Chapter 14

Real Options
Real Options

• Real options is the analysis of investment decisions, taking into account the ability to revise future operating decisions.

• When valuing real assets, it is often helpful to use discounted cash flow (DSF) and option pricing methods to perform a valuation.
DCF and Option Valuation for a Single Cash Flow

- Example: Project 1
  
  - A project will generate a single cash flow, $X$, occurring at time $T$. The cash flow will be $X_u$ if the economy is doing well—an event with probability $p$—and $X_d$ if the economy is doing poorly. Projects with comparable risk have systematic risk $B$, therefore an effective annual expected rate of return of

  $$\alpha = r + \beta (r_m - r)$$

  - Where $r$ is the risk-free rate and $r_m$ the expected return on the market.
DCF and Option Valuation for a Single Cash Flow (cont’d)

• **DCF Valuation**

• The standard discounted cash flow method for finding the value of the project, $V$, calls for computing the expected cash flow, $C$, and using as a discount rate the expected return on a project of comparable risk:

$$V = \frac{E(X)}{(1 + \alpha)^T} = \frac{pX_u + (1 - p)X_d}{(1 + \alpha)^T}$$  \hspace{1cm} (14.1)
The expected return on an asset with the same risk as Project 1 is

\[ \alpha = 0.06 + 1.25 \times (0.10 - 0.06) = 0.11 \]

The expected cash flow is

\[ E(X) = 0.571 \times 51.361 + 0.429 \times 31.152 = 42.692 \]

Using equation 14.1, the present value of the project cash flow is

\[ V = \frac{42.692}{1 + 0.11} = 38.461 \]
DCF and Option Valuation for a Single Cash Flow (cont’d)

- DCF and risk-neutral valuation give the same answer, but in a given circumstance one may be easier than the other to implement.
- Even if we plan to perform a risk-neutral valuation, a DCF valuation may be necessary in order to obtain a forward price.
- The appropriate DCF discount rate will vary with project characteristics, whereas the risk-neutral probability remains the same as along as the economic variable determining the cash flow ($X$) remains the same.
Multiperiod Valuations

- A preliminary DCF valuation is required in order to characterize the risk of the project.
- Static DCF, using a constant discount rate, is a correct discounting method only when risk resolves at a constant rate over time.
Real Options in Practice

• The decision about whether and when to invest in a project ~ call option

• The ability to shut down, restart, and permanently abandon a project ~ project + put option

• Strategic options: the ability to invest in projects that may give rise to future options ~ compound option

• Flexibility options: ability to switch between inputs, outputs, or technologies ~ rainbow option
Commodity Extraction

• Incur the extraction costs to realize the value of the resource: defer investment and stop and start production

• Example: single-barrel extraction under certainty
  – A plot of land contains one barrel of oil that can be extracted by paying $13.60. Currently, oil sells for $15/barrel, effective annual lease rate $\delta$ is 4%, and effective annual risk-free rate is 5%
  – How much is the land worth?
    • $1.40 (= $15 – $13.60) [no real options, immediate extraction]
    • $1.796 [considering real options, trigger price $16.918 at $t = 12.575$]

• If the lease rate of an extractive commodity is zero, it is best to leave the commodity underground forever
Real Options in Practice (cont’d)

- Peak-Load Electricity Generation
  - Plant idle when price of electricity is less than the cost of fuel
  - Plant online when electricity price spikes or fuel price declines
  - Similar to owning a strip of call options on electricity expiring daily, with a strike price of cost of variable inputs
  - Spark spread: $S_{elec} - H \times S_{gas}$, where $H$ is plant efficiency measure
  - Profit = $\max(S_{elec} - H \times S_{gas}, 0)$, a European exchange option

\[
\text{Value of Plant} = \sum_{i=1}^{n} BSCall(F_{E,t_i}, H \times F_{G,t_i}, \hat{\sigma}_{t_i}, r_i, t_i, r)
\]

- By rewriting using put-call parity

\[
= \sum_{i=1}^{n} e^{-r t_i} (F_{E,t_i} - H \times F_{G,t_i}) + \sum_{i=1}^{n} BSPut(F_{E,t_i}, H \times F_{G,t_i}, \hat{\sigma}_{t_i}, r_i, t_i, r)
\]

Static NPV Option Not to Operate
Real Options in Practice (cont’d)

- Pharmaceutical Research and Development
  - Costs incurred to acquire technology to be used in future projects
  - Projects in the future only undertaken if they have positive NPV

**Figure 14.2** The development process for a new drug. Probabilities are the percentage of pharmaceutical drugs proceeding from one stage to the next. For example, 74% of drugs submitted for FDA approval receive it.

Real Options in Practice (cont’d)

**Figure 14.3** An example of staged investment. The value of the project, if developed, is in the top line at each node. The value of the option to develop the project is shown below the value of the project. In each year, it is necessary to pay the amount in the Investment row to keep the project alive in the next period. The tree is generated as a forward tree assuming $S = $100, $\sigma = 0.50$, $r = 0.10$, and $\delta = 0.15$. 

