ME470: Uncertainty Quantification Outline #13
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PDEs with Random Inputs

- Random field models are often used to represent uncertain inputs to a physical system.
- PDE models with random inputs are useful for representing physical systems with uncertainty.
- The random inputs add new dimensions to the solution, and discretization procedures must account for these added dimensions. Some call this discretizing the stochastic space (but they're really just input parameters).
- All of the polynomial techniques we have discussed apply the outputs of the PDE models with random inputs.

Goals for class

- See how PDE models with random inputs are characterized and discretized.
- Understand how the polynomial approximation methods are applied in this context.

Notation & Definitions

- A random field $\alpha = \alpha(x, \omega)$ with finite variance admits a decomposition

$$\alpha(x, \omega) = \mu(x) + \sum_{i=1}^{n} \sqrt{\lambda_i}\phi_i(x)\eta_i(\omega),$$

  where $\mu(x)$ is the mean, $(\lambda_i, \phi_i(x))$ are the eigenpairs of the covariance function of the field, and $\eta_i(\omega)$ are zero-mean, uncorrelated random variables.

- For PDEs with random inputs, we seek a solution $u = u(x, s)$ defined on the domain $D \times \mathcal{S}$ (where D is the spatial domain and $\mathcal{S}$ is the stochastic space) such that

$$\mathcal{L}(u, s) = f$$

  for all $s \in \mathcal{S}$ subject to boundary conditions $u = g$ on ∂D.