_x \hat{\boldsymbol{a}}_x \qquad {}^{A}\vec{\boldsymbol{\omega}}^{B} = -\dot{\theta}_y \hat{\boldsymbol{b}}_y$$

$${}^{N}\vec{\boldsymbol{\omega}}^{B} = {}^{N}\vec{\boldsymbol{\omega}}^{A} + {}^{A}\vec{\boldsymbol{\omega}}^{B} = \cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_x - \dot{\theta}_y \hat{\boldsymbol{b}}_y - \sin\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_z$$

$$\boxed{{}^{N}\vec{\boldsymbol{\omega}}^{B} = \dot{\theta}_x \hat{\boldsymbol{n}}_x - \cos\theta_x\,\dot{\theta}_y \hat{\boldsymbol{n}}_y - \sin\theta_x\,\dot{\theta}_y \hat{\boldsymbol{n}}_z}$$

$${}^{N}\vec{\boldsymbol{v}}^{B_{cm}} = \frac{{}^{N}d\vec{\boldsymbol{r}}^{B_{cm}/N_O}}{dt} = \frac{{}^{B}d\vec{\boldsymbol{r}}^{B_{cm}/N_O}}{dt} + {}^{N}\vec{\boldsymbol{\omega}}^{B} \times \vec{\boldsymbol{r}}^{B_{cm}/N_O} == -\frac{1}{2} l_{Link}\dot{\theta}_y \hat{\boldsymbol{b}}_x - \frac{1}{2} l_{Link}\cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_y$$

$$\boxed{\begin{aligned}{}^{N}\vec{\boldsymbol{v}}^{B_{cm}} = & -\frac{1}{2} l_{Link}\cos\theta_y\,\dot{\theta}_y \hat{\boldsymbol{n}}_x + \frac{1}{2} l_{Link}\left(\sin\theta_x \sin\theta_y\,\dot{\theta}_y - \cos\theta_x\cos\theta_y\,\dot{\theta}_x\right)\hat{\boldsymbol{n}}_y \\ & -\frac{1}{2} l_{Link}\left(\sin\theta_x\cos\theta_y\,\dot{\theta}_x + \sin\theta_y\cos\theta_x\,\dot{\theta}_y\right)\hat{\boldsymbol{n}}_z\end{aligned}}$$

Link 2 (Body C), Angular velocity in frame N, Translational velocity of mass center in frame N:

$${}^{N}\vec{\boldsymbol{\omega}}^{C} = \dot{\theta}_x \hat{\boldsymbol{a}}_x$$

$$\boxed{{}^{N}\vec{\boldsymbol{\omega}}^{C} = \dot{\theta}_x \hat{\boldsymbol{n}}_x}$$

$${}^{N}\vec{\boldsymbol{v}}^{C_{cm}} = \frac{{}^{N}d\vec{\boldsymbol{r}}^{C_{cm}/N_O}}{dt} = \frac{{}^{B}d\vec{\boldsymbol{r}}^{C_{cm}/N_O}}{dt} + {}^{N}\vec{\boldsymbol{\omega}}^{B} \times \vec{\boldsymbol{r}}^{C_{cm}/N_O} == -l_{Link}\dot{\theta}_y \hat{\boldsymbol{b}}_x - l_{Link}\cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_y$$

$$\boxed{\begin{aligned}{}^{N}\vec{\boldsymbol{v}}^{B_{cm}} = & -l_{Link}\cos\theta_y\,\dot{\theta}_y \hat{\boldsymbol{n}}_x + l_{Link}\left(\sin\theta_x \sin\theta_y\,\dot{\theta}_y - \cos\theta_x\cos\theta_y\,\dot{\theta}_x\right)\hat{\boldsymbol{n}}_y \\ & -l_{Link}\left(\sin\theta_x\cos\theta_y\,\dot{\theta}_x + \sin\theta_y\cos\theta_x\,\dot{\theta}_y\right)\hat{\boldsymbol{n}}_z\end{aligned}}$$

Link 3 (Body D), Angular velocity in frame N, Translational velocity of mass center in frame N:

$${}^{N}\vec{\boldsymbol{\omega}}^{A} = \dot{\theta}_x \hat{\boldsymbol{a}}_x \qquad {}^{A}\vec{\boldsymbol{\omega}}^{D} = -\dot{\theta}_y \hat{\boldsymbol{b}}_y$$

$${}^{N}\vec{\boldsymbol{\omega}}^{D} = {}^{N}\vec{\boldsymbol{\omega}}^{A} + {}^{A}\vec{\boldsymbol{\omega}}^{D} = \cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_x - \dot{\theta}_y \hat{\boldsymbol{b}}_y - \sin\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_z$$

