MS&E 233: HW #1

Spring 2012-13, Prof. Ashish Goel, Mukund Sundararajan

Due in-class Tuesday, April 23

1. Consider a slight variant of the model for long tail phenomena described in class: Suppose the t-th new user in a system consumes one new product, which we will call product t. This user also consumes k existing products, each chosen with probability proportional to the number of times that product has been consumed already. Let $m_i(t)$ denote the number of times product i has been used up to and including time t. Derive an (approximate) expression for $m_i(t)$, and determine whether the popularity of products follows a long tail. If it does, how does the exponent e of i in your expression change with k? What is its limiting value as k tends to ∞?

2. The file movies-hw1.txt is an extract from somewhat old netflix data. The file contains the number of times the i-th movie was watched. Determine whether this data represents a long tail by finding the best fit of the form $a(b+i)^{-e}$. Explain your reasoning.

3. Consider a bidder who values a good at $10 and is participating in a sealed bid second-price auction. Consider three scenarios:

   (a) She bids $12.00

   (b) She bids $8.00

   (c) She bids $10.00

   Let x denote the highest bid made by the remaining bidders.

   (i) For each of the ranges $0 < x < 8; 8 < x < 10; 10 < x < 12; 12 < x$, write down the utility derived by the bidder as a function of x for each of the three scenarios.

   (ii) Use this to show that the bidder is never worse off bidding $10 compared to $8 or $12

(iii) Find a value of x such that the bidder is STRICTLY better off bidding \$10 compared to \$8

(iv) Find a value of x such that the bidder is STRICTLY better off bidding \$10 compared to \$12

(v) How do (i)-(iv) above change if we have a reserve price in the auction?

4. Recall in the lecture that we showed that the Vickrey auction is incentive-compatible and efficient. Show that it is the unique single-item auction with both these properties.

5. Your grandmother passed away and left you an antique rifle. You have no value for the rifle. A local collector is willing to pay a \$250 for it. eBay has an insertion fee of \$2 per listing, and takes a 10% cut on the eventual sales price. You are guaranteed to get one bidder on eBay (the bonus question makes the problem harder and more meaningful), whose willingness to pay is drawn from uniform distribution with range [0,1000]. If the rifle does not get sold on eBay the local collector will be still around the buy it from you.

   What is your optimal strategy, and what is your expected profit from it? We are seeking an optimal strategy of the form of picking a reserve price on eBay, or selling directly to the collector. **Bonus:** Suppose instead that you have 5 potential buyers with valuations distributed IID in the range uniform[0,1000]. What is your optimal strategy then? Note that we insist on an incentive compatible auction, i.e., you must run a second price auction with a reserve.

6. Either prove that these single-item auctions are incentive compatible or supply an example that demonstrates otherwise:

   (a) Given a weight w_i for each buyer, give item to bidder with highest product $w_i b_i$, charge $w_2 b_2 / w_1$, where b_2 is the second highest bid.

   (b) Give item to highest bidder, charge second highest bid if second highest bid is at least half of the highest bid. Charge half the highest bid otherwise.

   (c) Fix a sequence on the bidders. Inspect the first three bids in this sequence. Run a second price auction among the other bidders with a reserve price = max of the first three bids.