ENGINEERING RISK ANALYSIS
(M S & E 250 A)

VOLUME 1

CLASS NOTES

SECTION 2

ELEMENTS OF DECISION ANALYSIS

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PART 2

DECISION ANALYSIS

SECTION 5

INTRODUCTION

5.1 RISK ANALYSIS AND DECISION ANALYSIS

RISK ANALYSIS + DECISION CRITERION → DECISION

EXAMPLE: "MIN. MAX LOSS," OR P(F) PER YEAR < THRESHOLD

5.2 THE DECISION ANALYSIS CYCLE

<u>DETERMINISTIC PHASE</u>

- DEFINE RELEVANT VARIABLES (DECISION VARIABLES AND SYSTEM'S PARAMETERS)
- CHARACTERIZE THE RELATIONSHIPS BETWEEN THEM
- ASSIGN VALUES TO THE POSSIBLE OUTCOMES
- PERFORM SENSITIVITY ANALYSIS

<u>PROBABILISTIC PHASE</u>

- ASSIGN PROBABILITY DISTRIBUTIONS TO THE CRITICAL VARIABLES
- DERIVE PROBABILITY DISTRIBUTION FOR THE OUTCOMES
- ENCODE RISK ATTITUDE AND UTILITY FROM THE DECISION MAKER
- CHOOSE THE BEST ALTERNATIVE (MAX. EXPECTED UTILITY)

<u>INFORMATIONAL PHASE</u>

- COMPUTE COST AND VALUE OF ADDITIONAL INFORMATION
- IF DESIRABLE, GATHER INFORMATION AND REPEAT THE FIRST THREE PHASES

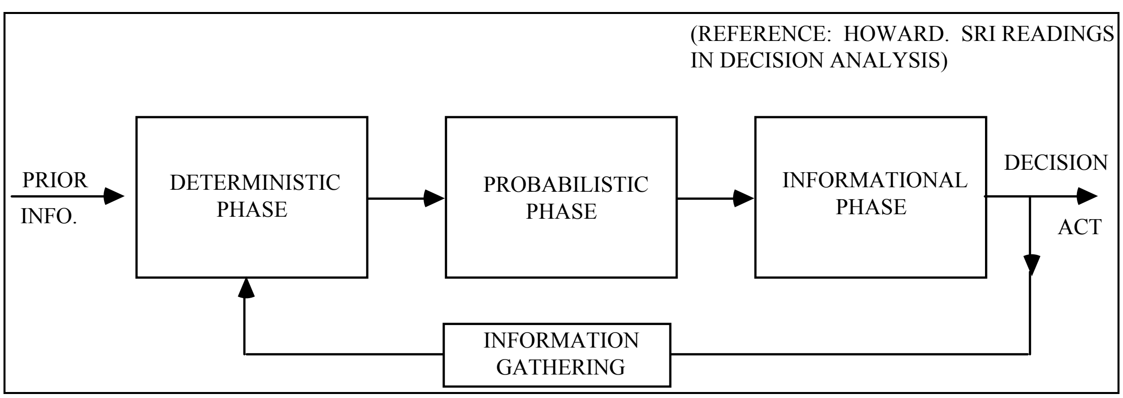


FIG. 5.1: THE DECISION ANALYSIS CYCLE

TWO TYPES OF PROBABILISTIC DEPENDENCIES:

FUNCTIONS: $X = f(Y)$: UNCERTAINTY IN Y $\Rightarrow$ UNCERTAINTY IN X

CONDITIONAL RELATIONS: $p(X \mid Y)$, $p(X \mid NY)$, $p(Y)$

$$\Rightarrow p(X) = p(X, Y) + p(X, NY)$$
$$= p(Y) \times p(X \mid Y) + p(NY) \times p(X \mid NY)$$

THE THEORY IS:

- NORMATIVE
- AXIOMATIC
- RESULTS IN SEPARATION OF FACTS AND PREFERENCES (VALUES) AND MAXIMIZATION OF EXPECTED UTILITY
- DESIGNED FOR AN INDIVIDUAL DECISION MAKER

5.3 DECISION MAKING. DESCRIPTIVE MODEL

DECISION MAKING (DESCRIPTIVE)
From Howard: MS & E 252

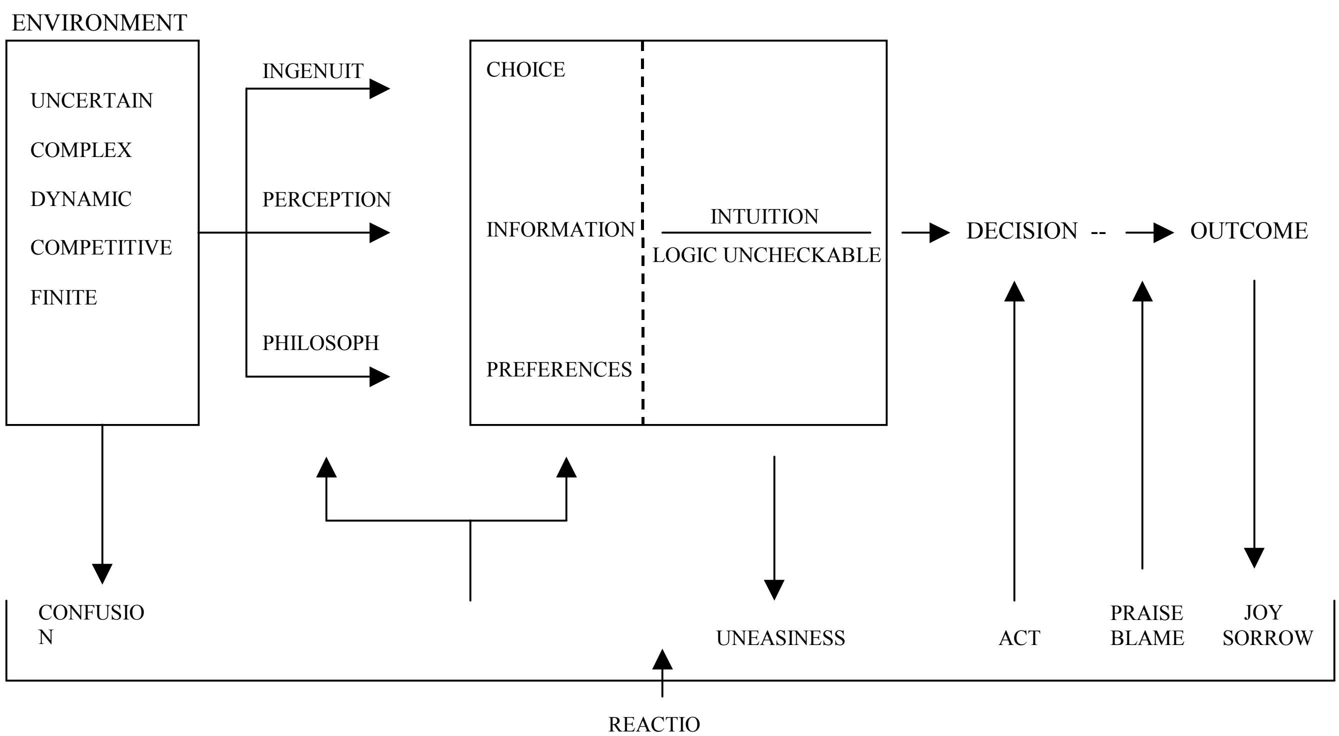


FIG. 5.2: DECISION MAKING: A DESCRIPTIVE MODEL

5.4 DECISION MAKING. NORMATIVE OR PRESCRIPTIVE MODEL

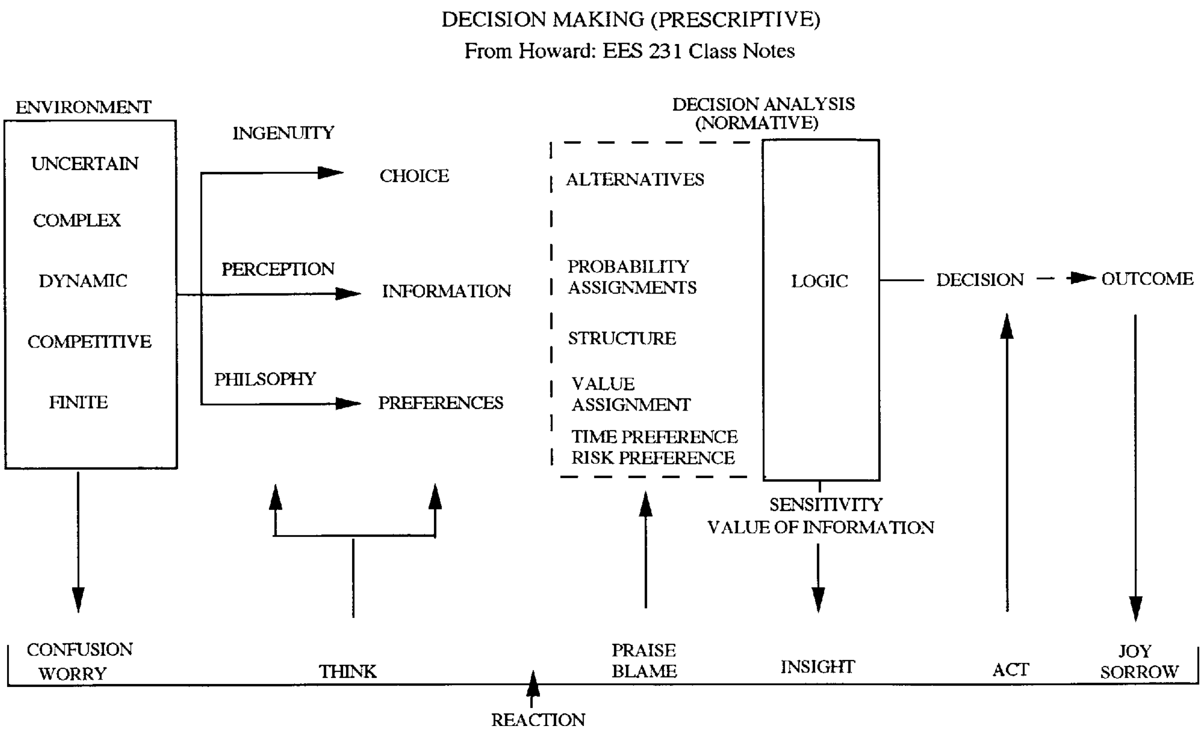


FIG. 5.3: DECISION MAKING: A PRESCRIPTIVE MODEL

5.5 INDIVIDUAL CHOICES UNDER UNCERTAINTY

- VALUE OF OUTCOMES AT A GIVEN TIME

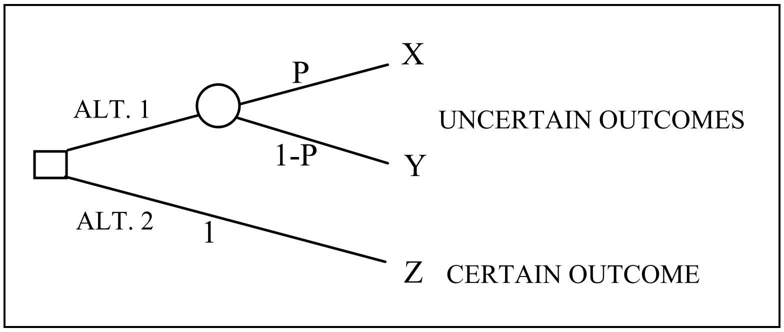


FIG. 5.4: A LOTTERY AT TIME T

• VALUE OF OUTCOMES AT DIFFERENT TIMES

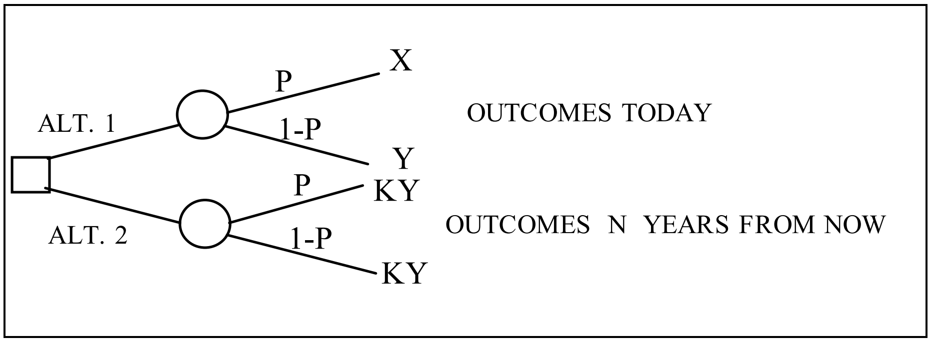


FIG. 5.5: A LOTTERY AT DIFFERENT TIMES

5.6 ELEMENTS OF DECISION THEORY

- STRUCTURE OF THE PROBLEM
  - DEFINITION OF THE <u>BOUNDARIES</u>
  - IDENTIFICATION OF ALTERNATIVES
  - LOGICAL RELATIONSHIP AMONG ELEMENTS
- PROBABILITY OF OUTCOMES
  BAYESIAN VS. CLASSICAL STATISTICS
- DECISION CRITERIA
  VALUATION OF OUTCOMES, ATTRIBUTES UNDER UNCERTAINTY (EX: LIVES, MONEY). LOTTERIES.
  RISK ATTITUDE FOR DIFFERENT ATTRIBUTES (IDENTICAL FOR ALL ATTRIBUTES IN A SINGLE OBJECTIVE APPROACH)
  TIME PREFERENCE

5.7 CHARACTERISTICS OF THE THEORY

- EMPHASIS ON GOOD DECISIONS (VS. GOOD OUTCOMES)
- FOCUSES ON DECISION OUTCOMES (ALTERNATIVES, INFORMATION AND PREFERENCES) AND NOT ON DECISION PROCESS
- THE THEORY IS NORMATIVE AND NOT DESCRIPTIVE; NORM BASED ON PREFERENCE AXIOMS (REF. VON NEUMAN, SAVAGE, DE FINETTI, HOWARD)
- THEORY IS DESIGNED FOR ONE DECISION MAKER OR HOMOGENOUS GROUP

5.8 CONSTRUCTION OF A DECISION TREE

- LIST ALL DECISION ALTERNATIVES (NO PROBABILITY THERE).
    - FOR EACH ALTERNATIVE LIST ALL POSSIBILITIES FOR FIRST EVENT. ASSESS PROBABILITY OF EACH POSSIBILITY.
    - FOR EACH CASE, CONSIDER ALL POSSIBILITIES FOR SECOND EVENT. ASSESS PROBABILITIES OF EACH, CONDITIONAL ON THE OCCURRENCE OF FIRST EVENT.
- ETC. FOR ALL SUBSEQUENT EVENTS.
    - WHEN ALL POSSIBLE SCENARIOS HAVE BEEN COMPLETELY DESCRIBED, ASSESS OUTCOME OF EACH SCENARIO.

PROBABILITY: SEE APPENDIX 1.

SCENARIO: (PATH IN THE TREE FROM DECISION TO OUTCOME)

- ALTERNATIVE: A
- EVENTS: B, C, D
- OUTCOME X

$$p(\text{OUTC. } X \mid \text{ALT. } A, B, C, D) = p(B \mid \text{ALT. } A) \times p(C \mid \text{ALT. } A, B) \times p(D \mid \text{ALT. } A, B, C) \times p(X \mid \text{ALT. } A, B, C, D)$$

SECTION 6

INVESTMENT AND ENGINEERING EXAMPLES

WE ASSUME FIRST A PARTICULAR TYPE OF DECISION MAKER WHO MAXIMIZES THE EXPECTED VALUE OF THE OUTCOMES. WE WILL SEE LATER THAT THERE IS A CONTINUUM OF RISK ATTITUDES CHARACTERIZED BY UTILITIES.

6.1 EXAMPLE 1: INVESTMENT DECISION

SINGLE STATE VARIABLE

A FIRM HAS IDENTIFIED THREE POTENTIAL OUTCOMES FOR AN INVESTMENT OF $1,000,000. THE TOTAL REVENUE FROM EACH INVESTMENT (INCLUDING THE PROFIT THAT WILL OCCUR IN LESS THAN A YEAR) ARE THE FOLLOWING:

A = $1,400,000	P(A) = 0.2
B = $1,200,000	P(B) = 0.5
C = $ 500,000	P(C) = 0.3

QUESTION:

SHOULD THEY INVEST IF THEY WANT TO MAXIMIZE EXPECTED VALUE OF OUTCOMES (EXPECTED-VALUE DECISION MAKERS)?

