ME469A

Numerical Methods for Fluid Mechanics

Handout #1

Gianluca Iaccarino

ME469A: Objective

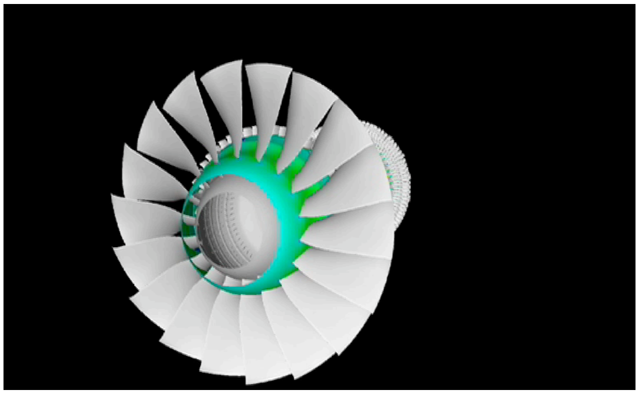
- Analysis of computational approaches applied to the simulation of fluid motion
  - Key ingredients
    - Approximations: from the real world to a physical/mathematical representation)
    - Computers: from a continuous mathematical model to computer algorithms and, finally, to a discrete solution
- Emphasis is on the numerical algorithms although the analysis and understanding of the physics introduces guidelines and constraints
- Computer use and programming obviously of primary importance

What is CFD?

- Computational Fluid Dynamics is a branch of computer-based science that provides predictions of fluid flows
  - Mathematical modeling (typically a system on non-linear, coupled PDEs)
  - Numerical methods (discretization and solution techniques)
  - Software tools
- CFD is used in a growing number of disciplines
- Several CFD software tools are commercially available, but still extensive research and development is ongoing to improve the methods, physical models, etc.

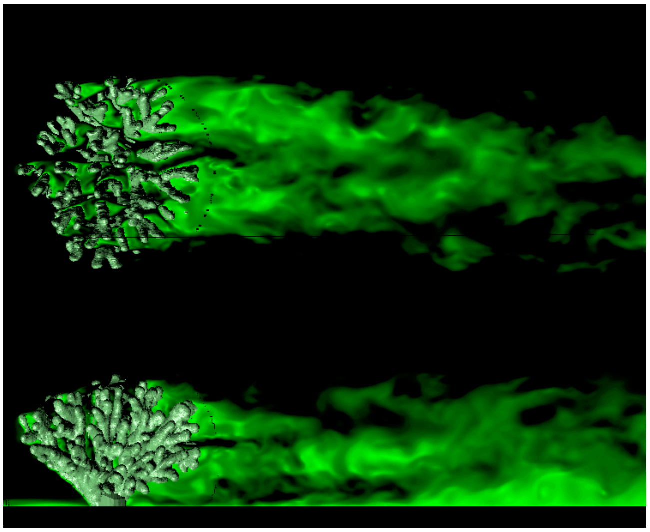
“Fluid flow” disciplines

Engineering design: aerodynamics, propulsion,etc.



