Lecture 2: eBay, Auctions

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Last Lecture: Long tail phenomena

- Last lecture: Descriptive formalization of long-tail, along with one example process
- Long tail of query frequency in: Search
- i^{-2} versus 2^{-i}
- Kim Kardashian (not on h/w or midterm)
- Online retailers (Large fraction of sales from good not in physical stores for online retailers.)

Importance of Math in Analysis:

- Interview of an analyst at Google: Interview

What problem does eBay solve?

- Creation Myth: eBay created to help founder's fiancee sell Pez dispensers.
- What sells on eBay?
  - Let us investigate: http://labs.ebay.com/Catman/
  - Which categories are easier to price?
  - Collectibles, Art: Hard to identify inherent value for such items, or to price them

Single-item Auctions

- Let us model the 'eBay Problem'.
- "All models are wrong, but some are useful..." P.E. Box.
- There is a single-item on sale
- We have a set B of potential buyers. Every $i \in B$ has a *value* v_i for the item
  - *value* = "maximum willingness to pay"
- First-cut: Ask for values, give item to buyer with highest value, charge this price
- Problem: People are unlikely to bid their value
  - Modify model: Every bidder $i \in B$ has a *bid* b_i
  - b_i need not equal v_i
  - v_i is `private' information[1]
- Let us analyze 'first-cut'
  - What would you bid?
    - If I expect to be the winner, I would undercut my bid so long as I don't lose

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[1] By 'private', we mean that the buyer is willing to reveal this information so long as it does not hurt him. We will later study a different model where the agent would not like his information revealed under any circumstances.

- Every bidder must have a handle on other bidder's willingness to pay, and their strategies
- Every bidder is doing this simultaneously
- What a pain!
- Can we set up the auction to *incentivize* bidders to set $b_i = v_i$?
    - That is, truth-telling maximizes utility for all possible bids b_{-i} of other bidders
        - *utility* = $(v_i - p_i)\, x_i$
        - $x_i = 1$ if item is sold to bidder i, 0 otherwise
    - *Called Incentive-compatibility/truthfulness/strategyproofness*

Strategyproofness in Single-item auctions

- Observation: Offering each bidder a preset price p is *incentive-compatible*
- That is, offer the item at this price to each buyer till one of them accepts the good.
    - This is informally what we call a Take-it-Or-Leave-It (TILI) offer.
    - TILIs are incentive compatible (informal)
        i. Claim 1: Suppose value is less than price, then bidding anything less than the price yields zero utility. Bidding larger than the price yields negative utility. So why lie?
        ii. Claim 2: Suppose value is larger than price, then bidding larger than the price wins the item at the fixed price. and yields the same positive utility. Whereas bidding less than price yields zero utility. So why lie again?
- Notice: This is the same as standard pricing mechanisms
- This does not do any value discovery :(
- A more formal definition of a TILI offer:
    - Property 1: There is a fixed, bid-independent price p.
    - Property 2: If the bidder bids more than p, allocate the item to the bidder and charge this price. Else don't allocate.
- Clearly, we could condition each bidders TILI on other bidders' bids, and still have an incentive compatible mechanism by essentially the same proof as above
- For instance, offer each bidder the median of the the other bidder's bids.
    - Problem: Breaks the constraint that we only have one item to sell
    - E.g.: Consider 5 buyers with bids: 5,4,3,2,1. We end up selling the item to two buyers.
- We could instead offer each bidder a the max of the the other bidder's bids
    - Assuming there are no ties, supply constraint is fine
    - E.g.: Consider 5 buyers with bids: 5,4,3,2,1. We sell the item to the first buyer at price=4.
- Does anyone recognize this auction?
- This is the celebrated Vickrey or 'second-price' auction
- eBay auction is a dynamic variant of this auction. Do you see why?
    - See section on 'Incremental Bidding' here: http://pages.ebay.ca/help/buy/bidding-overview.html