(1) DECISION TREE

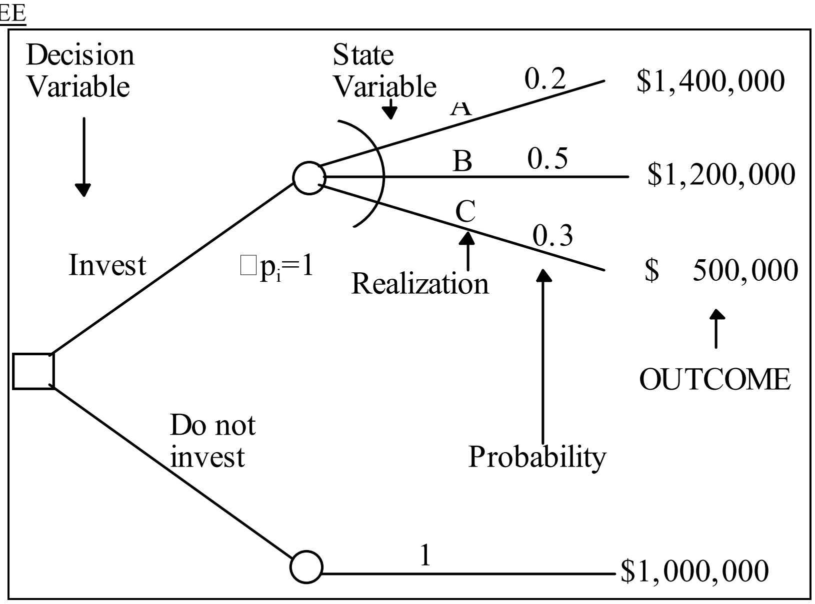


FIG. 6.1: AN INVESTMENT DECISION AT TIME T

(2) RESOLUTION

EV FOR THE DECISION TO INVEST: $1{,}400{,}000 \times 0.2 + 1{,}200{,}000 \times 0.5 + 500{,}000 \times 0.3 = \$1{,}030{,}000$

EV FOR THE DECISION NOT TO INVEST: $1{,}000{,}000 \times 1 = \$1{,}000{,}000$

(3) CONCLUSION:

THE EV DECISION MAKER ("RISK INDIFFERENT") WILL PREFER TO INVEST.

THE MOST "RISK-PRONE" DECISION MAKER WHO DECIDES TO MAXIMIZE THE MAXIMUM RETURN (1.4M, REGARDLESS OF PROBABILITY) WILL ALSO INVEST.

THE MOST "RISK-AVERSE" DECISION MAKER WHO WANTS TO MINIMIZE THE MAXIMUM POSSIBLE LOSS WILL DECIDE NOT TO INVEST (HE PREFERS 1M FOR SURE TO A POSSIBLE LOSS OF 500,000 REGARDLESS OF PROBABILITY). IN BETWEEN, THERE IS A CONTINUUM OF RISK ATTITUDES.

TWO STATE VARIABLES

ASSUME NOW THAT A, B, AND C CORRESPOND TO THREE DIFFERENT SALE VOLUMES CONDITIONAL ON THE STATE OF THE ECONOMY WITH THE FOLLOWING CONDITIONAL PROBABILITIES (THE TOTAL REVENUES FOR OUTCOMES A, B, AND C ARE THE SAME AS IN THE PREVIOUS EXAMPLE):

DATA:

STATES OF THE ECONOMY:	GOOD	(PROB. 0.25)	← DATA
	OK	(PROB. 0.45)	FROM YOUR
	BAD	(PROB. 0.3)	BEST EXPERT

DATA:

SALES CONDITIONAL ON THE STATE OF THE ECONOMY:

LEVEL A:	$P(A \mid G) = 0.6$	
	$P(A \mid OK) = 0.4$	
	$P(A \mid BD) = 0.1$	
LEVEL B:	$P(B \mid G) = 0.3$	← DATA
	$P(B \mid OK) = 0.45$	FROM YOUR
	$P(B \mid BD) = 0.3$	BEST EXPERT
LEVEL C:	$P(C \mid G) = 0.1$	
	$P(C \mid OK) = 0.15$	
	$P(C \mid BD) = 0.6$	

QUESTION: SHOULD THE EXPECTED-VALUE DECISION MAKER INVEST OR NOT?

(1) <u>DECISION TREE</u>

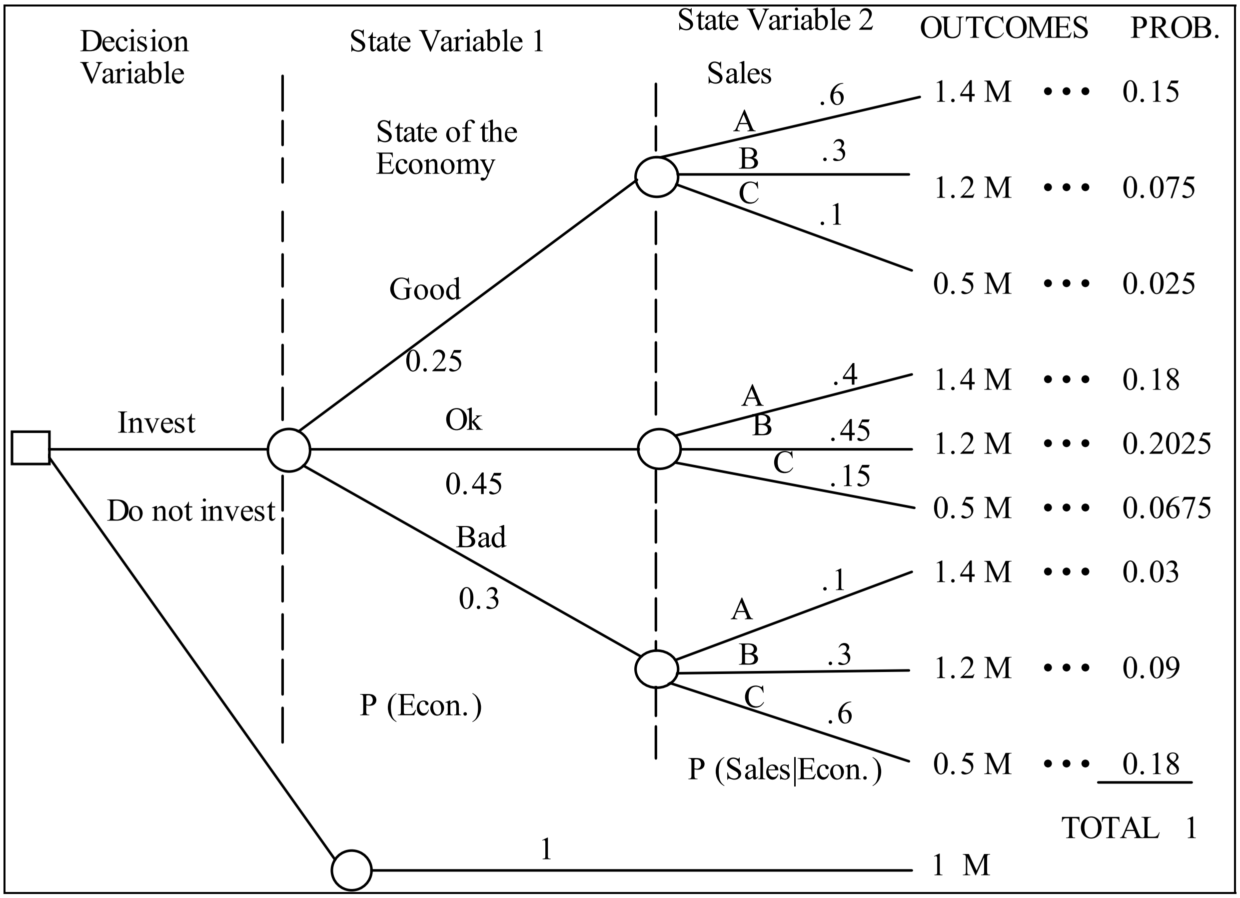


FIG. 6.2: DECISION TREE FOR THE CASE OF TWO STATE VARIABLES

(2) <u>RESOLUTION OF THE DECISION TREE:</u>

- IDENTIFY AND QUANTIFY OUTCOME OF EACH SCENARIO
- COMPUTE PROBABILITY OF EACH SCENARIO, EX. HERE:

P (ECON. IS GOOD AND SALES AT LEVEL A) $= P(G, A)$

$= P(G) \times P(A \mid G)$

$= 0.25 \times 0.6$

$= 0.15$

P (SCENARIO) = Π PROBABILITIES ON "PATH" THAT CHARACTERIZES THE SCENARIO

- COMPUTE EXPECTED VALUE FOR EACH DECISION BRANCH BY SUMMING PRODUCTS OF OUTCOMES AND PROBABILITY FOR EACH SCENARIO.

HERE: EV (INVEST) = 1.4 (0.15 + 0.18 + 0.03)
+ 1.2 (0.075 + 0.2025 + 0.09)
+ 0.5 (0.025 + 0.0675 + 0.18)
= $1,081,250

EV (DO NOT INVEST) = $1,000,000

(3) CONCLUSION:

THE EXPECTED-VALUE DECISION MAKER WILL INVEST, SO WILL THE MOST RISK-PRONE BUT NOT THE MOST RISK-AVERSE.

6.2 EXAMPLE 2: SEISMIC RISK MITIGATION

FACTS OF THE CASE:

A COMPANY IN THE SILICON VALLEY WANTS TO MAKE A SEISMIC SAFETY DECISION FOR A BUILDING CONTAINING VALUABLE EQUIPMENT.

ALTERNATIVES:

- REINFORCE THE BUILDING
- PURCHASE EARTHQUAKE INSURANCE
- DO NOTHING AND CONSIDER THAT YOU ARE "SELF-INSURED"

POSSIBLE EVENTS:

1. SEISMIC EVENTS EVERY YEAR
   - EARTHQUAKE OCCURS DURING THE YEAR: PROBABILITY 1/70 (ASSUME ONLY ONE EQ CAN OCCUR PER YEAR)
   - NO EARTHQUAKE OCCURS DURING THE YEAR: PROBABILITY 69/70

2. PERFORMANCE OF THE BUILDING IF THE EQ OCCURS:

WITHOUT REINFORCEMENT:

- HEAVY LOSSES: p(H | EQ) = 0.75
- LIGHT DAMAGE: p(L | EQ) = 0.2
- NO DAMAGE: p(Z | EQ) = 0.05

WITH REINFORCEMENT:

- HEAVY LOSSES H
  PROBABILITY OF H CONDITIONAL ON EQ OCCURRENCE = p'(H | EQ) = 0.3
- LIGHT DAMAGE L
  PROBABILITY OF L CONDITIONAL ON EQ OCCURRENCE = p'(L | EQ) = 0.5
- NO DAMAGE AT ALL Z
  PROBABILITY OF Z CONDITIONAL ON EQ OCCURRENCE = p'(Z | EQ) = 0.2

OUTCOMES:

- HEAVY LOSSES: $2,000,000
- LIGHT LOSSES: $ 700,000
- NO DAMAGE: $ 0

INSURANCE COST: $ 30,000 PER YEAR

INSURANCE REIMBURSEMENT: $100,000 DEDUCTIBLE

COST OF REINFORCEMENT: EQUIVALENT UNIFORM ANNUAL COST = $5,000

QUESTION: WHAT SHOULD BE THE DECISION OF THE EV DECISION MAKER? WHAT SHOULD BE THE DECISION OF THE MOST RISK-AVERSE AND THE MOST RISK-PRONE DECISION MAKER?

NOTE: THE TIME FRAME IS ONE YEAR BOTH FOR MARGINAL PROBABILITIES AND FOR THE ECONOMIC DESCRIPTION OF THE OUTCOMES.

(1) DECISION TREE OUTCOME

PROBABILITY

FOR ONE YEAR

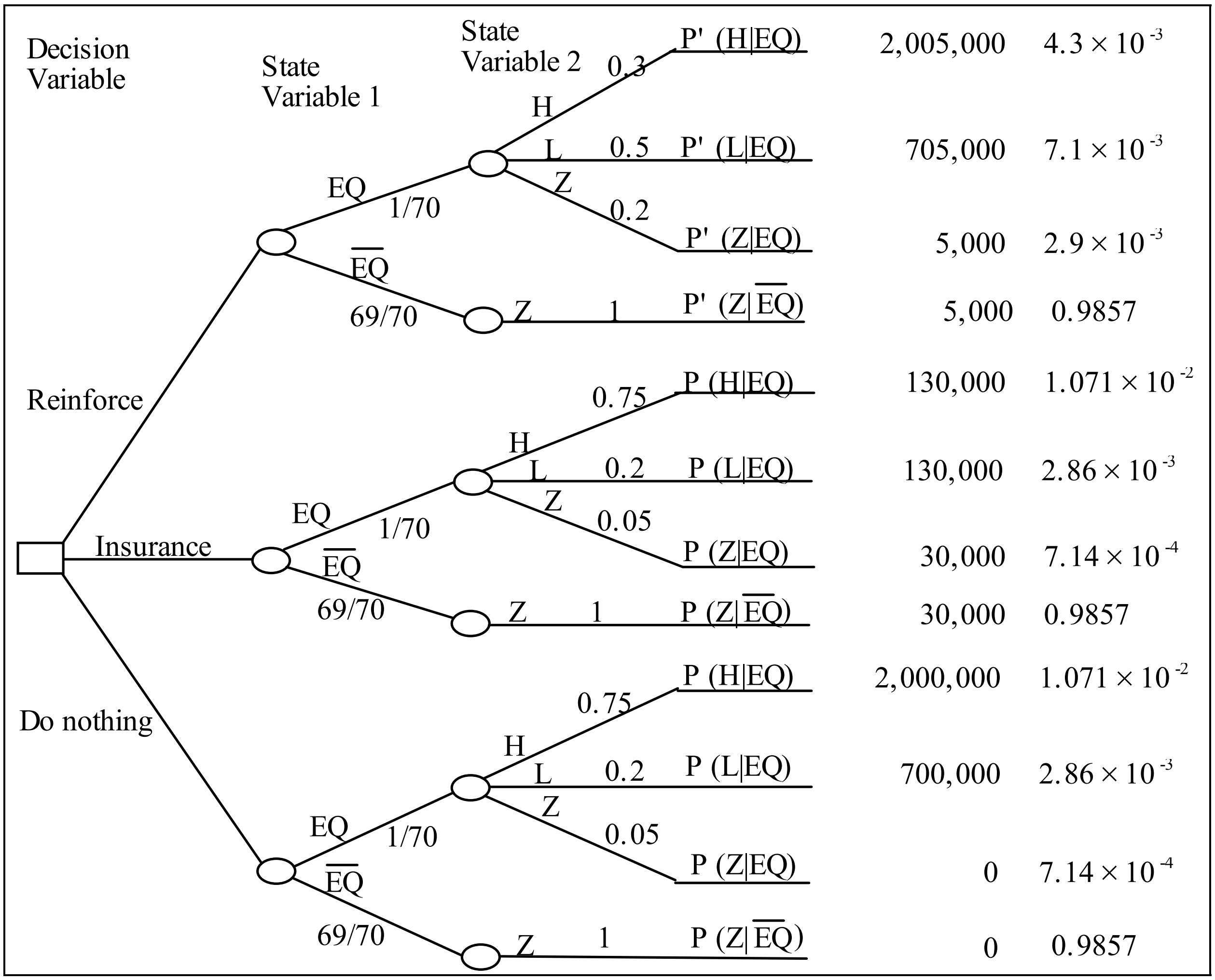


FIG. 6.3: DECISION TREE FOR THE SEISMIC RISK REDUCTION PROBLEM

NOTE: CONSISTENCY OF TIME UNITS (COSTS AND PROBABILITIES)

(2) RESOLUTION

$$
\begin{aligned}
\text{EV(REINFORCEMENT)} &= 2{,}005{,}000 \times 4.3 \times 10^{-3} \\
&+ 705{,}000 \times 7.1 \times 10^{-3} \\
&+ 5{,}000 \times 2.9 \times 10^{-3} \\
&+ 5{,}000 \times 0.9857 \\
&= \$\ 18{,}571.43 \ \text{(PER YEAR)}
\end{aligned}
$$

EV (INSURANCE) = \$31,364.18

EV (DO NOTHING) = \$23,428

3) CONCLUSION

THE EV DECISION MAKER CHOOSES REINFORCEMENT SOLUTION.

THE MOST RISK-PRONE (MAXIMAX) DM DOES NOTHING.

THE MOST RISK-AVERSE (MINIMAX) DM CHOOSES THE INSURANCE.

6.3 EXAMPLE 3: SEQUENTIAL DECISIONS AND INFORMATION GATHERING

FACTS OF THE CASE:

THE SAME FIRM AS IN SECTION 6.1 FACES THE SAME INVESTMENT PROBLEM (1 MILLION DOLLARS INVESTMENT; OUTCOME OF SALES A (HIGH), B (MEDIUM) AND C (LOW) WITH RESPECTIVE PROBABILITIES 0.2, 0.5, AND 0.3; RETURN AND PROFIT IN LESS THAN ONE YEAR: \$1,400,000 FOR A, \$1,200,000 FOR B, AND \$500,000 FOR C).

THIS TIME, THE DECISION MAKER CAN HIRE A CONSULTANT FOR THE PRICE OF \$50,000. THE CONSULTANT'S RECORD SHOWS THE FOLLOWING CORRESPONDENCE BETWEEN PREDICTIONS AND OCCURRENCE OF STATES: (NOTATION FOR THE MESSAGE: "X")

DATA	OCCURRENCE OF STATE				PROBABILITY OF PREDICTION
PREDICTION	A	B	C		
"A"	0.667	0.208	0.125		P("A") = 0.24
"B"	0.038	0.849	0.113	= P(X | "Y")	P("B") = 0.53
"C"	0.087	0.0	0.913		P("C") = 0.23

NOTE: CONSISTENCY WITH DATA OF SECTION 6.1:

$P(A) = P(A, \text{"A"}) + P(A, \text{"B"}) + P(A, \text{"C"}) = \sum P(\text{"Y"}) \times P(X{=}A \mid \text{"Y"}) = 0.2$

$P(\text{"A"}) = P(\text{"A"}, A) + P(\text{"A"}, B) + P(\text{"A"}, C) = 0.24$

UPDATING MECHANISMS:

$$p(X \mid \text{"Y"}) = \frac{p(X, \text{"Y"})}{p(\text{"Y"})} = \frac{p(X) \times p(\text{"Y"} \mid X)}{p(\text{"Y"})}$$

$p(X)$: priors

$p(\text{"Y"} \mid X)$: likelihood function

$p(X \mid \text{"Y"})$: posterior

$p(\text{"Y"})$: pre-posterior

NOTE: THE DATA ARE GIVEN EITHER UNDER THE FORM OF P(Y) AND P("X"|Y) (THE LIKELIHOOD FUNCTIONS) OR P("Y") AND P(X|"Y"). THE RELATIONSHIPS ARE THE SAME.