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<td>$0,000$</td>
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Chapter 14

Additional Art
Equation 14.1

\[ V = \frac{\mathbb{E}(X)}{(1 + \alpha)^T} = \frac{p X_u + (1 - p) X_d}{(1 + \alpha)^T} \]
Equation 14.2

Forward price = $F_1 = (1.06) \times $38.461 = $40.769$
Equation 14.3

\[ V \beta (r_M - r) = 38.461 \times 1.25 \times (0.04) = 1.923 \]
Equation 14.4

\[ F_{0,1} = E(X) - V \beta (r_M - r) = 42.692 - 1.923 = 40.769 \]
Equation 14.5

\[ V = \frac{E(X)}{1 + r + \beta (r_M - r)} \]
Equation 14.6

\[ F_{0,1} = V(1 + r) = E(X) - \beta V(r_M - r) \]
Equation 14.7

\[ V = \frac{p^* X_u + (1 - p^*) X_d}{(1 + r)^T} \]
Equation 14.8

\[ p^* X_u + (1 - p^*) X_d = F_1 \]
Equation 14.9

\[ p^* = \frac{F_1 - X_d}{X_u - X_d} \]
Equation 14.10

\[ p^* = \frac{40.769 - 31.152}{51.361 - 31.152} = 0.476 \]
Equation 14.11

\[
\frac{X_{2u} - X_{2d}}{X_u - X_d} = \frac{\$16.361 - (-\$3,848)}{\$51.361 - \$31.152} = 1
\]
Equation 14.12

Elasticity = \frac{X_{2u} - X_{2d}}{X_u - X_d} \frac{V_1}{V_2} = \frac{16.361 - (-3,848)}{51.361 - 31.152} \times \frac{38.461}{5.443} = 7.067
Equation 14.13

\[
\frac{0.571 \times 16.361 + (1 - 0.0571) \times (-3.848)}{1 + 0.06 + 8.833 \times (0.10 - 0.06)} = 5.443
\]
Table 14.2  Cash flows, true probabilities, and risk-neutral probabilities from investments that pay risky cash flows for 3 years. Project 2 cash flows are project 1 cash flows less $35. Project 3 cash flows are the greater of project 2 cash flows and zero. The probabilities are the unconditional time 0 probabilities of reaching a given node, computed using either true or risk-neutral probabilities.

<table>
<thead>
<tr>
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<th>Project 2</th>
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<th>Project 3</th>
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Expected Cash Flows (true probabilities)

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<td>48.633</td>
<td>7.692</td>
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Expected Cash Flows (risk-neutral probabilities)

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True Probabilities

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Risk-Neutral probabilities

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| 0.079 | 1.144 |

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Equation 14.14

\[
\frac{42.692}{1.11} + \frac{45.566}{1.11^2} + \frac{48.633}{1.11^3} = 111.00
\]
Equation 14.15

$$\frac{\$40.769}{1.06} + \frac{\$41.553}{1.06^2} + \frac{\$42.352}{1.06^3} = \$111.00$$
Equation 14.16

\[
\frac{5.769}{1.06} + \frac{6.553}{1.06^2} + \frac{7.352}{1.06^3} = 17.448
\]
Equation 14.17

\[
\frac{7.786}{1.06} + \frac{9.503}{1.06^2} + \frac{11.180}{1.06^3} = 25.190
\]
Equation 14.18

\[ F_{0,T} = S_0 \left( \frac{1 + r}{1 + \delta} \right)^T \]
Equation 14.19

\[
\frac{S_T - x}{(1 + r)^T}
\]
Equation 14.20

\[
\frac{S}{X} < \frac{r_d}{\delta_d} \cdot \frac{1}{1 + \delta_d}
\]
Equation 14.21

Price of perpetual call = \frac{K}{h - 1} \left( \frac{h - 1}{h} \frac{S}{K} \right)^h
**Figure 14.1** Forward price (top panel) and volatility (middle panel) curves for electricity and natural gas. The bottom panel depicts the spark spread implied by the forward price curves, assuming a heat rate of 9000.
Equation 14.22

\[
\text{Value of plant} = \sum_{i=1}^{n} \text{BSCall}(F_{E,i}, H \times F_{G,i}, \hat{o}_{t_i}, r, t_i, r)
\]
Equation 14.23

\[
\sum_{i=1}^{n} \left[ e^{-r_{i}}(F_{E,t_{i}} - H \times F_{G,t_{i}}) + \text{BSPut}(F_{E,t_{i}}, H \times F_{G,t_{i}}, \hat{\sigma}_{t_{i}}, r, t_{i}, r) \right]
\]

\[
= \sum_{i=1}^{n} e^{-r_{i}}(F_{E,t_{i}} - H \times F_{G,t_{i}}) + \sum_{i=1}^{n} \text{BSPut}(F_{E,t_{i}}, H \times F_{G,t_{i}}, \hat{\sigma}_{t_{i}}, r, t_{i}, r)
\]

Static NPV

Option not to operate
Equation 14.23

\[\sum_{i=1}^{n} \left[ e^{-rt_i}(F_{E,t_i} - H \times F_{G,t_i}) + \text{BSPut}(F_{E,t_i}, H \times F_{G,t_i}, \hat{\sigma}_t, r, t_i, r) \right]\]

\[= \sum_{i=1}^{n} e^{-rt_i}(F_{E,t_i} - H \times F_{G,t_i}) + \sum_{i=1}^{n} \text{BSPut}(F_{E,t_i}, H \times F_{G,t_i}, \hat{\sigma}_t, r, t_i, r)\]

Static NPV  
Option not to operate
Equation 14.24

\[
\frac{pX_{2u} + (1 - p)X_{2d} - \left\{ \left[ \frac{(X_{2u} - X_{2d})}{(X_u - X_d)} \right] V\beta (r_M - r) \right\}}{1 + r} = V_2
\]
Equation 14.25

\[
\frac{p^*X_{2u} + (1 - p^*)X_{2d}}{1 + r} = V_2
\]

where

\[
p^* = p - \frac{V \beta (r_M - r)}{X_u - X_d}
\]