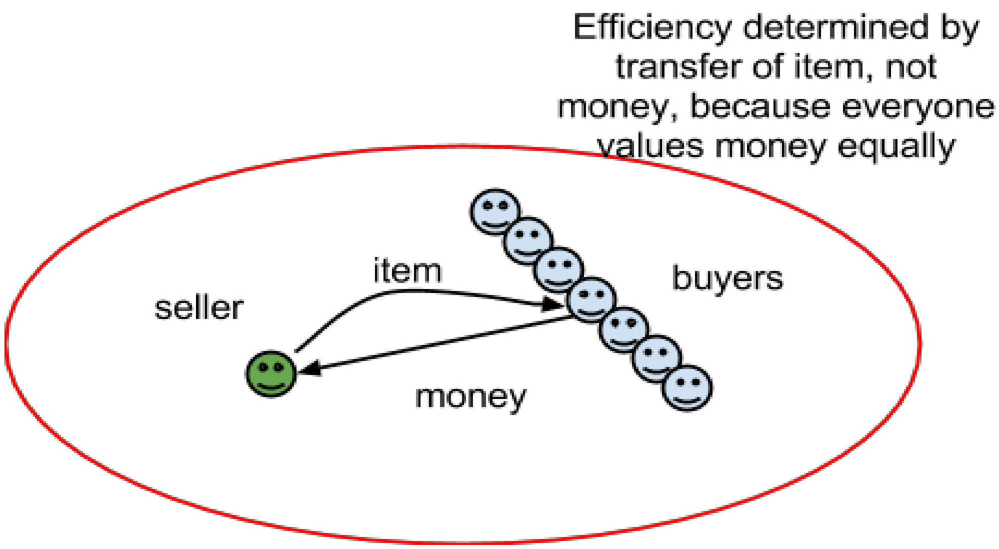
- But clearly, there are other incentive-compatible auctions. So why this one?
- Before we discuss which auction to pick, let us identify the space of auctions that satisfy incentive compatibility
- An auction is a protocol that takes bids as input, and produces an allocation and prices for the winners as output.
- A **threshold based** auction is one where there is a threshold specific to each bidder that is independent of its bid (i.e., depends only on other bidder's bids). The bidder wins if and only if it bids above the threshold, and is charged this threshold if and only if it wins.
- Lemma: An auction satisfies incentive compatibility if and only if it is threshold based.
- Proof:
    - Notice that we already hand waved the 'if' direction above. We now prove the other direction.
    - Claim 1: Fix a bidder i. Fix other bids b_{-i}. Bidder i's winning price is invariant of its bid. That is the winning price is a function of b_{-i} : $p_i(b_{-i})$
        i. Example that necessitates this: For a fixed set of bids by the other bidders, Suppose bidder i has two winning bids of 10$ and 15$, but pays 6$ in the first case and 8$ in the second, it would benefit by lying when its value is 15$
    - Claim 2: Notice that it is not enough for the winning price to be bid independent for it to be a threshold based auction. We also need to assert *when* the item is allocated to the bidder. So bidder i must win the item whenever its bid is higher than $p_i(b_{-i})$, and lose otherwise.
        i. Example that necessitates this: For a fixed set of bids by the other bidders, Suppose bidder i is charged a price 10$ on winning, but does not win the item when it bids 12$, but does win the item when it bids 15$, then it benefits by lying when its value is 12$.
- Note that Claim 1 and Claim 2 parallel Property 1 and Property 2 mentioned in the formal definition of a TILI offer.
- Check: Which claim does the first price auction not satisfy?
- Check that the second price auction satisfies both claims
- Notice this does not take care of the supply constraint

Additional references (be wary of notational differences):

- See Section 9.3.1. in this book for a proof of the incentive compatibility of the second price auction.

Economically 'Efficient' Auctions

- Efficiency = total value realized



- Assume that cost of manufacture = 0, and seller has no zero value for item
- To maximize efficiency, we must allocate item to buyer with highest value
- Notice that second price auction maximizes efficiency and is incentive compatible!
- **Notice that you cannot make a similar claim for the first price auction; do you see why?**

How does eBay add value?

- How does eBay increase efficiency?
    - Connects buyers and sellers globally (This is generally true of many e-commerce companies)
    - Implements a value discovery mechanism for sellers
    - Implements item search for buyers
    - Enables trades that may otherwise not occur

- Possible to make money as a side-effect. We will make this explicit in the context of auction soon. **Focus on adding value, and money will come**. e.g. Google, Facebook, Twitter.

Other questions to think about:

- Compare Amazon and eBay markets
- When will the efficient auction not make money?