QUESTIONS: WHAT SHOULD BE THE STRATEGY OF THE EV DECISION MAKER? HIRE THE CONSULTANT?

INVEST ACCORDING TO THE PREDICTION?

(1) DECISION TREE

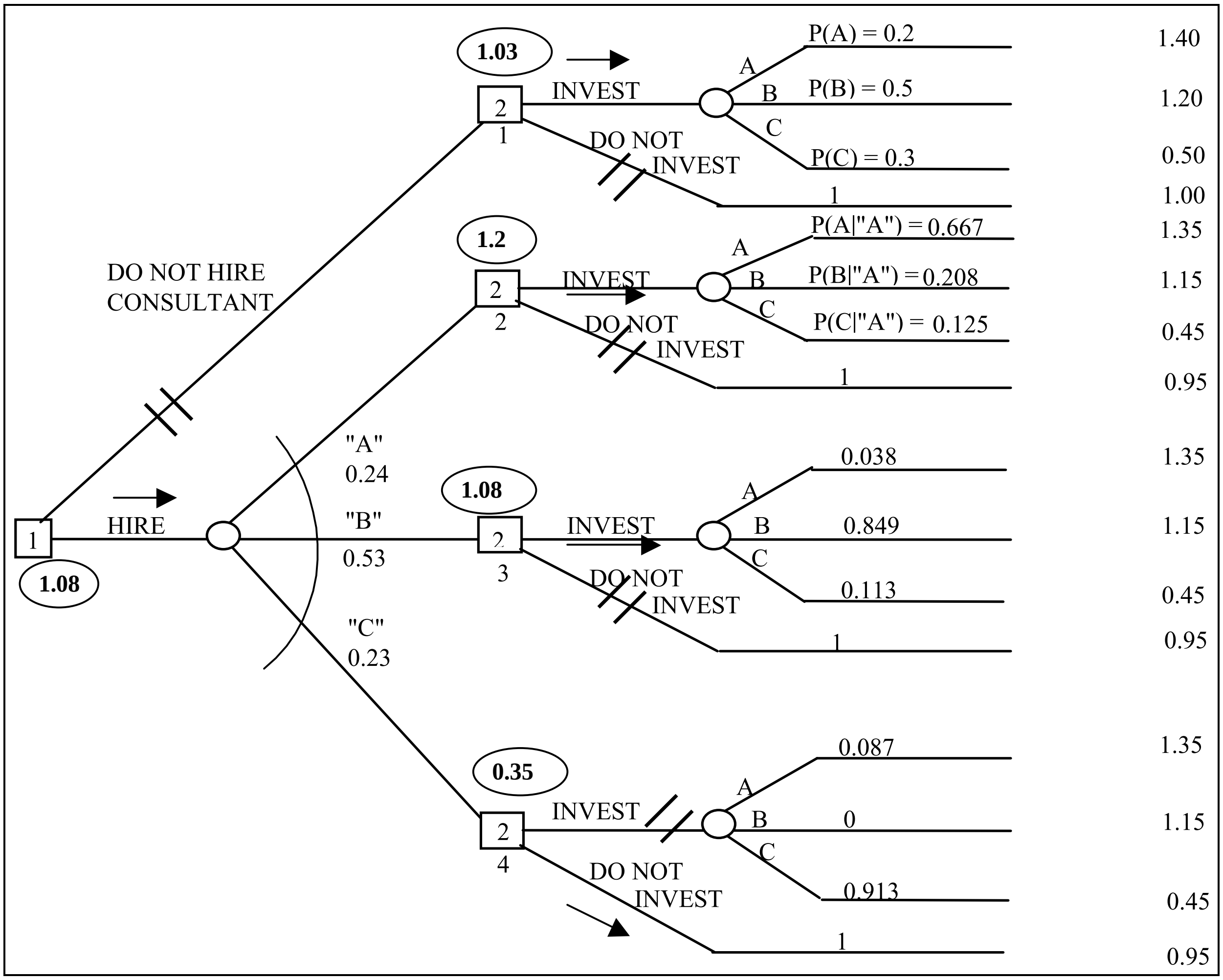


FIG. 6.4: DECISION TREE FOR THE SEQUENTIAL DECISION CASE
INFORMATION GATHERING

(2) RESOLUTION

- START WITH DECISION NODES ON THE RIGHT-HAND SIDE OF THE TREE.
- FOR EACH OF THEM COMPUTE THE EV OF EACH ALTERNATIVE.
- SELECT THE ALTERNATIVE WITH HIGHEST EV.
- REPLACE DECISION NODE BY CHOSEN ALTERNATIVE (EV).
- COMPUTE EV OF THE NEXT LEFT DECISION NODE.

NODE 2.1

EV(INVEST) $= 1.4 \times 0.2 + 1.2 \times 0.5 + 0.5 \times 0.3$
$= 1.03M$

EV(DO NOT INVEST) = 1M

REPLACE NODE BY 1.03M WITH PROBABILITY 1.

NODE 2.2

EV(INVEST) $= 1.35 \times 0.667 + 1.15 \times 0.208 + 0.45 \times 0.125$
$= 1.2M$

EV (DO NOT INVEST) = 0.95M

REPLACE NODE BY 1.2M WITH PROBABILITY 1.

NODE 2.3

EV(INVEST) $= 1.35 \times 0.038 + 1.15 \times 0.849 + 0.45 \times 0.113$
$= 1.08M$

EV(DO NOT INVEST) = 0.95M

REPLACE NODE BY 1.08M WITH PROBABILITY 1.

NODE 2.4

EV(INVEST) $= 1.35 \times 0.087 + 1.15 \times 0.0 + 0.45 \times 0.913$
$= 0.53M$

EV(DO NOT INVEST) = 0.95M

REPLACE NODE BY 0.95M WITH PROBABILITY 1.

NODE 1

EV(DO NOT HIRE CONS.) = 1.03M (INVEST)

EV(HIRE CONSULTANT) $= 0.24 \times 1.2 + 0.53 \times 1.08 + 0.23 \times 0.95$
$= 1.08M$

VALUE OF THE INFORMATION PROVIDED BY THE CONSULTANT

= THE MAXIMUM PRICE THE INVESTOR IS WILLING TO PAY

= 50K + (1.08M - 1.03M) = $100,000

(3) CONCLUSION

OPTIMAL STRATEGY (SEQUENCE OF DECISIONS) FOR THE *EV DECISION MAKER*:

- HIRE THE CONSULTANT.
- IF THE CONSULTANT PREDICTS A OR B, INVEST.
- IF THE CONSULTANT PREDICTS C, DO NOT INVEST.
  (A **STRATEGY** IS A SEQUENCE OF DECISIONS)

THE *MOST RISK-AVERSE* DECISION MAKER (WHO MINIMIZES THE MAXIMAL LOSS) WILL NOT HIRE THE CONSULTANT AND WILL NOT INVEST.

THE *MOST RISK-PRONE* DECISION MAKER (WHO MAXIMIZES THE MAXIMAL GAIN) WILL INVEST WITHOUT HIRING THE CONSULTANT.

6.4 ORDER OF VARIABLES IN A DECISION TREE

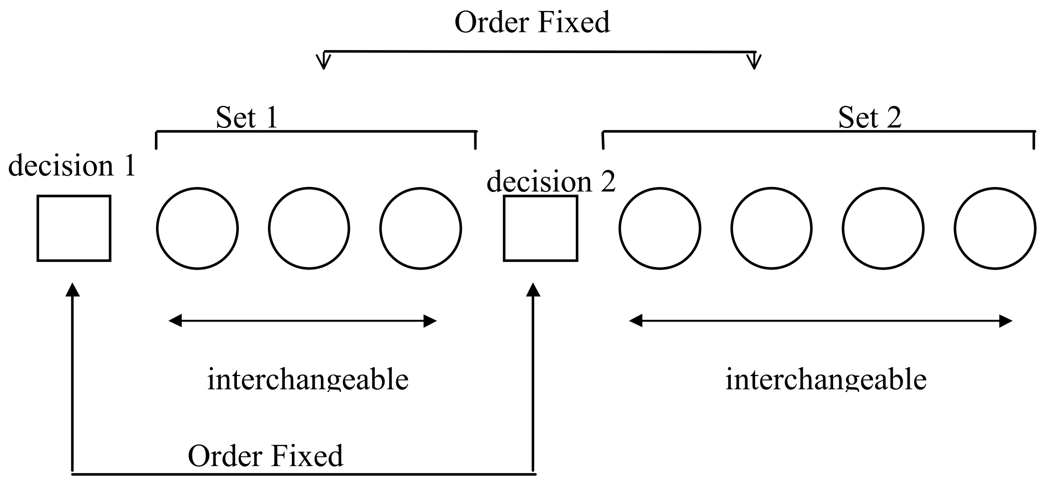


FIG. 6.5 ORDER OF VARIABLES IN A DECISION TREE

- ORDER OF DECISION VARIABLES: FIXED
- ORDER OF STATE VARIABLES BETWEEN TWO DECISION VARIABLES: OPTIONAL BECAUSE JOINT PROBABILITIES ARE INDEPENDENT OF THE ORDER OF EVENTS OR VARIABLES
- SET OF VARIABLES BETWEEN DECISIONS: FIXED

DEPENDENCY ≠ CAUSALITY

6.5 "FLIPPING" TREES

WE NEED P("X") <u>BEFORE</u> MAIN DECISION AND REALIZATION P(X | "X")

- ASSESSMENT ("NATURAL") ORDER

  <u>DATA</u>:

  PRIORS $P(A)$ $(\Rightarrow P(\bar{A}))$

  LIKELIHOODS $P(\text{"A"} \mid A)$ <u>AND</u> $P(\text{"A"} \mid \bar{A})$

- NEEDED IN VALUE OF INFORMATION PROBLEMS: INFERENCE ORDER

  $p(\text{"A"})$ and $\begin{cases} p(A \mid \text{"A"}) \\ p(A \mid \text{"}\bar{A}\text{"}) \end{cases}$

RESULTS:

$$\begin{cases} p(\text{"A"}) & = p(\text{"A"}, A) + p(\text{"A"}, \overline{A}) \\ & = p(A) \times p(\text{"A"} \mid A) + p(\overline{A}) \times p(\text{"A"} \mid \overline{A}) \\ p(A \mid \text{"A"}) & = \dfrac{p(A, \text{"A"})}{p(\text{"A"})} = \dfrac{p(A) \times p(\text{"A"} \mid A)}{p(\text{"A"})} \end{cases}$$

$$p(A \mid \text{"}\overline{A}\text{"}) = \frac{p(A, \text{"}\overline{A}\text{"})}{p(\text{"}\overline{A}\text{"})} = \frac{p(A) \times p(\text{"A"} \mid \overline{A})}{p(\text{"}\overline{A}\text{"})}$$

Section 7

INFLUENCE DIAGRAMS

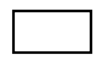
<u>Readings:</u> Howard, "Influence Diagrams"

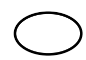
7.1 DEFINITION

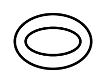
- Directed graphs representing dependencies among decision alternatives, events, random variables and outcomes
- Probabilistic information: marginal and conditional probability (distributions)
- Outcome quantification

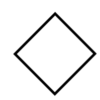
7.2 ELEMENTS OF INFLUENCE DIAGRAMS AND EXAMPLES

- Nodes

    Decision nodes are represented by rectangles. 

    Chance nodes are represented by ovals. 

    Deterministic nodes are represented by double ovals. 

    Outcome or value nodes are represented by a diamond. 

    (other forms may apply)

- Arcs

    Arcs into decision nodes represent knowledge before decision.
    Arcs into chance nodes represent probabilistic dependence, (not always causality).
    Arcs into value nodes represent relevant attributes.

- Tables of realizations of decision variables, state variables and outcomes.
  Include: tables of probabilities (marginal and conditional), tables of outcomes for each scenario.

- EXAMPLE: SEISMIC RISK OF SECTION 6.2

  TWO COMPONENTS;

  DIAGRAM

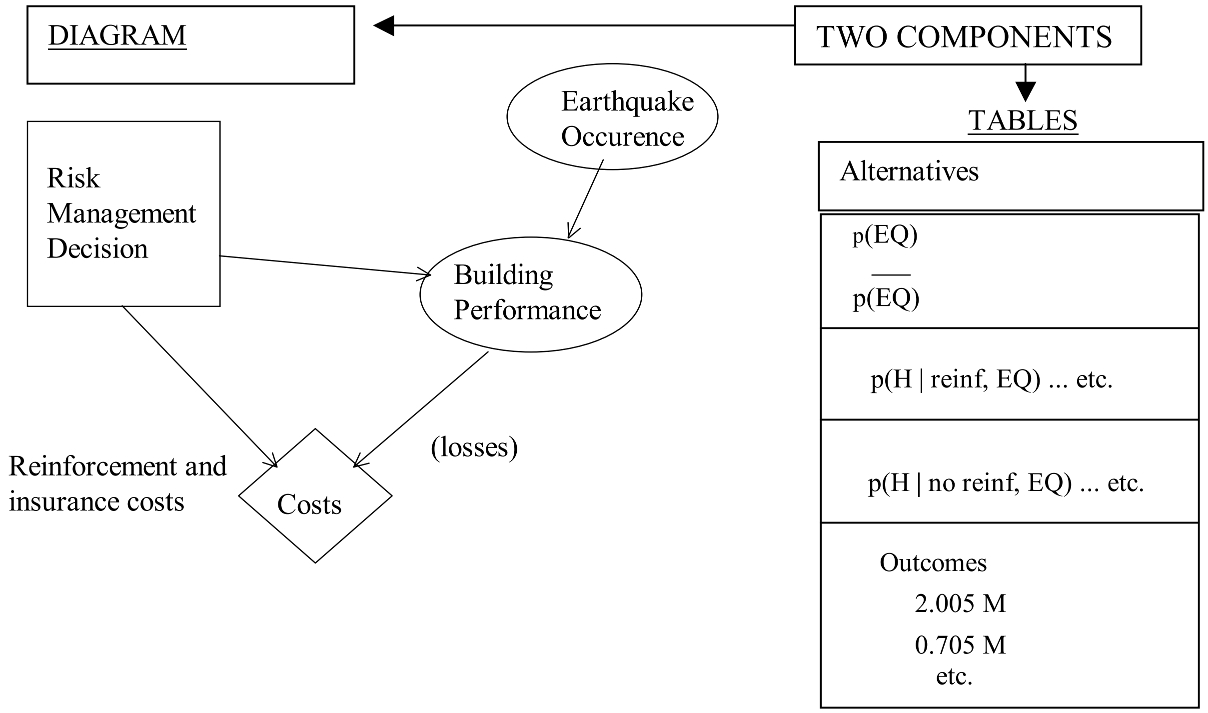


FIG. 7.1: INFLUENCE DIAGRAM FOR SEISMIC RISK MANAGEMENT DECISION

7.3 CONSTRUCTING AN INFLUENCE DIAGRAM

- IDENTIFY THE DECISION(S)
- WHAT IS THE OBJECTIVE (THIS WILL BE THE CONSEQUENCE OR VALUE NODE)?
- ADD THE RELEVANT STATE VARIABLES, EVENTS AND DEPENDENCIES

EXAMPLE 1:

- FAVORITE INFLUENCE DIAGRAM (TAKEN FROM SHACHTER, R.D., "MAKING DECISIONS IN INTELLIGENT SYSTEMS," MANUSCRIPT IN PROGRESS)

VACATION DECISION: RV OR HIKING (TENT)?

KEY STATE VARIABLE: THE WEATHER (ACTUAL)

POOR WEATHER AND TENT: NOT NICE

GREAT WEATHER AND RV: NOT NICE

KEY INFORMATION VARIABLE: THE WEATHER FORECAST

WEATHER FORECAST IS NOT PERFECT (I.E. THERE IS A CHANCE OF RAIN, EVEN IF THE FORECAST SAYS "SUN").

