Lecture 5: The Incorrectly Generalized Generalized Second Price Auction, Reserve Prices

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Reserve Prices

- Reserve prices are useful in a variety of situations:
  - seller values item
  - seller has a ‘offline’ buyer lined up
  - item costs money to produce
  - there are too few bidders
  - top bid is way higher than second highest one

- We focus for the moment on the last two bullets
- Simple exercise: One bidder, value drawn from distribution F. We would like to maximize expected revenue.
  - Recall incentive compatible auction must set bid-independent threshold
  - So all we can do here is to set a price!
  - To optimize revenue, maximize p (1-F(p))
  - In general, higher prices will yield low probability of sale, but better revenue on sale. We need to find the optimal trade-off between revenue on sale and probability of sale.
  - Example: For uniform distribution U[0,1], optimize p (1-p)). Differentiate, equate to zero: 2p=1, so optimal price is a half, probability of sale is a half, revenue is 1/4.
- How should we set a reserve within a single item auction?
  - Consider a single item auction. Suppose we employ a second price auction with a reserve. That is sale occurs if and only if the highest bidder’s bid is higher than the reserve, in which case it pays the maximum of the second highest bid and the reserve.
  - Notice that reserves are not always binding, second price can raise revenue too
  - Consider the model where bidder’s bids are drawn independently and identically from a specific distribution. How do you expect the optimal reserve to vary as a function of the number of bidders?
  - **First Surprise:** The optimal reserve price does not vary with the number of bidders!
  - Recall that the for the uniform distribution U[0,1], with one bidder, the optimal reserve price was 1/2. Let us see what the optimal reserve price is with 2 bidders.
    - 2*r *F(r) [1-F(r)] +2*r1 t*f(t)*[1-F(t)] dt

        - = 2*r*r [1-r] + 2 *r1 t[1-t] dt
        - = 2*r2 * (1-r) + 2 [1/2 - 1/3 - r2/2 + r3/3]
        - Now differentiate and set to zero: - 4r2 + 2*r = 0
        - Optimal reserve price is still 1/2!
    - In general, it is not even clear that a second price auction with a reserve is the best way to maximize revenue.
    - **Second Surprise:** The revenue maximizing single-item auction with values distributed IID is a second price auction with a reserve!
    - The form of the revenue optimal auction was identified in a seminal paper by Roger Myerson, who won a Nobel Prize in economics in 2007.

- How much did we gain by setting a reserve with bids drawn IID from U[0,1]?
    - When there was one bidder, revenue goes from 0 to 1/4.
    - When there are two bidders, revenue goes from 1/3 to 1/3 + 1/12.
- In general, as intuition would suggest, the gain from reserve prices decreases as the number of bidders goes up.
- In practice setting good reserves only possible if general range of bids is known. Probably better to err on the lower side, because we make revenue of second pricing anyway. Also, it is very important to encourage bidding. This is useful for efficiency and revenue.

- **On eBay, there are two forms or reserves:**
    - http://ebay.com/help/sell/starting_price.html
    - **Start price**
        - Used to shorten bidding process or as a reserve
    - **Secret reserve**
        - Empirically observed that low starting price encourages bidding. So, secret reserve prevents sale below certain price.

The Generalized Second Price Search Auction

- Initially, Yahoo used a first-price auction, which resulted in volatile bidder behavior
    - see Page 3 of this paper
- Google switched to a second-price auction
    - second price auction for each position, remove winner repeat with next position
    - As we discussed in last lecture, this is not incentive compatible
- Surprisingly, Bing, Yahoo, and Google all still use this auction
- One explanation for this is that the second price auction is stable enough
    - Recall payments of the incentive compatible auction: ith highest bidder pays: j > i(qj-1 - qj) bj
    - In contrast, in a second price auction, the bidder pays: j > i(qj-1 - qj) bi+1
    - Suppose that the position weights and bids both fall off geometrically. E.g., the

bids of the bidders are 16, 8, 4, 2, 1. And the position weights are 1, 0.5, 0.25, 0.125, 0.0625. A first price auction would charge the highest bidder 16, a second price auction would charge the bidder 8, the incentive compabtible auction will charge the bidder 5.25. Notice that the reduction in price from the first-price auction to the second price auction goes most of the way towards incentive compatible price.

End of lecture