$$\boxed{{}^{N}\vec{\boldsymbol{\omega}}^{D} = \dot{\theta}_x \hat{\boldsymbol{n}}_x - \cos\theta_x\,\dot{\theta}_y \hat{\boldsymbol{n}}_y - \sin\theta_x\,\dot{\theta}_y \hat{\boldsymbol{n}}_z}$$

$${}^{N}\vec{\boldsymbol{v}}^{D_{cm}} = \frac{{}^{N}d\vec{\boldsymbol{r}}^{D_{cm}/N_O}}{dt} = \frac{{}^{B}d\vec{\boldsymbol{r}}^{D_{cm}/N_O}}{dt} + {}^{N}\vec{\boldsymbol{\omega}}^{B} \times \vec{\boldsymbol{r}}^{D_{cm}/N_O} == -\frac{1}{2} l_{Link}\dot{\theta}_y \hat{\boldsymbol{b}}_x - \frac{1}{2} l_{Link}\cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_y$$

$$\boxed{\begin{aligned}{}^{N}\vec{\boldsymbol{v}}^{D_{cm}} = & -\frac{1}{2} l_{Link}\cos\theta_y\,\dot{\theta}_y \hat{\boldsymbol{n}}_x + \frac{1}{2} l_{Link}\left(\sin\theta_x \sin\theta_y\,\dot{\theta}_y - \cos\theta_x\cos\theta_y\,\dot{\theta}_x\right)\hat{\boldsymbol{n}}_y \\ & -\frac{1}{2} l_{Link}\left(\sin\theta_x\cos\theta_y\,\dot{\theta}_x + \sin\theta_y\cos\theta_x\,\dot{\theta}_y\right)\hat{\boldsymbol{n}}_z\end{aligned}}$$

Link 4 (Body E), Angular velocity in frame N, Translational velocity of mass center in frame N:

$${}^{N}\vec{\boldsymbol{\omega}}^{A} = \dot{\theta}_x \hat{\boldsymbol{a}}_x \qquad {}^{A}\vec{\boldsymbol{\omega}}^{E} = -\dot{\theta}_y \hat{\boldsymbol{b}}_y$$

$${}^{N}\vec{\boldsymbol{\omega}}^{E} = {}^{N}\vec{\boldsymbol{\omega}}^{A} + {}^{A}\vec{\boldsymbol{\omega}}^{E} = \cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_x - \dot{\theta}_y \hat{\boldsymbol{b}}_y - \sin\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_z$$

$$\boxed{{}^{N}\vec{\boldsymbol{\omega}}^{E} = \dot{\theta}_x \hat{\boldsymbol{n}}_x - \cos\theta_x\,\dot{\theta}_y \hat{\boldsymbol{n}}_y - \sin\theta_x\,\dot{\theta}_y \hat{\boldsymbol{n}}_z}$$

$${}^{N}\vec{\boldsymbol{v}}^{E_{cm}} = \frac{{}^{N}d\vec{\boldsymbol{r}}^{E_{cm}/N_O}}{dt} = \frac{{}^{B}d\vec{\boldsymbol{r}}^{E_{cm}/N_O}}{dt} + {}^{N}\vec{\boldsymbol{\omega}}^{B} \times \vec{\boldsymbol{r}}^{E_{cm}/N_O} = -\frac{1}{2}(l_{Link} - 2d_z)\dot{\theta}_y \hat{\boldsymbol{b}}_x - \frac{1}{2}(l_{Link} - 2d_z)\cos\theta_y\,\dot{\theta}_x \hat{\boldsymbol{b}}_y - \dot{d}_z \hat{\boldsymbol{b}}_z$$

$$\boxed{\begin{aligned}{}^{N}\vec{\boldsymbol{v}}^{E_{cm}} = & \left[\sin\theta_y\,\dot{d}_z - \frac{1}{2}(l_{Link} - 2d_z)\cos\theta_y\,\dot{\theta}_y\right]\hat{\boldsymbol{n}}_x \\ & + \left[\sin\theta_x\cos\theta_y\,\dot{d}_z + \frac{1}{2}\sin\theta_x\sin\theta_y\,(l_{Link} - 2d_z)\dot{\theta}_y - \frac{1}{2}\cos\theta_x\cos\theta_y\,(l_{Link} - 2d_z)\dot{\theta}_x\right]\hat{\boldsymbol{n}}_y \\ & + \left[-\cos\theta_x\cos\theta_y\,\dot{d}_z - \frac{1}{2}\sin\theta_x\cos\theta_y\,(l_{Link} - 2d_z)\dot{\theta}_x - \frac{1}{2}\sin\theta_y\cos\theta_y\,(l_{Link} - 2d_z)\dot{\theta}_y\right]\hat{\boldsymbol{n}}_z\end{aligned}}$$