ID NOTES:

- OBJECTIVE: SATISFACTION (*HAVE A NICE VACATION*)
- DECISION: GO HIKING OR USE RV
- CHANCE VARIABLES: WEATHER AND WEATHER FORECAST

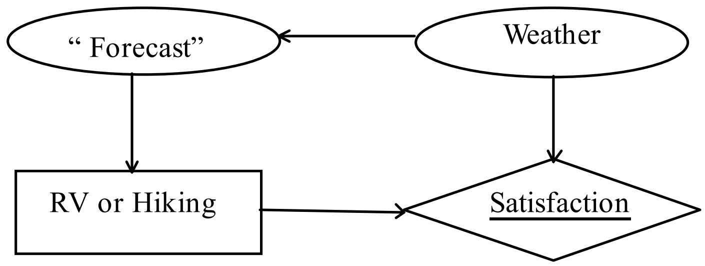


FIG. 7.2: EXAMPLE OF INFLUENCE DIAGRAM

<u>EXAMPLE 2:</u> INVESTMENT DECISION OF SECTION 6.1: INFLUENCE DIAGRAMS (1 OR 2 STATE VARIABLES)

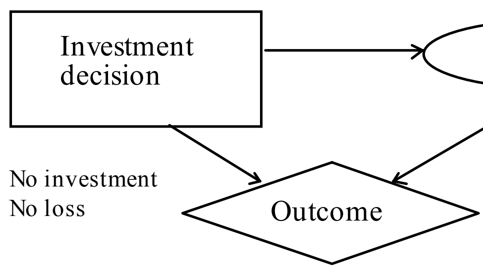


FIG. 7.3: INVESTMENT BASED ON ANTICIPATED SALES

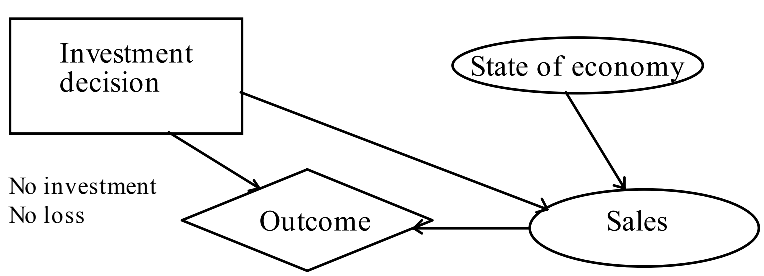


FIG. 7.4: INVESTMENT BASED ON STATE OF ECONOMY AND SALES

EXAMPLE 3: INVESTMENT DECISION OF SECTION 6.3 (INFERENCE FORM)

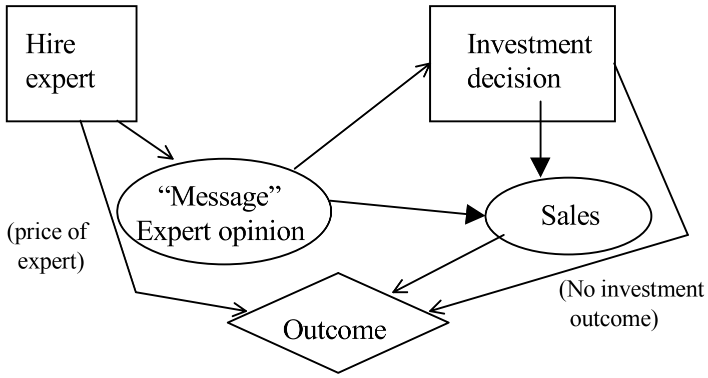


FIG. 7.5: INFLUENCE DIAGRAM FOR INVESTMENT DECISION WITH EXPERT INFORMATION

7.4 KEY ADVANTAGES AND PITFALLS

- COMMUNICATION TOOL (KNOWLEDGE ELICITATION)
- COMPACTNESS OF REPRESENTATION
- RELATIONSHIP WITH DECISION TREES
- INFLUENCE DIAGRAMS ARE **NOT FLOW CHARTS**!
- THERE ARE **NO LOOPS** IN INFLUENCE DIAGRAMS. THE ARROWS BETWEEN CHANCE NODES REPRESENT **PROBABILISTIC DEPENDENCIES**
- INFLUENCE DIAGRAMS ARE EASY TO UNDERSTAND BUT **CAN BE DIFFICULT TO CONSTRUCT**
- OTHER NAMES: BAYESIAN NETWORKS, KNOWLEDGE DIAGRAMS, RELEVANCE DIAGRAMS

7.5 INFLUENCE DIAGRAM MANIPULATIONS BY A COMPUTER OR AN ANALYST

SAME RULES AS ORDER OF VARIABLES IN DECISION TREES (HOMOMORPHISM).

- ARCS INTO CHANCE NODES AND DETERMINISTIC NODES CAN BE REVERSED BECAUSE THE UNDERLYING JOINT DISTRIBUTION REMAINS UNCHANGED
- REVERSING ARCS THAT LEAD INTO A CHANCE NODE IS DONE BY APPLYING BAYES THEOREM
- REVERSING AN ARC THAT LEADS INTO A DETERMINISTIC NODE MIGHT RESULT IN CHANGING THE DETERMINISTIC NODE INTO CHANCE NODE(S) (E.G., SQUARE ROOT)
- REVERSING AN ARC THAT LEADS INTO A DECISION NODE MEANS CHANGING THE STATE OF INFORMATION AT DECISION TIME (DON'T DO IT UNLESS IT IS THE CASE).

SOMETIMES, YOU HAVE THE CHOICE OF ORDER OF VARIABLES

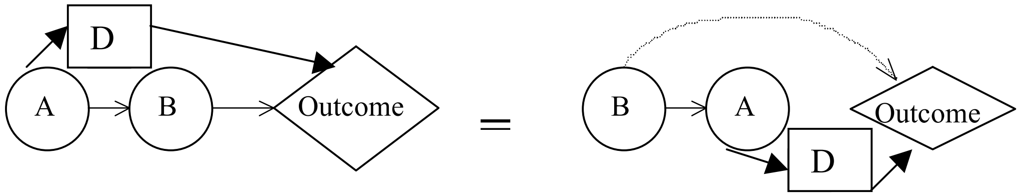


FIG. 7.6: ARC REVERSALS IN INFLUENCE DIAGRAMS

CHOOSE THE MOST CONVENIENT ORDER IN TERMS OF INFORMATION. ADD ARC B → OUTCOME IF OUTCOME DEPENDS ON A AND B.

<u>ILLUSTRATION OF ARC REVERSALS FOR THE CAMPING EXAMPLE</u>:

- ASSESSMENT FORM AND INFERENCE FORM:

ASSESSMENT FORM: PROBABILITY OF A PARTICULAR FORECAST GIVEN WEATHER

INFERENCE FORM: PROBABILITY OF WEATHER GIVEN A PARTICULAR FORECAST

➔ THE ARCS ARE REVERSED

"W" ≡ W

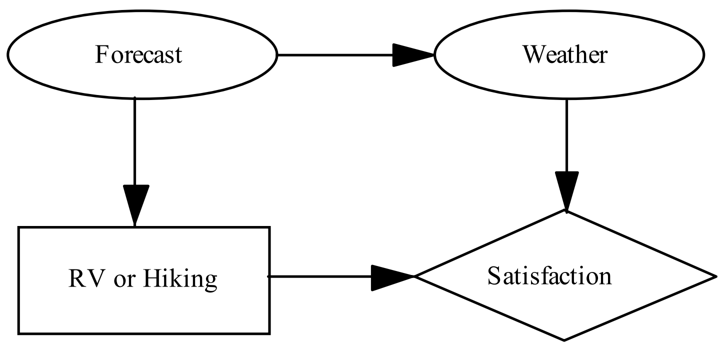


FIG. 7.7: INFERENCE FORM FOR THE EXAMPLE

- **CLAIRVOYANCE** (KNOWING WHAT WILL HAPPEN IN THE FUTURE): JUST ADD AN ARC FROM THE CHANCE NODE REPRESENTING THE VARIABLE OF INTEREST TO THE RELEVANT DECISION

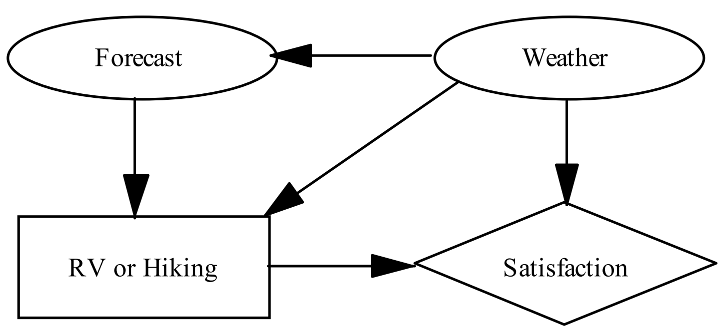


FIG. 7.8: INFLUENCE DIAGRAM GIVEN PERFECT INFORMATION ABOUT THE WEATHER

Influence Diagram above represents clairvoyance about "Weather." If we assume an expected value Decision maker, we can calculate the expected value for this situation and for the situation without the Clairvoyant. the difference is the "Value of Clairvoyance" or "Value of Perfect Information."

7.6 QUANTIFYING AN INFLUENCE DIAGRAM

- Filling in the data
- Evaluating the diagram

Influence diagrams have three hierarchical levels:

- Relation: which are the nodes that are connected, i.e. relevant to each other?
- Function: the functions of shared conditional distributions, asymmetries in the decision problem
- Number: the possible realizations of the different variables and the corresponding probabilities or probability distributions

quantifying an influence diagram ("filling in the numbers.")

It encompasses two tasks:

- Definition of the possible outcomes
  (e.g. "given that there is low demand for your product, what will your estimated sales be" and "given that there is high demand for your product, what will your estimated sales be?")
- Assessment of the relevant probabilities
  (e.g., "what is the probability for *high* demand?")
  a number of techniques are in use; e.g., the probability wheel: a circle colored with two colors, one of which represents the expertís state of belief about the assessed probability. Other elicitation methods use the idea of reference gambles where the unknown probability is compared to known probabilities (e.g., "is *X* as likely as rolling a 6 on a fair dice?").

Heuristics and Biases:

Have to be dealt with when assessing probabilities. These are cognitive and psychological limitations that can be overcome in certain instances, as long as the elicitor is aware of them (see the section on "Expert Judgment").

ONCE THE PROBABILITIES HAVE BEEN ASSESSED AND THE POSSIBLE OUTCOMES HAVE BEEN DETERMINED, THE INFLUENCE DIAGRAM CAN BE EVALUATED. THIS IS DONE BY USING THE NUMBERS AND THE CORRESPONDING FUNCTIONS AS WELL AS THE RELATIONS EXPRESSED IN THE DIAGRAM Ñ FOR DETAILS ON THE EVALUATION OF INFLUENCE DIAGRAMS SEE SHACHTER, 1986.

7.7 INFLUENCE DIAGRAMS AND DECISION TREES

- EVERY DECISION TREE CAN BE TRANSFORMED INTO AN INFLUENCE DIAGRAM
- INFLUENCE DIAGRAMS HAVE TO BE BROUGHT INTO "DECISION FORMAT" OR "CANONICAL FORM" (INFERENCE FORM) BEFORE THEY CAN BE TRANSFORMED INTO A DECISION TREE.

DECISION TREES INCLUDE DECISION NODES. A DECISION NODE BRINGS THE CONCEPT OF TIME INTO PLAY: ANY NODE THAT PRECEDES THE DECISION NODE IS REVEALED (I.E., ITS OUTCOME IS KNOWN) TO THE DECISION MAKER.

EXAMPLE:

THE INFLUENCE DIAGRAM OF FIG. 7.7 (INFERENCE FORM) CAN BE TRANSFORMED INTO A DECISION TREE (NOTE: IT IS ASSUMED HERE THAT THE INFLUENCE DIAGRAM IS FULLY SPECIFIED, I.E., THAT THE LEVELS OF RELATION, FUNCTION AND NUMBER HAVE BEEN SPECIFIED).

TO TRANSFORM THE INFLUENCE DIAGRAM INTO A DECISION TREE, START WITH THE ROOT NODE (THE ONE NODE THAT HAS NO ARROWS POINTING TO IT) AND PROCEED FROM THERE; BECAUSE THE INFLUENCE DIAGRAM REPRESENTS A JOINT DISTRIBUTION, THE ORDER OF NODES THAT REPRESENT STATE VARIABLES IS IRRELEVANT AS LONG AS THEY ARE DOWNSTREAM FROM THE DECISION NODE.

THE CORRESPONDING DECISION TREE HAS THREE NODES ON EACH SCENARIO (A SCENARIO IS REPRESENTED BY ONE BRANCH OF THE TREE); NOTE THE ORDER OF THE NODES: THE "FORECAST" NODE PRECEDES THE DECISION NODE, WHICH IS FOLLOWED BY THE "WEATHER" NODE.

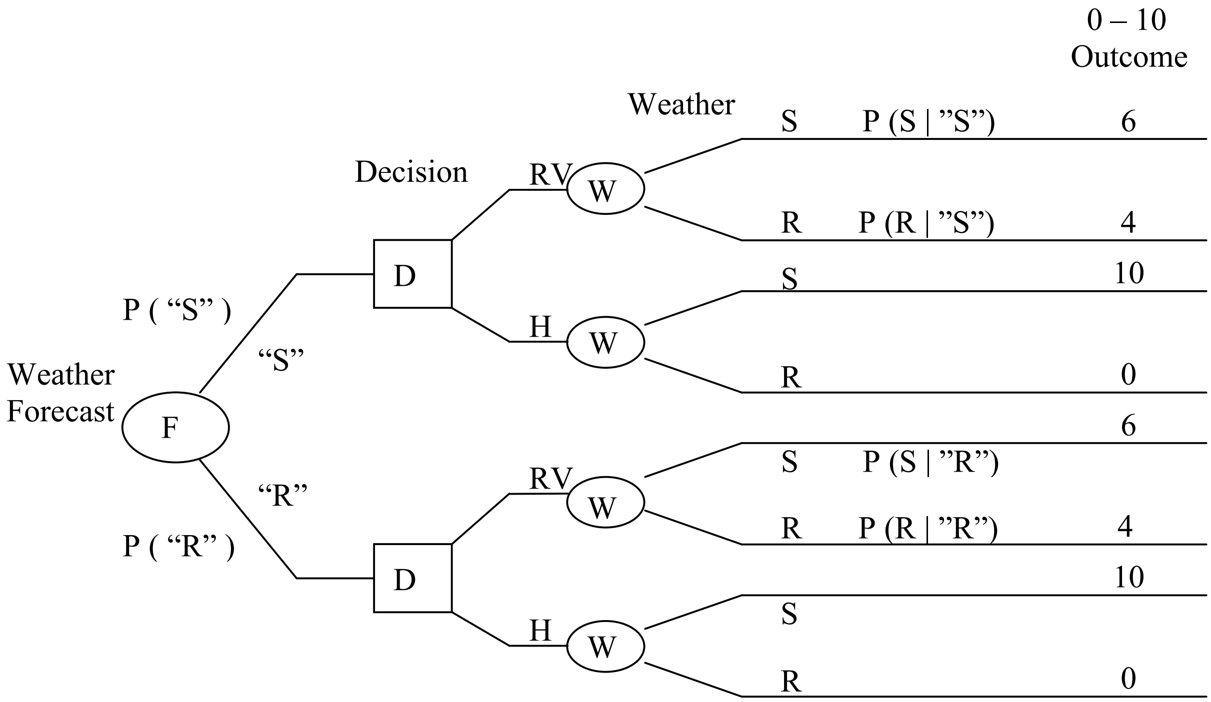


FIG. 7.9: DECISION TREE FOR THE INFLUENCE DIAGRAM OF FIG. 7.7

ONCE THE DECISION TREE IS CONSTRUCTED, THE NUMBERS ELICITED FOR THE INFLUENCE DIAGRAM CAN BE INCLUDED; IN DOING SO, ONE HAS TO BE CAREFUL ABOUT THE ORDER OF CONDITIONING.

<u>POTENTIAL PROBLEM: ASYMMETRIES</u>

DIFFICULTY OF CONSTRUCTING IDS FOR ASYMMETRIC TREES

7.8 ENGINEERING EXAMPLE: RISK OF SHIP GROUNDING DUE TO LOSS OF PROPULSION

INFLUENCE DIAGRAM (ALL NODES ARE CHANCE NODES; THE OUTCOME NODE IS THE SOURCE TERM)
NO DECISION HERE. RISK ONLY

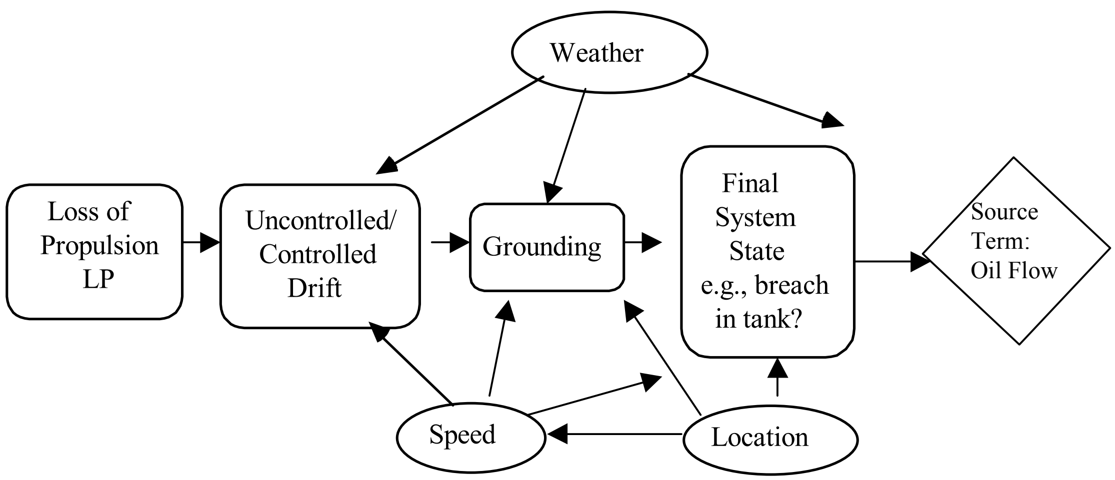


FIG. 7.10: EXAMPLE OF INFLUENCE DIAGRAM FOR AN ENGINEERING RISK PROBLEM (SOURCE: PATÉ-CORNELL, 1997). NOTE; NO DECISION HERE, ONLY RISK ANALYSIS.