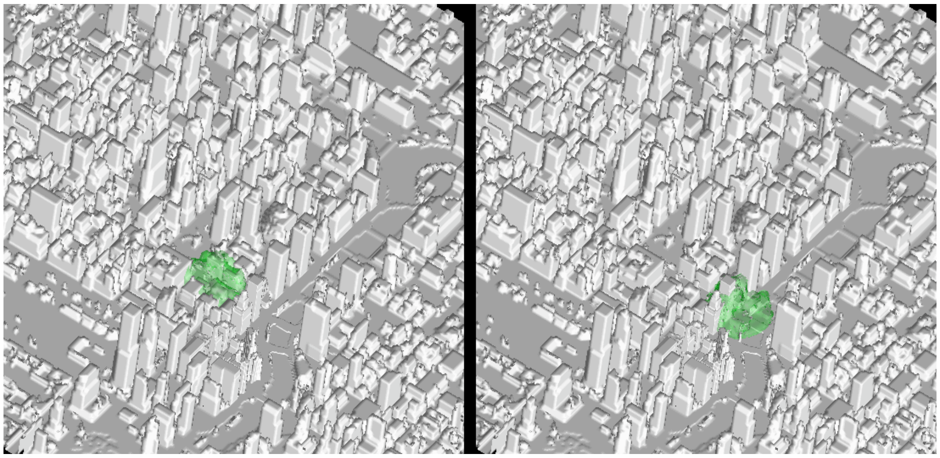
“Fluid flow” disciplines

Biological systems: nutrient transport,etc.



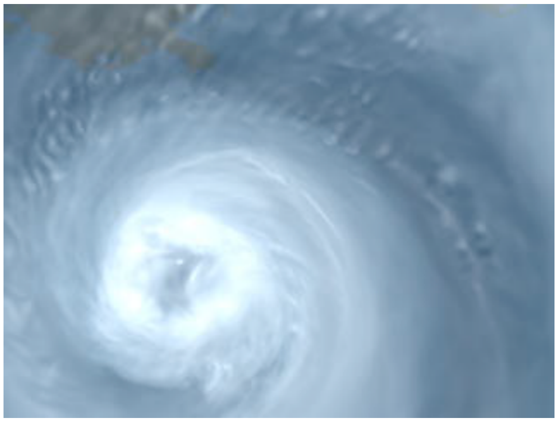
“Fluid flow” disciplines

Homeland security: hazard dispersion,etc.



“Fluid flow” disciplines

Meteorology: wind, hurricanes, tsunamis,etc.



http://woodall.ncsa.uiuc.edu/dbock/projects/Opal/

Structure of the course

- Introduction, numerical models and properties
- Flow equations and approximation levels
- Finite volume approach
- Solution of linear and non-linear systems
- The Navier-Stokes equations
- Numerical methods for incompressible flows
- Verification and validation

Ferziger & Peric “Computational Methods dor fluid Dynamics”, Springer.

Areas of Focus, I

- Solution of linear system
  - Basic construction
  - Incomplete LU decomposition
  - Splitting methods
  - Descent methods for symmetric matrices
    - Steepest descent, Conjugate gradient, Preconditioning
  - Descent methods for general matrices
    - Bi-conjugate gradient, GMRES
  - Multigrid
    - Geometric and Algebraic multigrid
  - Solvers for parallel computers
  - Unstructured grids
- Solution of non linear systems
  - Newton technique
  - Deferred-Correction approach

Areas of Focus, II

- Solution of the Navier-Stokes equations
  - Conservation properties
  - Location of the unknowns
    - Collocation vs. staggering
    - Cell center vs. vertex based discretization
  - Time integration
    - Explicit
    - Implicit
    - Time-spectral
  - Pressure-based methods
    - Pressure-correction method
    - Fractional step method
    - Stream function-vorticity formulation
    - Artificial compressibility
  - Density-based methods

Coursework

- Homeworks (30%)
  - short exercises
  - Algorithm development and tests
- Mid-term (20%)
  - True/False questions
  - 1 more involved exercise
  - Only material directly covered in class
- Final project (50%)
  - Problem of your choice but directly related to the material presented in class