SECTION 8

UTILITY THEORY AND RISK ATTITUDE

8.1 NOTATIONS

PRIZES A, B

$A \succ B$	I PREFER A TO B
$A \sim B$	I AM INDIFFERENT BETWEEN A AND B
$A \succcurlyeq B$	I LIKE A AT LEAST AS MUCH AS B

8.2 LOTTERY

A LOTTERY IS A SET OF PRIZES (PROSPECTS) WITH ASSOCIATED PROBABILITIES

ONE POSSIBLE LOTTERY:

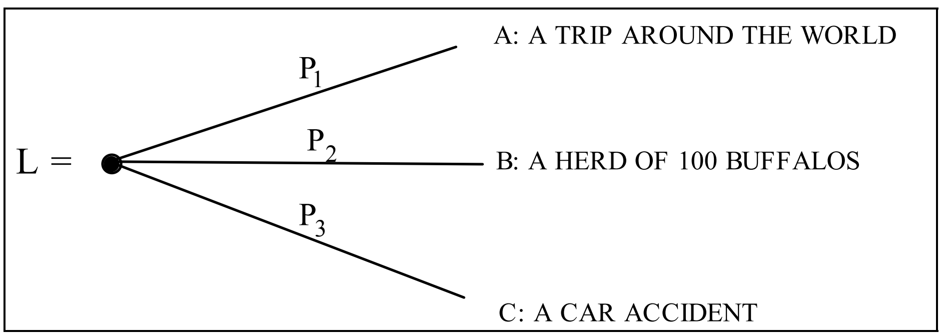


FIG. 8.1: A WEIRD LOTTERY

$L = (p_1, A; p_2, B; p_3, C)$

WITH $p_1 + p_2 + p_3 = 1$

IF THE PRIZES ARE ALL MEASURED IN THE SAME UNIT (E.G., DOLLARS), A LOTTERY CAN BE CONSIDERED AS THE PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE.

DISCRETE SET OF OUTCOMES:

DESCRIPTION BY A <u>MASS FUNCTION</u>

$$\sum_i p_I = 1$$

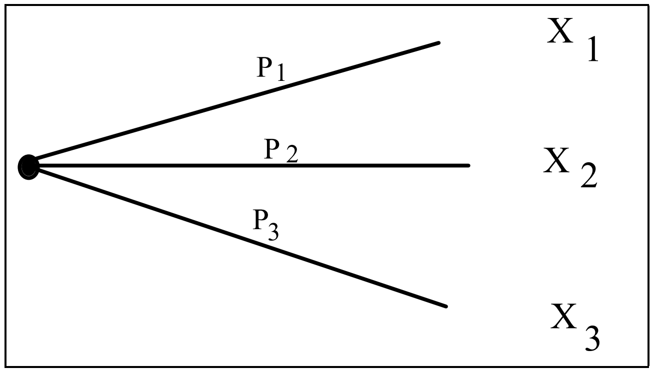


FIG. 8.2: A DISCRETE LOTTERY

CONTINUOUS SET OF OUTCOMES:

DESCRIPTION BY A <u>CONTINUOUS DENSITY</u> OF PROBABILITIES

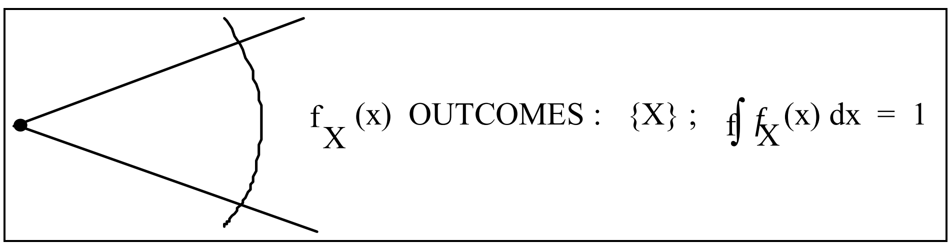


FIG. 8.3: A CONTINUOUS LOTTERY

8.3 CERTAIN EQUIVALENT

THE CERTAIN EQUIVALENT OF A LOTTERY IS A PRIZE SUCH THAT THE INDIVIDUAL IS INDIFFERENT BETWEEN RECEIVING THE PRIZE AND PARTICIPATING IN THE LOTTERY.

NOTATION:

L IS THE CERTAIN EQUIVALENT OF LOTTERY L.

EXAMPLE:

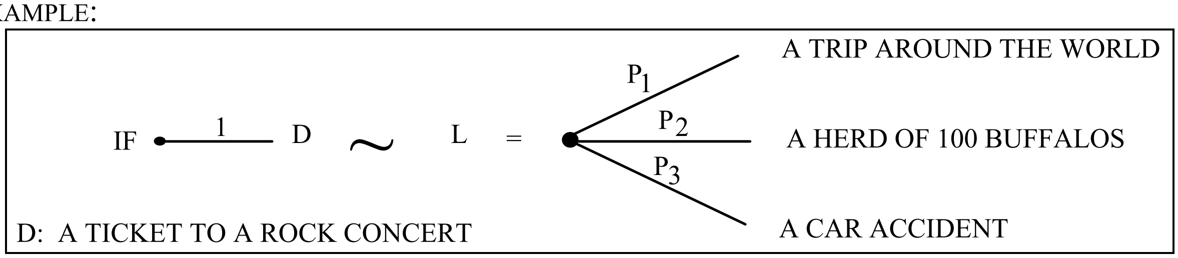


FIG. 8.4: CERTAIN EQUIVALENT

THEN L = D (THE CERTAIN EQUIVALENT OF THE LOTTERY IS A TICKET TO A ROCK CONCERT).

IF THE LOTTERY IS A RANDOM VARIABLE, X, THE CERTAIN EQUIVALENT IS NOTED X.

8.4 AXIOMS OF RATIONAL CHOICES

REFERENCES: VON NEUMAN AND MORGENSTERN, 1947; HOWARD, 1970.

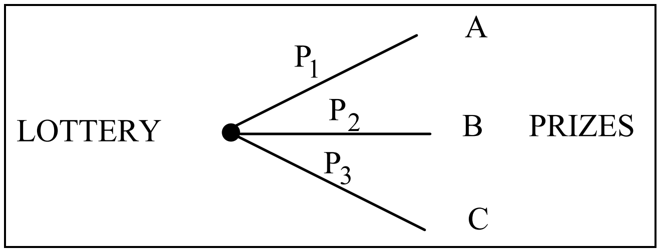


FIG. 8.5: LOTTERY

(1) ORDERABILITY OF PRIZES

$A > B$, $A < B$, $A \sim B$, $A \leq B$, OR $A \geq B$

TRANSITIVITY: IF $A > B$ AND $B > C$, THEN $A > C$

(2) CONTINUITY

IF $A > B > C$

THEN FOR SOME P

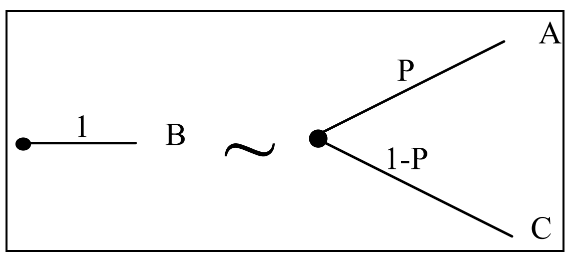


FIG. 8.6: CONTINUITY AXIOM

B IS THE CERTAIN EQUIVALENT OF THE LOTTERY.

(3) SUBSTITUTABILITY

A LOTTERY AND ITS CERTAIN EQUIVALENT ARE INTERCHANGEABLE WITHOUT AFFECTING PREFERENCES.

(4) MONOTONICITY

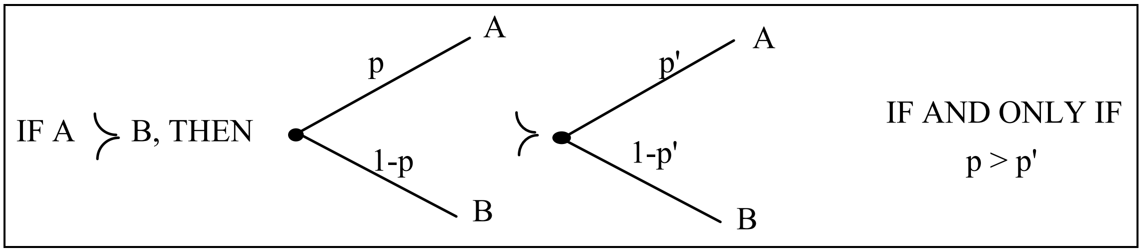


FIG. 8.7: MONOTONICITY AXIOM

(5) DECOMPOSABILITY

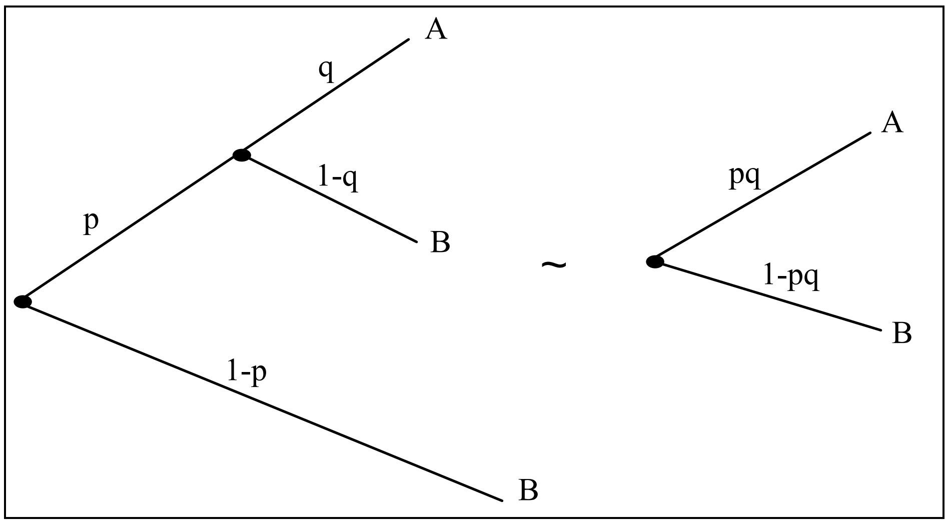


FIG. 8.8: DECOMPOSABILITY AXIOM

THESE AXIOMS LEAD TO THE MAXIMIZATION OF EXPECTED UTILITY.

<u>PROOF:</u>

1. CONSIDER LOTTERY A(ALT$_1$)

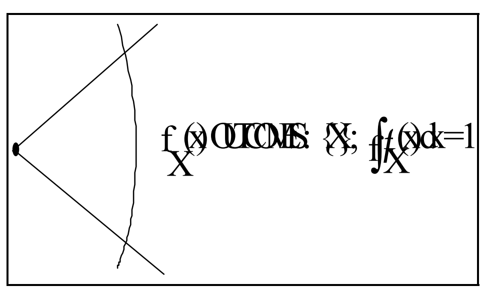


FIG. 8.9: LOTTERY A

RANK ALL OUTCOMES OF A AND OTHER LOTTERIES FACED IN DECISION

$X_{MIN} < X_i < X_j < \ldots < X_{MAX}$ <= ORDERABILITY

2. For each outcome X_i of lottery A, there is a probability U_i that makes the DM indifferent between X_i and $[X_{max}, U_i, X_{min}, (1-U_i)]$

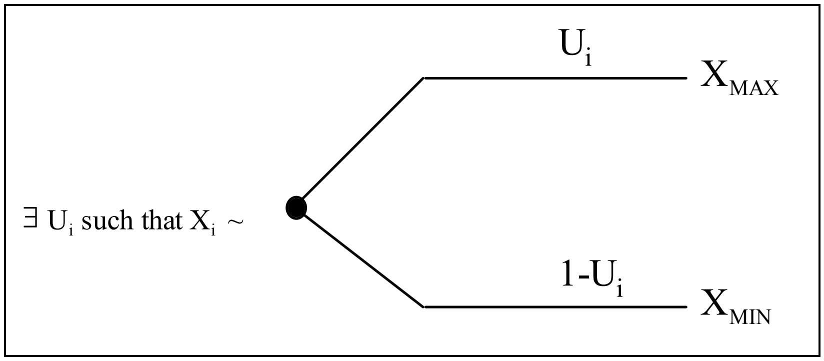


Fig. 8.10: Continuity

3. Replace each X_i in A by this lottery

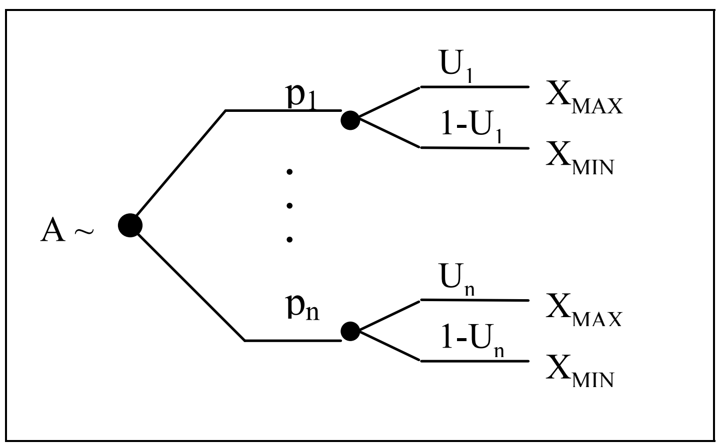


Fig. 8.11: Substitutability

4. "Fold" this lottery according to decomposability axiom

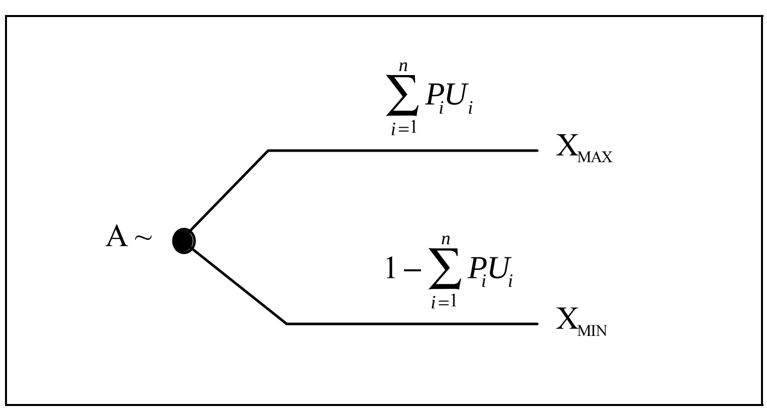


Fig. 8.12: Decomposability

5. ASSUME THAT IN A DECISION, DM FACES TWO ALTERNATIVES.
   ALTERNATIVE 1 LEADS TO LOTTERY A.
   ALTERNATIVE 2 LEADS TO LOTTERY B. (SAME MAX MIN OUTCOME.)

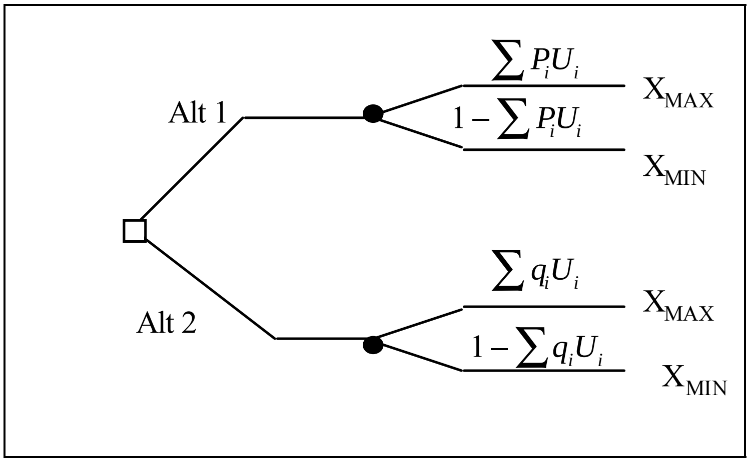


FIG. 8.13: MONOTONICITY

BY VIRTUE OF MONOTONICITY AXIOM, DM PREFERS THE ALTERNATIVE THAT MAXIMIZES THE PROBABILITY OF GETTING X_{MAX}: $\sum \text{prob}_i \times U_i$

U_i IS CALLED THE UTILITY OF OUTCOME X_i.

⇒ MAXIMIZATION OF EXPECTED UTILITY

⇒ DECISIONS ARE UNCHANGED BY A LINEAR TRANSFORMATION OF THE UTILITY FUNCTION

8.5 UTILITY FUNCTION FOR AN INDIVIDUAL

CONSEQUENCE OF THE AXIOMS

IF THOSE AXIOMS ARE SATISFIED, THE VALUE ATTACHED BY AN INDIVIDUAL TO THE OUTCOMES CAN BE DEFINED BY A **UTILITY FUNCTION.** THAT FUNCTION CHARACTERIZES THE PREFERENCES OF A SPECIFIC PERSON.

→ CONCLUSION OF PROOF: THE GOAL OF A DECISION MAKER WHO SUBSCRIBES TO THE VON NEUMANN AXIOMS IS TO **MAXIMIZE HIS EXPECTED UTILITY** (<= ABOVE AXIOMS OF RATIONAL BEHAVIOR).

→ DEFINITION: THE UTILITY OF A LOTTERY (TO THE DECISION MAKER) IS THE EXPECTED UTILITY OF ITS OUTCOMES.

→ FROM AXIOMS: THE DECISION MAKER PREFERS LOTTERY 1 TO LOTTERY 2 IF AND ONLY IF $u(L_1) > u(L_2)$ WHICH IS EQUIVALENT TO: $Eu(L_1) > Eu(L_2)$ (NOTED HERE $L_1 > L_2$).

UTILITY

ASSUME THAT ALL POSITIVE OUTCOMES CAN BE MEASURED AS A FUNCTION OF ONE ATTRIBUTE (E.G., DOLLAR VALUE)

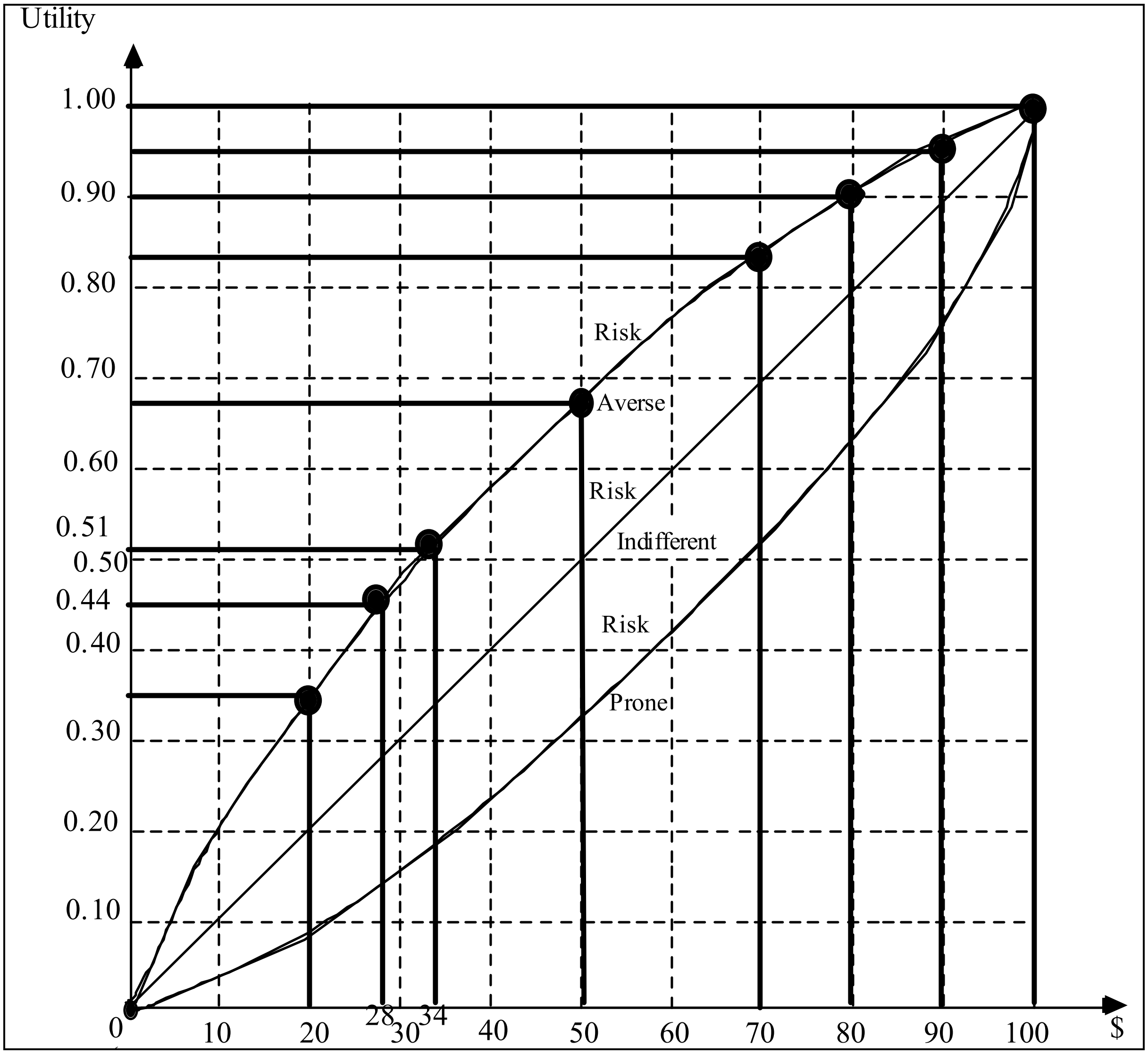


FIG. 8.14: DIFFERENT TYPES OF RISK ATTITUDES

FOR GAINS:

RISK AVERSE: CONCAVE UTILITY CURVE

RISK INDIFFERENT: STRAIGHT LINE

RISK PRONE: CONVEX UTILITY CURVE

8.6 EXPECTED UTILITY OF A LOTTERY AND CERTAIN EQUIVALENT

U (L) = EU (OUTC.) = U (CE) ⇐ INDIFFERENCE

DEFINITION: ⇒ **CE (L) = U^{-1}(U (L))**

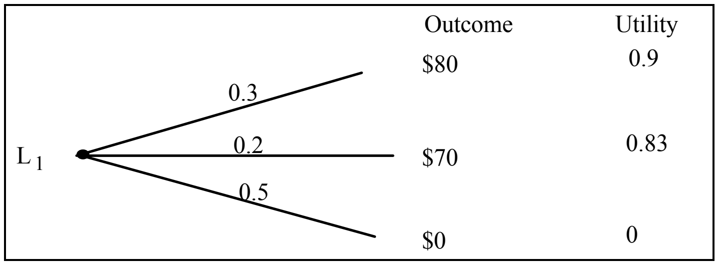


FIG. 8.15: LOTTERY 1 FOR THE RISK AVERSE DM OF FIG. 8.14 (CALL HIM TOM)

$U(L_1) = 0.3 \times 0.9 + 0.2 \times 0.83 + 0.5 \times 0$

$= 0.436 \sim 0.44$

CERTAIN EQUIVALENT = QUANTITY OF MONEY THAT PROVIDES THE SAME UTILITY AS THE LOTTERY

$CE(L_1) = U^{-1}(U(L_1)) = \$28 \quad < \quad \neq \quad EV(L_1) = \$38$ ⇐ CONCAVITY OF UTILITY CURVE

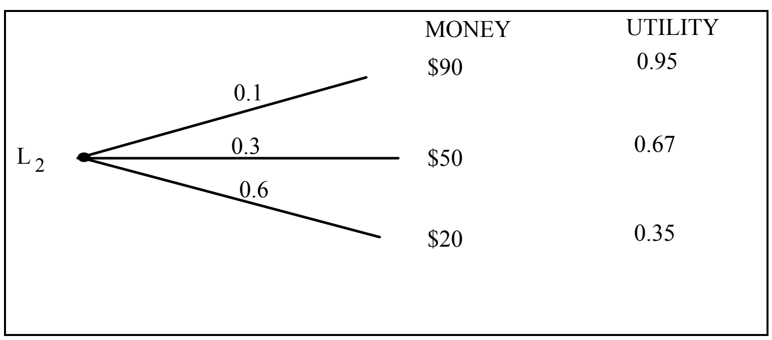


FIG. 8.16: LOTTERY 2 FOR TOM

$U(L_2) = 0.51$

CERTAIN EQUIVALENT $CE(L_2) = U^{-1}(U(L_2))$ = $34

= SELLING PRICE OF LOTTERY

EXPECTED VALUE OF THE LOTTERY (IN MONETARY TERMS) = $36

LOTTERY L_2 IS PREFERRED BY THE RISK-AVERSE DECISION MAKER EVEN THOUGH THE EXPECTED VALUE OF LOTTERY L_2 ($36) IS LOWER THAN THE EXPECTED VALUE OF LOTTERY L_1 ($38).

RISK AVERSION ⇒ CE < EV

8.7 GENERAL DEFINITION OF THE RISK-AVERSION COEFFICIENT

$$R(X) = \frac{-U''(X)}{U'(X)}$$

$U''(X)$: SECOND DERIVATIVE OF U

$U'(X)$: FIRST DERIVATIVE OF U

$R > O \Rightarrow$ RISK AVERSION (CONCAVE UTILITY CURVE)

CHARACTERIZES THE CURVE CONCAVITY. CAN CHANGE ALONG THE X AXIS.

EXAMPLE OF VARIABLE RISK-AVERSION COEFFICIENT:

$$U(X) = \ln(X+A)$$

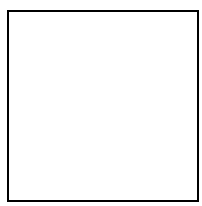
$$R(X) = \frac{1}{X+A}$$

RISK AVERSION DECREASES WHEN CAPITAL INCREASES

IN THE GENERAL CASE, THE RISK AVERSION COEFFICIENT VARIES OVER THE RANGE OF POSSIBLE OUTCOMES.

RISK-AVERSION COEFFICIENT AND CONVEXITY OF THE UTILITY CURVE

U



8.17: RISK ATTITUDES AND CONVEXITY OF THE UTILITY CURVE

8.8 THE DELTA PROPERTY: CONSTANT RISK ATTITUDE

CONSIDER A POSSIBLE ADDITIONAL ASSUMPTION: THE DELTA PROPERTY. AN INCREASE OF ALL PRIZES IN A LOTTERY BY AN AMOUNT Δ INCREASES THE CERTAIN EQUIVALENT BY Δ.

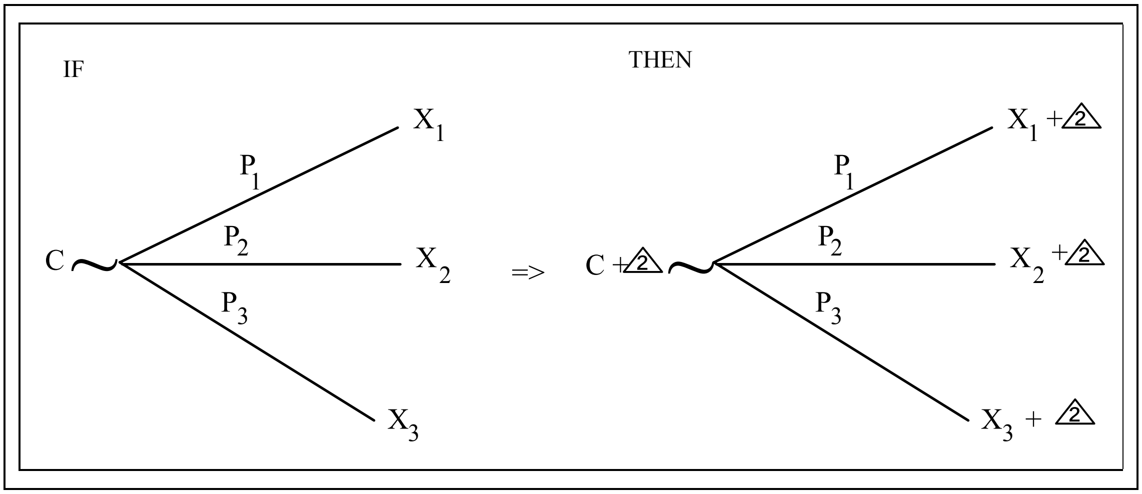


FIG. 8.18: DELTA PROPERTY

<u>CONSEQUENCES OF THE DELTA PROPERTY</u>:

(1) THE RISK ATTITUDE IS THE SAME ALONG THE UTILITY CURVE.
THE UTILITY CURVE MUST BE EITHER A STRAIGHT LINE OR AN EXPONENTIAL.

$$-R(x) = \frac{U''(x)}{U'(x)} = ct = \gamma \quad \text{Linear differential equation}$$

$$\begin{cases} \Rightarrow \quad u(x) = a + b \exp(-\gamma x) \quad \text{(constant risk attitude)} \\ \text{or } u(x) = a + b x \qquad \gamma > 0 \Rightarrow \text{Risk aversion} \end{cases}$$

Convenient form $\quad \mathbf{u}(\mathbf{x}) = \dfrac{1 - \exp(-\gamma \mathbf{x})}{1 - \exp(-\gamma)}$

for which: $\quad : \begin{cases} u(0) = 0 \\ u(1) = 1 \\ \lim_{\gamma \to 0} \mathbf{u}(\mathbf{x}) = \mathbf{x} \end{cases} \quad$ (normalization)

NOTE: DECISIONS ARE UNCHANGED BY LINEAR TRANSFORMATIONS OF THE UTILITY FUNCTION ($\Leftarrow$ RANKING BY ORDER OF EU).

$\Rightarrow$ THE NORMALIZED FORM IS EQUIVALENT TO THE ORIGINAL UTILITY FUNCTION.

WHEN THE RISK AVERSION COEFFICIENT γ IS 0, THE UTILITY CURVE IS A STRAIGHT LINE AND THE INDIVIDUAL IS RISK-INDIFFERENT.

WHEN γ IS POSITIVE, HE IS RISK-AVERSE.

WHEN γ IS NEGATIVE, HE IS RISK-PRONE.

(2) THE BREAK-EVEN PAYMENT (B) FOR A LOTTERY WILL BE THE SAME AS THE CERTAIN EQUIVALENT (C) $\Rightarrow$ (BUYING PRICE = SELLING PRICE)

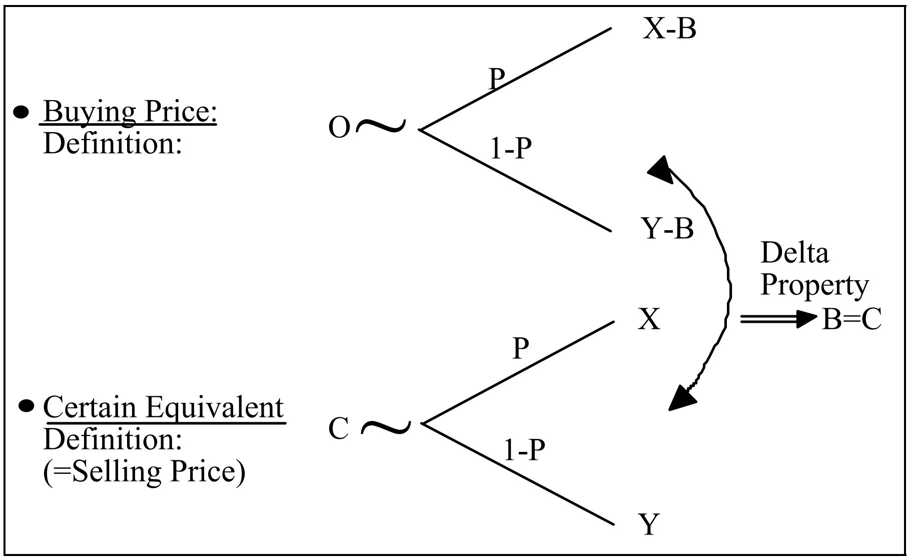


FIG. 8.19: SELLING PRICE AND BUYING PRICE OF A LOTTERY FOR CONSTANT RISK ATTITUDES

NOTE: CASE OF A "DISUTILITY" CURVE

EXAMPLE: WILLINGNESS TO PAY TO AVOID A LOSS. THE CONVEXITY OF THE DISUTILITY CURVE (DEPENDING ON THE RISK ATTITUDE) IS INVERTED

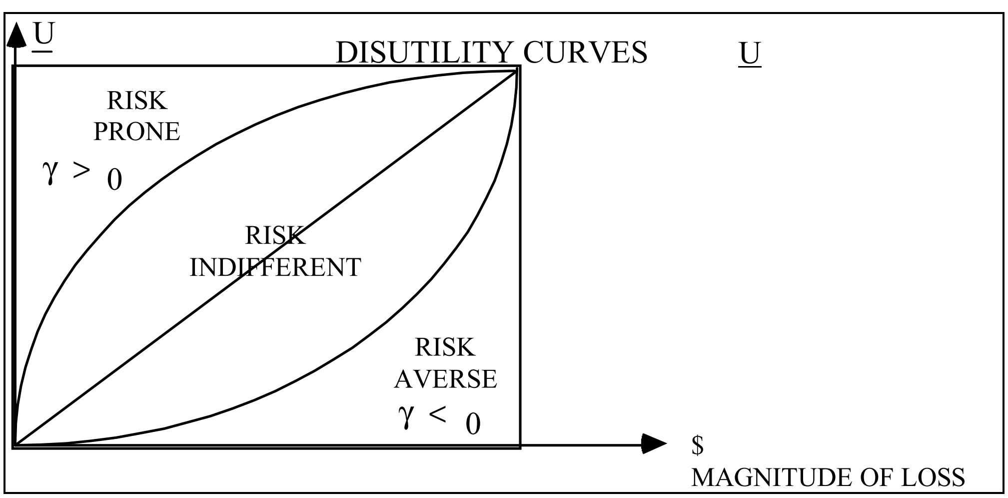


FIG. 8.20: DISUTILITY CURVES

SECTION 9

VALUE OF INFORMATION
TEST EXAMPLES

9.1 A PARTY PROBLEM. DECISION ANALYSIS AND VALUE OF INFORMATION

THE PARTY LOCATION DECISION PROBLEM (SOURCE: HOWARD)