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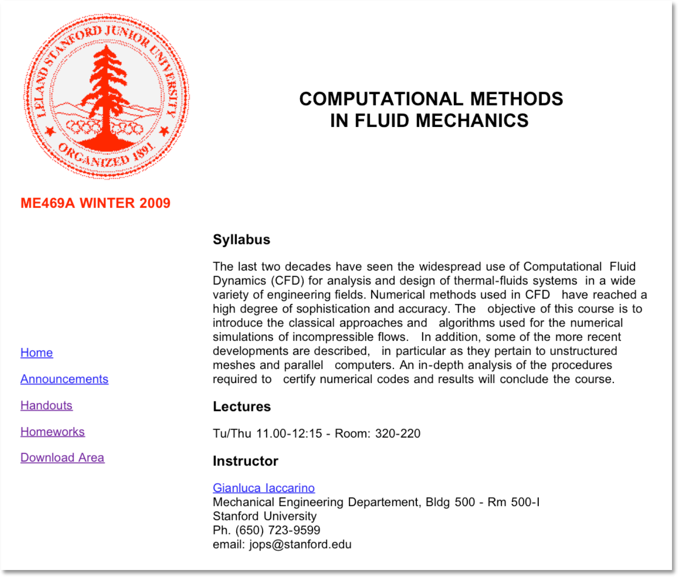

COMPUTATIONAL METHODS IN FLUID MECHANICS

ME469A WINTER 2009

Syllabus

The last two decades have seen the widespread use of Computational Fluid Dynamics (CFD) for analysis and design of thermal-fluids systems in a wide variety of engineering fields. Numerical methods used in CFD have reached a high degree of sophistication and accuracy. The objective of this course is to introduce the classical approaches and algorithms used for the numerical simulations of incompressible flows. In addition, some of the more recent developments are described, in particular as they pertain to unstructured meshes and parallel computers. An in-depth analysis of the procedures required to certify numerical codes and results will conclude the course.

Home

Announcements

Handouts

Homeworks

Download Area

Lectures

Tu/Thu 11.00-12:15 - Room: 320-220

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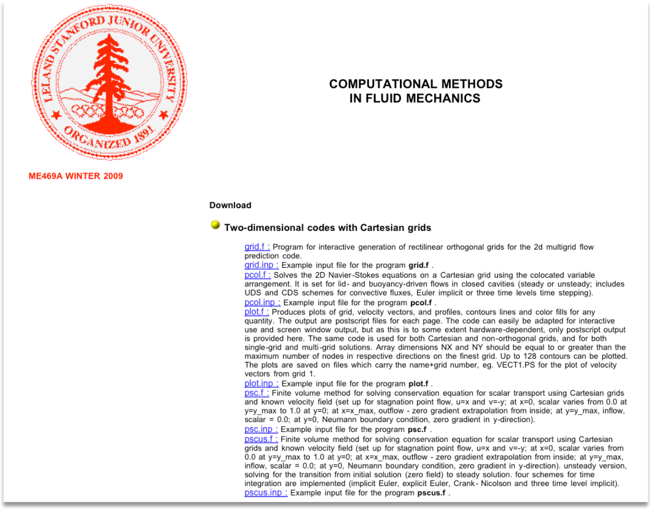