ALTERNATIVES	STATE VARIABLE	VALUE (DEGREE OF SATISFACTION)	
PARTY LOCATION	WEATHER	O, S	100
O: OUTDOORS	S: SUNSHINE	O, R	0
P: PORCH	R: RAIN	PO, S	90
I: INDOORS		PO, R	20
		I, S	40
		I, R	50

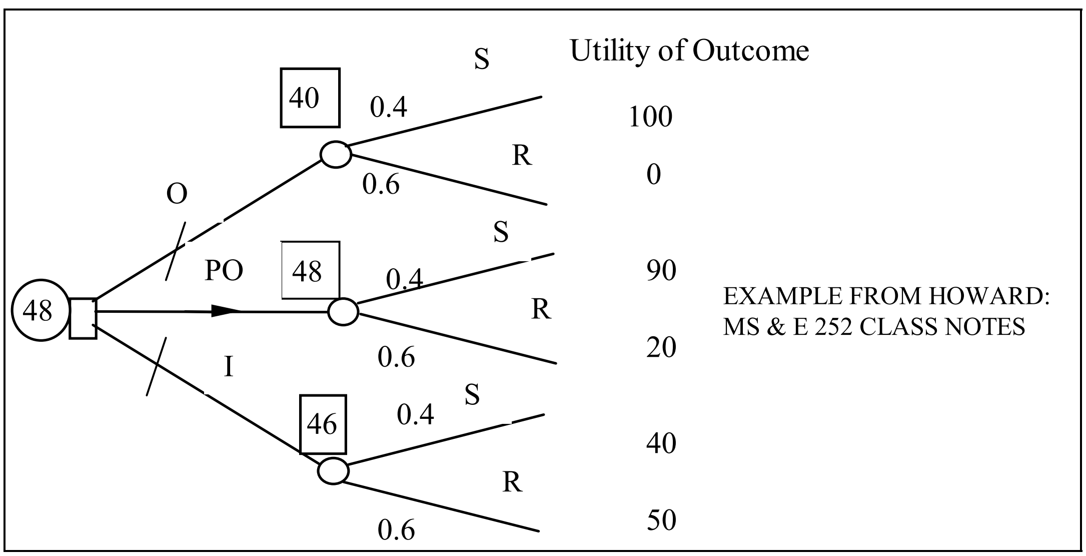


FIG. 9.1: THE PARTY PROBLEM

VALUE OF "CLAIRVOYANCE" (PERFECT INFORMATION) FOR AN E V DECISION MAKER ("W"=W)

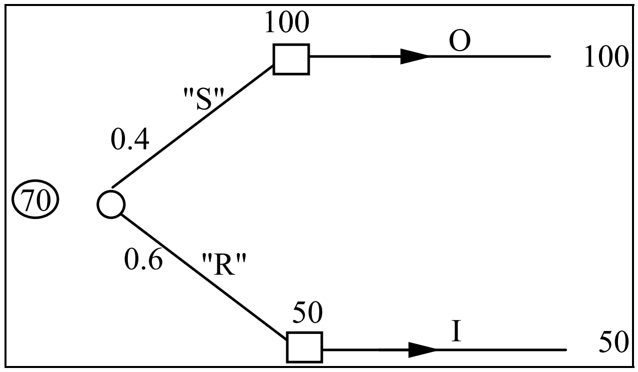


FIG. 9.2: VALUE OF CLAIRVOYANCE. SOURCE: HOWARD

EXPECTED UTILITY WITH PERFECT INFORMATION	=	70
EXPECTED UTILITY WITHOUT INFORMATION	=	48
EXPECTED VALUE OF PERFECT INFORMATION	=	22 (UTILS.)

THIS IS THE VALUE OF PERFECT INFORMATION.

NOTE: IN CASE OF PERFECT INFORMATION ("CLAIRVOYANCE"), "E" ≡ E. THE REALIZATIONS OF "E" AND E ARE INDENTICAL THEREFORE, THE PROBABILITY OF THE MESSAGE ("E") IS EQUAL TO THE PRIOR PROBABILITY OF THE EVENT P("E") = P(E)..

EFFECT OF RISK TOLERANCE ON THE VALUE OF PERFECT INFORMATION:

THE MOST RISK-AVERSE DECISION MAKER USES MORE CONSERVATIVE (INDOORS) SOLUTION IN THE ABSENCE OF INFORMATION. THEREFORE, THE RISK ATTITUDE AFFECTS VALUE OF INFORMATION (70-46=24)

EVEN WITH LESS EXTREME RISK AVERSION:

ASSUME: U(S, PO) = 70 ⇒ EU(PO) = 40 ⇒ BEST ALTERNATIVE = INDOORS

VALUE OF INFORMATION = 70 - 46 = 24 > 22

THE RISK-AVERSE IN BOTH CASES IS WILLING TO PAY MORE FOR THE INFORMATION ABOUT THE WEATHER

VALUE OF INFORMATION DEPENDS ON

- PROBABILITIES
- OUTCOMES
- UTILITIES

9.2 VALUE OF IMPERFECT INFORMATION

EXAMPLE 1: CHOICE OF A TEST FOR A RISK-INDIFFERENT DECISION MAKER

TEST 1 (COST C1) LESS INFORMATIVE (BUT LESS EXPENSIVE) THAN TEST 2 (COST C2)

DECISION: WHICH TEST TO ADOPT?

VARIABLE TO BE TESTED: STRENGTH OF A PART.

DECISION RULE: MINIMIZE EXPECTED COSTS.

IMPERFECT INFORMATION CHARACTERIZED BY CONDITIONAL PROBABILITIES:

$P(A \mid \text{“}A\text{”})$ AND $P(A \mid \text{“}\bar{A}\text{”})$ OR $P(\text{“}A\text{”} \mid A)$ AND $P(\text{“}A\text{”} \mid \bar{A})$

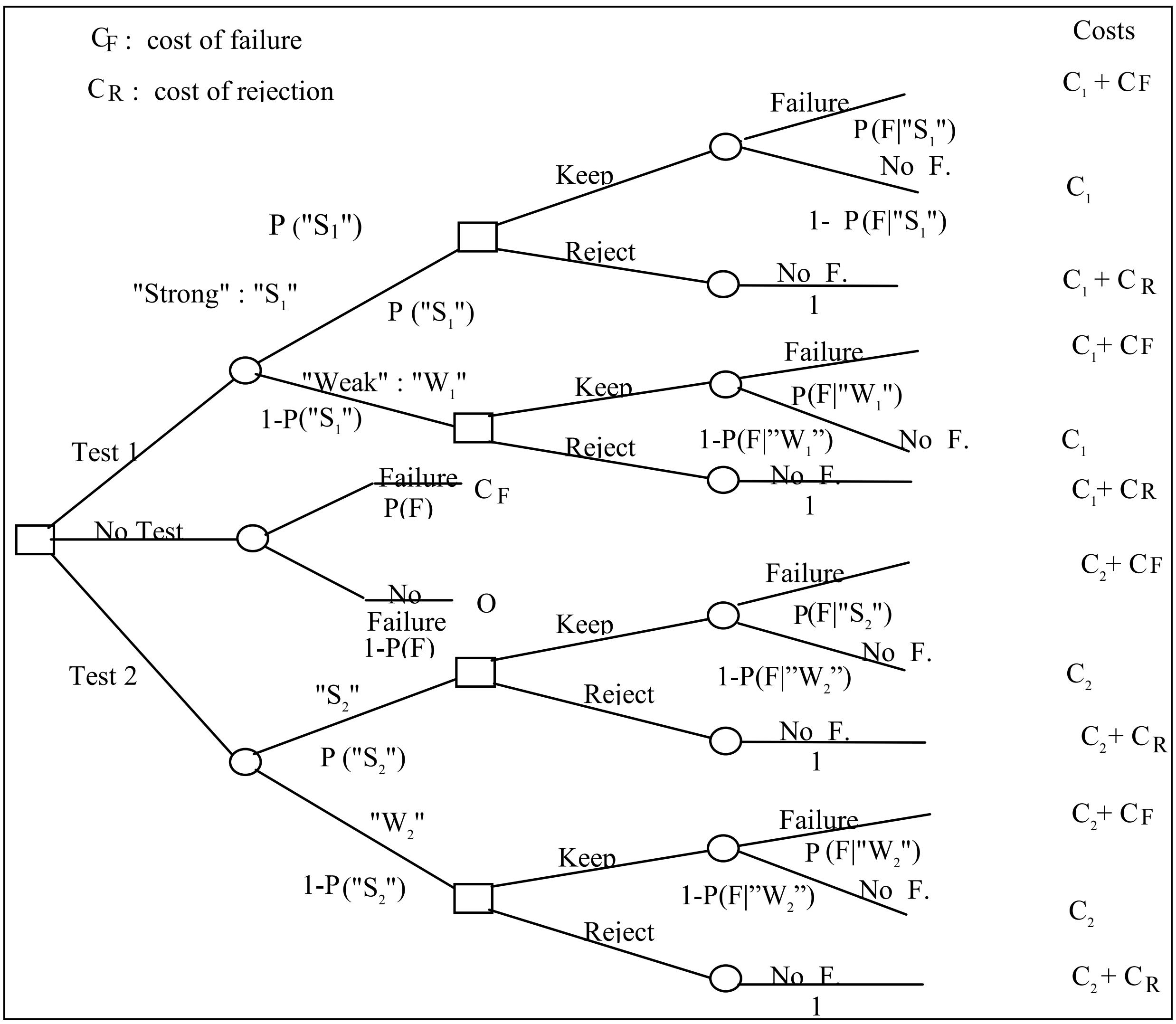


FIG. 9.3: TESTING A PART. VALUE OF IMPERFECT INFORMATION

NOTES:

- UNDER IMPERFECT INFORMATION, THE PRE-POSTERIOR (“X”) IS **NOT** EQUAL TO THE PRIOR (X).

- THE STRUCTURE OF THE TREE IS BASED ON **CONDITIONALITY** OF EACH EVENT WITH RESPECT TO PREVIOUS DECISIONS AND EVENTS IN THE TREE.

ASSUME THAT THE DECISION MAKER IS RISK-NEUTRAL (MINIMIZES EV(COSTS)).

(1) DECISION TO **KEEP** OR **REJECT** THE PART GIVEN TEST RESULTS:

FIRST DECISION NODE:

$P(F|\text{"}S_1\text{"}) \times (C_1 + C_F) + (1 - P(F|\text{"}S_1\text{"})) \times C_1 >? \; C_1 + C_R$

ASSUME $P(F|\text{"}S_1\text{"}) \times C_F \; > \; C_R$ ====> REJECT IF “W_1”

“ $< \; C_R$ ====> KEEP IF “S_1”

ASSUME, FOR EXAMPLE, THAT THE DATA ARE SUCH THAT, WHEN THE TEST’S RESULT IS “STRONG,” (1 OR 2) THE DECISION IS KEEP; REJECT OTHERWISE.

(2) DECISION TO ADOPT TEST 1 OR TEST 2: ASSUMING:

- EV(COSTS$_1$) $= P(\text{"}S_1\text{"}) \times [C_1 + C_F \times P(F|\text{"}S_1\text{"})]$ “S_1” $\Rightarrow$ K
  $+ \; [1 - P_1(\text{"}S_1\text{"})] \times (C_1 + C_R)$ “W_1” $\Rightarrow$ R
- EV(COSTS$_2$) $= P(\text{"}S_2\text{"}) \times [C_2 + C_F \times P(F|\text{"}S_2\text{"})]$ “S_2” $\Rightarrow$ K
  $+ \; [1 - P(\text{"}S_2\text{"})] \times (C_2 + C_R)$ “W_1” $\Rightarrow$ R
- EV(NO TEST) $= C_F \times P(F)$

CHOOSE THE ALTERNATIVE THAT MINIMIZES EV (COSTS) OVERALL (FIRST NODE)

(3) VALUE OF INFORMATION FOR THE RISK-NEUTRAL DECISION MAKER

FOR TEST 1 = MAXIMUM ONE IS WILLING TO PAY FOR THE TEST

= EV(NO TEST) - EV(COSTS$_1$)

$= C_F[P(F) - P(F, \text{"}S_1\text{"})] - C_R P(\text{"}S_1\text{"}) + C_R$

FOR TEST 2 = EV(NO TEST) - EV(COSTS$_2$)

$= C_F[P(F) - P_2(F, \text{"}S_2\text{"})] - C_R P_2(\text{"}S_2\text{"}) + C_R$

NUMERICAL ILLUSTRATION

RESOLUTION FOR THE EV DECISION MAKER

DATA

$$\text{Test 1} \begin{cases} P(\text{“}S_1\text{”}) = 0.5 \\ P(F \mid \text{“}S_1\text{”}) = 0.01 \\ P(\text{“}W_1\text{”}) = 0.5 \\ P(F \mid \text{“}W_1\text{”}) = 0.6 \end{cases}$$

$$\Rightarrow P(F) = P(F, \text{“}S_1\text{”}) + P(F, \text{“}W_1\text{”})$$

$$= P(\text{“}S_1\text{”}) \times P(F \mid \text{“}S_1\text{”}) + P(\text{“}W_1\text{”}) \times P(F \mid \text{“}W_1\text{”})$$

$$= 0.5 \times 10^{-2} + 0.5 \times 0.6$$

$$\Rightarrow PRIORS \begin{cases} P(F) = 0.305 \\ P(\bar{F}) = 0.695 \end{cases}$$

$$\text{Test 2} \begin{cases} P(\text{“}S_2\text{”}) = 0.6 \\ P(F \mid \text{“}S_2\text{”}) = 10^{-3} \\ P(\text{“}W_2\text{”}) = 0.4 \\ P(F \mid \text{“}W_2\text{”}) = 0.761 \end{cases}$$

[CHECK: $P(F) = 0.6 \times 10^{-3} + 0.4 \times 0.761 = 0.305$]

C_R = \$100,000

C_F = \$1,000,000

RESOLUTION OF KEEP - REJECT DECISION (BASED ON EXPECTED COSTS):

- FOR TEST 1, MESSAGE “S_1”

  EV (K) = 0.01 X 1M = \$10,000

  EV (R) = \$100,000

  ⇒ KEEP
- FOR TEST 1, MESSAGE “W_1”

  EV (K) = 0.6 X 1M = \$600,000

  EV (R) = \$100,000

  ⇒ REJECT
- FOR TEST 2, MESSAGE “S_2”

  EV (K) = 10^{-3} X 1M = \$1,000

  EV (R) = \$100,000

  ⇒ KEEP

- FOR TEST 2, MESSAGE "W_2"

  EV (K) = 0.761 X 1M = $761,000

  EV (R) = $100,000

  ⇒ REJECT

<u>RESOLUTION OF TEST DECISION</u>

REPLACE KEEP-REJECT NODES BY EV OF BEST ALTERNATIVE

- EV (COSTS OF NO TEST) = 0.305 X 1M = $305,000
- EV (COSTS OF TEST 1) = P("S_1") X EV (K)

  + P("W_1") X EV (R) + C_1

  = 0.5 X $10,000

  + 0.5 X $100,000 + C_1

  = $55,000 + C_1
- EV (COSTS OF TEST 2) = P("S_2") X EV (K)