COMPUTATIONAL METHODS
IN FLUID MECHANICS

ME469A WINTER 2009

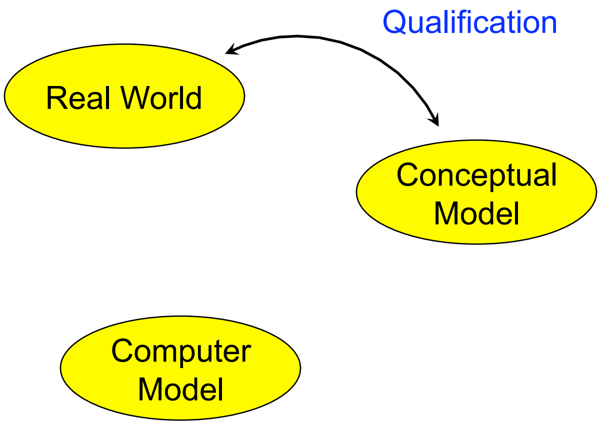
Download

- **Two-dimensional codes with Cartesian grids**

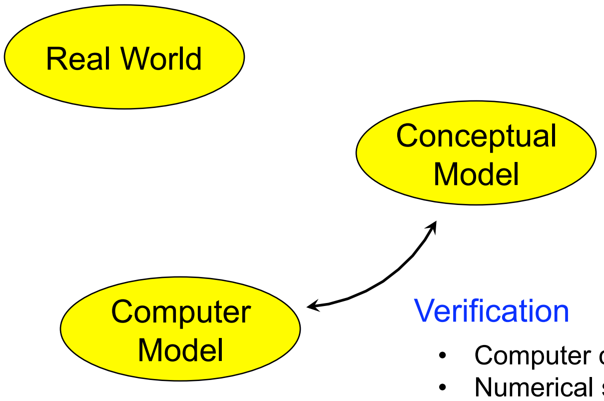
grid.f : Program for interactive generation of rectilinear orthogonal grids for the 2d multigrid flow prediction code.
grid.inp : Example input file for the program **grid.f** .
pcol.f : Solves the 2D Navier-Stokes equations on a Cartesian grid using the colocated variable arrangement. It is set for lid- and buoyancy-driven flows in closed cavities (steady or unsteady; includes UDS and CDS schemes for convective fluxes, Euler implicit or three time levels time stepping).
pcol.inp : Example input file for the program **pcol.f** .
plot.f : Produces plots of grid, velocity vectors, and profiles, contours lines and color fills for any quantity. The output are postscript files for each page. The code can easily be adapted for interactive use and screen window output, but as this is to some extent hardware-dependent, only postscript output is provided here. The same code is used for both Cartesian and non-orthogonal grids, and for both single-grid and multi-grid solutions. Array dimensions NX and NY should be equal to or greater than the maximum number of nodes in respective directions on the finest grid. Up to 128 contours can be plotted. The plots are saved on files which carry the name+grid number, eg. VECT1.PS for the plot of velocity vectors from grid 1.
plot.inp : Example input file for the program **plot.f** .
psc.f : Finite volume method for solving conservation equation for scalar transport using Cartesian grids and known velocity field (set up for stagnation point flow, u=x and v=-y; at x=0, scalar varies from 0.0 at y=y_max to 1.0 at y=0; at x=x_max, outflow - zero gradient extrapolation from inside; at y=y_max, inflow, scalar = 0.0; at y=0, Neumann boundary condition, zero gradient in y-direction).
psc.inp : Example input file for the program **psc.f** .
pscus.f : Finite volume method for solving conservation equation for scalar transport using Cartesian grids and known velocity field (set up for stagnation point flow, u=x and v=-y; at x=0, scalar varies from 0.0 at y=y_max to 1.0 at y=0; at x=x_max, outflow - zero gradient extrapolation from inside; at y=y_max, inflow, scalar = 0.0; at y=0, Neumann boundary condition, zero gradient in y-direction). unsteady version, solving for the transition from initial solution (zero field) to steady solution. four schemes for time integration are implemented (implicit Euler, explicit Euler, Crank- Nicolson and three time level implicit).
pscus.inp : Example input file for the program **pscus.f** .

Preliminaries

Real World	The intended application
Conceptual Model	Physical laws, hypothesis and models (e.g. a set of PDEs) and initial and boundary conditions
Computer Model	A set of computational algorithms that allow to build a numerical, approximated solution to the conceptual model



Qualification: Determination of the adequacy of the conceptual model to provide acceptable level of agreement for the intended applications



Real World

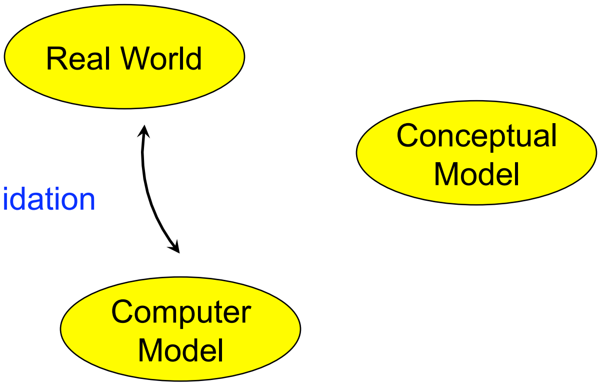
Conceptual Model

Computer Model

Verification

- Computer codes
- Numerical solutions

Verification: Process of determining that (1) the model implementation accurately represents the conceptual model and (2) the solution to the model is accurate



Real World

Conceptual Model

Validation

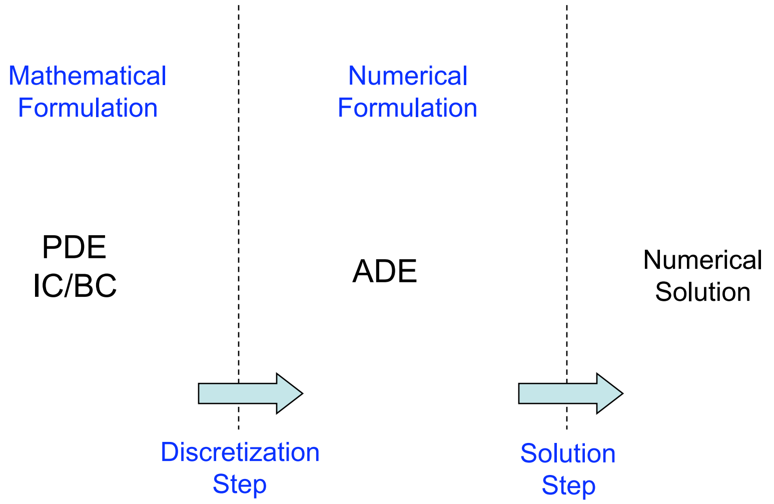
Computer Model

Validation: Process of determining the degree to which the model is an accurate representation of reality from the perspective of the intended application

ME469A

- This class is naturally focused on Verification
  - Deals with mathematics and numerics
  - We always assume that a mathematical representation of reality is available
  - We do not compare numerical results with reality!
- One specific requirement to perform verification is the knowledge of the “true” solution
  - Verification answers the question: “are we solving the equations correctly?”
  - Need to use simple mathematical models that can be solved exactly
  - Can we do something more? Manufactured solutions

Numerical Simulations



Mathematical Formulation

Numerical Formulation

PDE
IC/BC

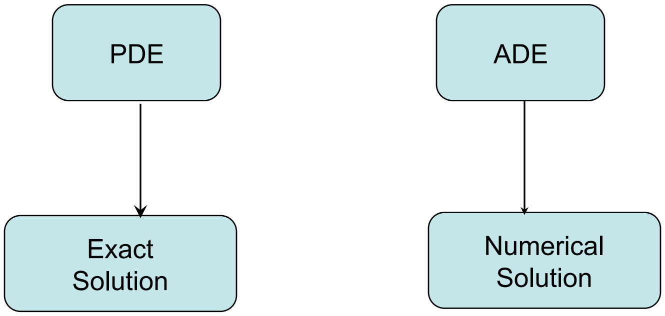
ADE

Numerical Solution

Discretization Step

Solution Step

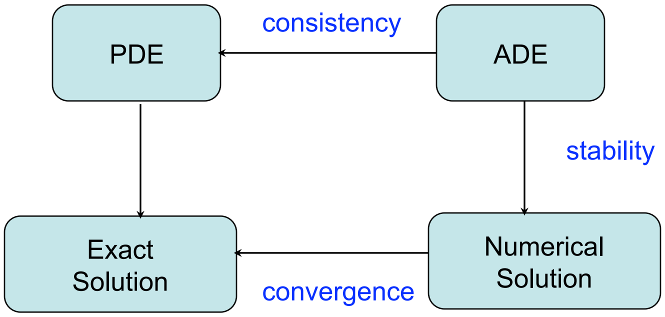
Numerical Simulations



What we would like…

What we will get!

Properties of Numerical Methods



What we would like…

What we will get!