  + P("W_2") X EV (R) + C_2

  = 0.6 X $1,000

  + 0.4 X $100,000 + C_2

  = $40,600 + C_2

VALUE OF INFO. OF TEST 1

= $305,000 - $55,000

= $250,000 = MAXIMUM ACCEPTABLE COST

VALUE OF INFO OF TEST 2

= $305,000 - $40,600

= $264,400 = MAXIMUM ACCEPTABLE COST

PREFERRED TEST? DEPENDS ON TEST COST

ASSUME C_1 = $50,000

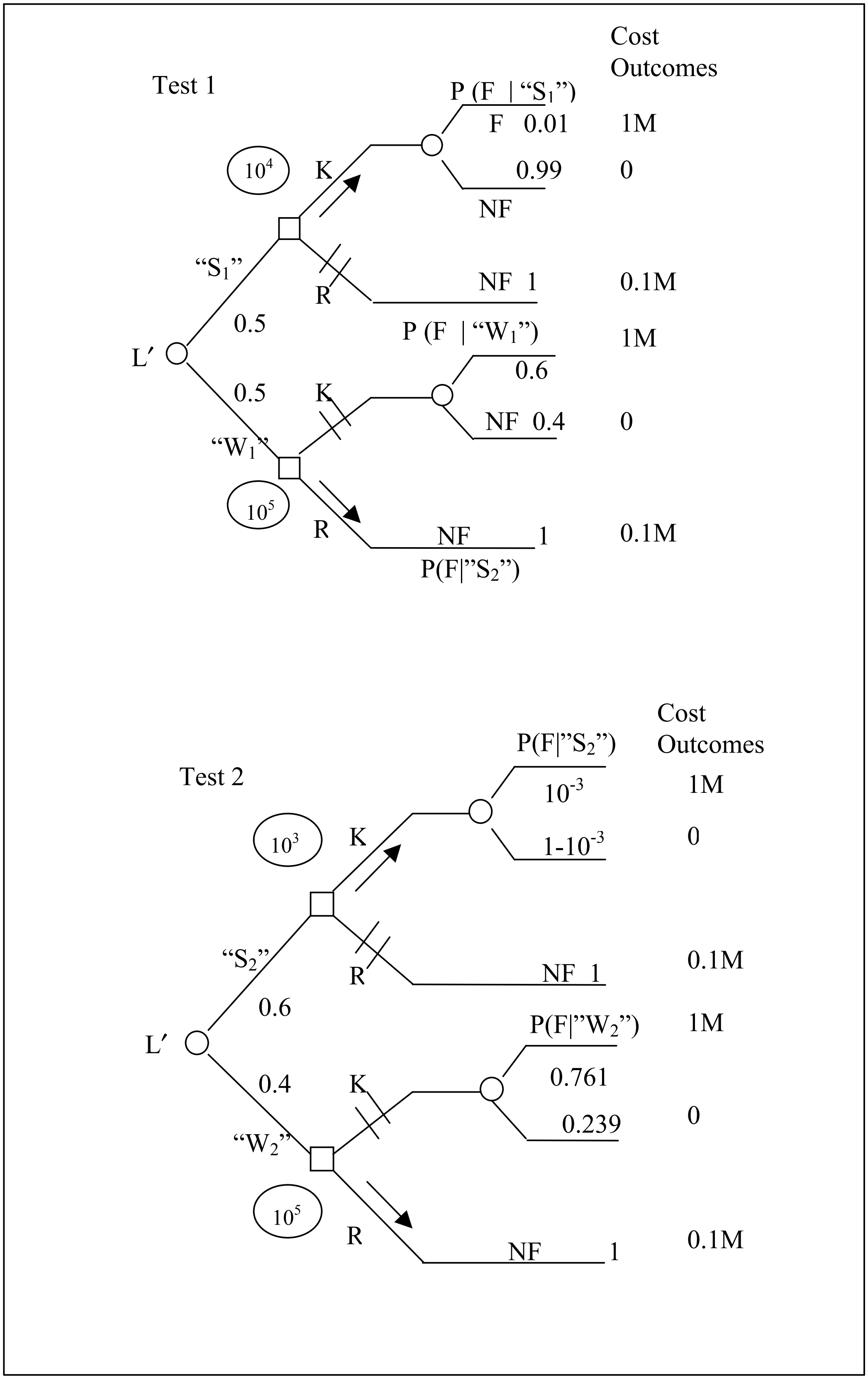
C_2 = $70,000

EV (COSTS OF TEST 1) = $105,000

EV (COSTS OF TEST 2) = $110,600

⇒ FOR THE RISK NEUTRAL, TEST 1 IS PREFERRED TO BOTH TEST 2 AND NO TEST

CHOICE OF A TEST: VALUE OF TESTS 1 AND 2 FOR A RISK-NEUTRAL DECISION MAKER



EXAMPLE 2: DRILLING FOR TEST OF LOCAL SEISMICITY BEFORE CONSTRUCTION OF A DAM

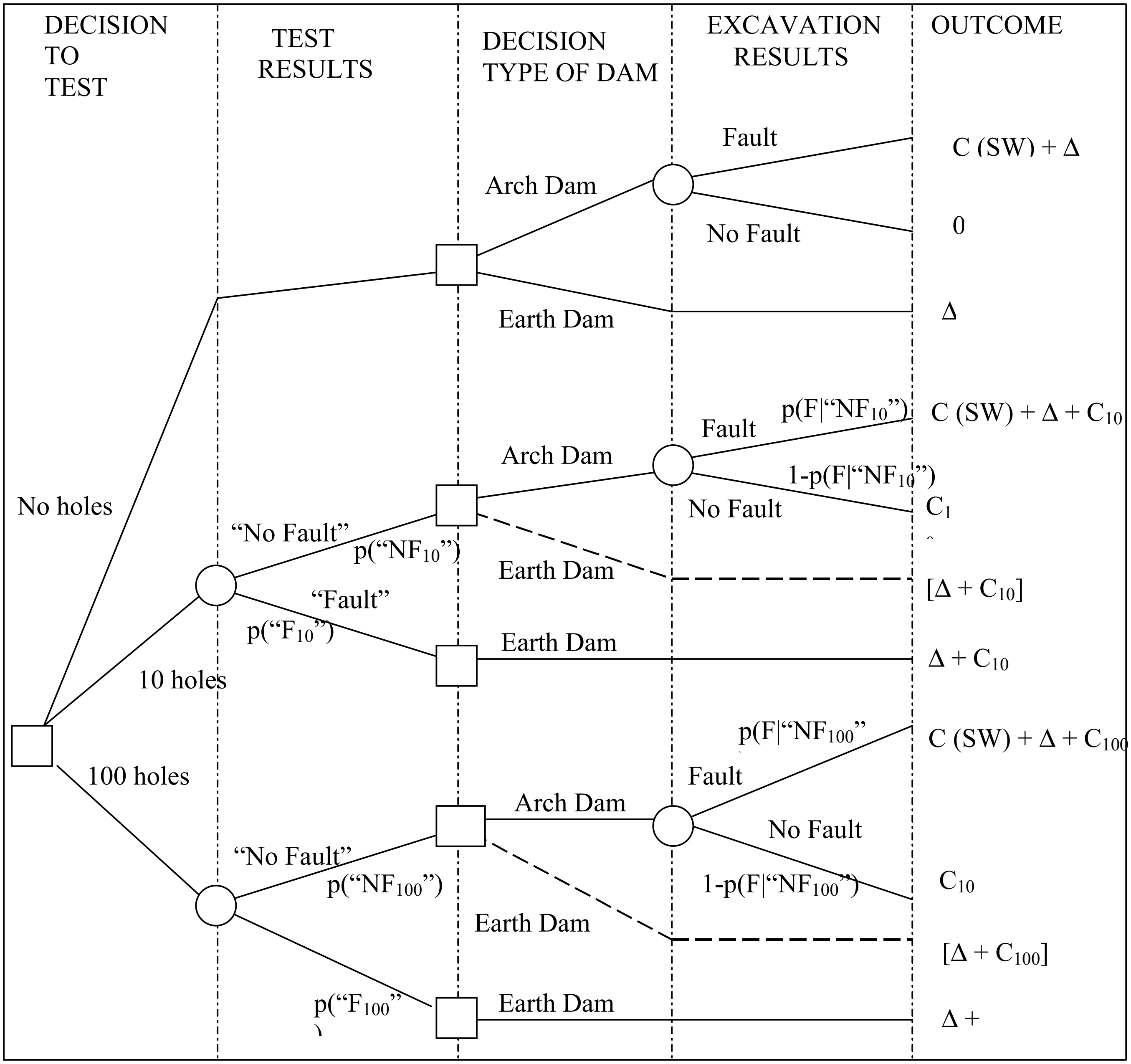


FIG. 9.4: SOIL TESTING BEFORE CONSTRUCTION OF A DAM. STRUCTURE OF THE DECISION TREE

C(SW): COST OF SWITCH; Δ: LOSS OF INCREMENTAL BENEFITS (EARTH DAM OR ARCH DAM);

C_{10}: COST OF DRILLING 10 HOLES; C_{100}: COST OF DRILLING 100 HOLES

NOTE ON DOMINANCE:

IF NO FAULT IS FOUND AFTER DRILLING 10 HOLES, AN ARCH DAM SHOULD BE PLANNED. OTHERWISE, NO DRILLING AND AN EARTH DAM WOULD DOMINATE THE ALTERNATIVE 10 HOLES AND EARTH DAM (COSTS WOULD BE LOWER WHATEVER THE OUTCOME OF THE EXCAVATION PHASE). THE SAME IS TRUE FOR DRILLING 100 HOLES AND FINDING NO FAULT.

9.3 VALUE OF INFORMATION FOR A GIVEN UTILITY

DEFINITION OF VALUE OF INFORMATION:

THE MAXIMUM PRICE ONE IS WILLING TO PAY (BREAKEVEN POINT) FOR THE "MESSAGE"; EQUAL TO THE AMOUNT X THAT YOU SUBTRACTED FROM THE OUTCOMES MAKES THE DECISION MAKER INDIFFERENT BETWEEN THE LOTTERIES WITH OR WITHOUT INFORMATION.

ALGORITHM: IN THE GENERAL CASE: LET L BE THE LOTTERY WITHOUT INFORMATION AND L' THE LOTTERY WITH INFORMATION AT COST X

- RESOLVE U (L' (X)) = U (L) TO FIND X
- NO ALGEBRAIC SOLUTION IN THE GENERAL CASE

  ⇒ TRIAL AND ERROR
- CHOOSE VALUE OF X, RESOLVE TREE AND COMPUTE U(L' (X))

EXAMPLE 1 - VALUE OF IMPERFECT INFORMATION FOR THE INVESTMENT DECISION AND A RISK-AVERSE DM

(SEE INVESTMENT PROBLEM INTRODUCED EARLIER)

FOR DECISION MAKER FRED (RISK AVERSE)

FRED'S UTILITY FUNCTION

U (1.4) = 1	U (1.0) = 0.88
U (1.3) = 0.98	U (0.9) = 0.85
U (1.2) = 0.95	U (0.5) = 0.6
U (1.1) = 0.91	U (0.4) = 0.53

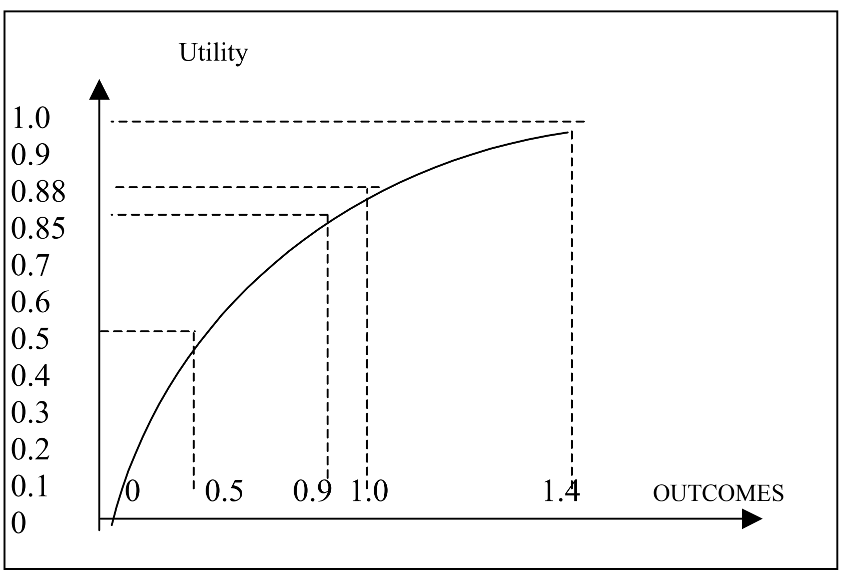


FIG. 9.7: FRED'S UTILITY CURVE

WITHOUT INFORMATION

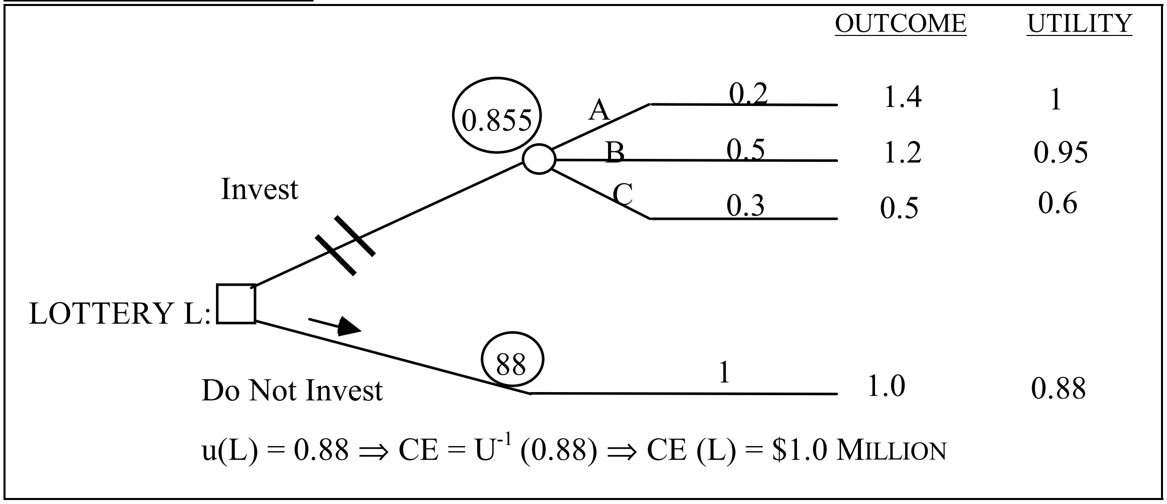


FIG. 9.6: DECISION TREE WITHOUT INFORMATION FOR FRED
(RISK AVERSE DECISION MAKER)

SOLUTION: FRED CHOOSES THE ALTERNATIVE NOT TO INVEST

WITH INFORMATION

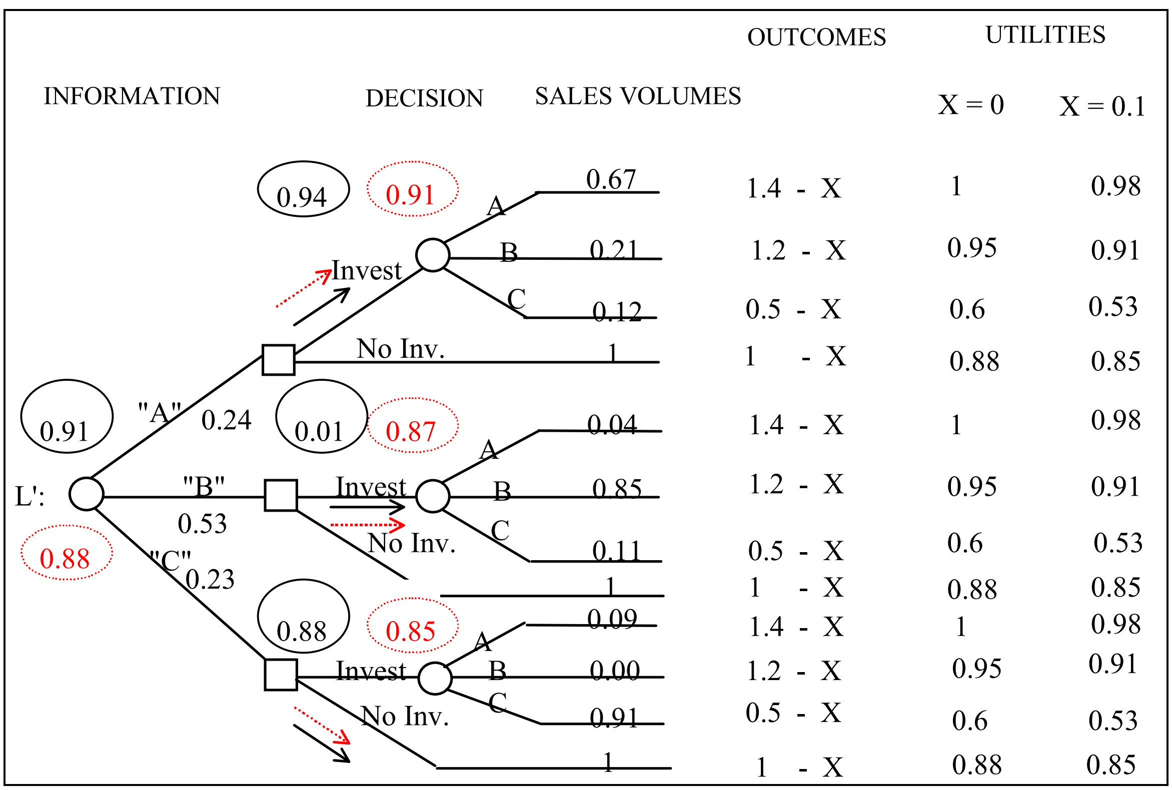


FIG. 9.8: DECISION TREE WITH IMPERFECT INFORMATION FOR FRED

DOTTED LINE ---- FOR X = 0.1 ($100,000);

SOLID LINE —— FOR X = 0 X IN THOUSAND DOLLARS

ALGORITHM: IN THE GENERAL CASE:

- RESOLVE U (L' (X)) = U (L) TO FIND X
- NO ALGEBRAIC SOLUTION

  TRIAL AND ERROR
- CHOOSE VALUE OF X, RESOLVE TREE AND COMPUTE U(L' (X))

HERE TRY X = 0.1

SOLUTION:	"A"	$\rightarrow$	INV.	EU = 0.91	EU(L' (0.1)) = 0.88
	"B"	$\rightarrow$	INV.	EU = 0.87	
	"C"	$\rightarrow$	DO NOT	EU = 0.85	

U (L' (0.1)) $\approx$ U (L) $\rightarrow$ X $\approx$ 0.1

IT HAPPENS HERE THAT EU (L'(X=0.1)) = EU (L WITHOUT INFORMATION)

$\Rightarrow$ X = 0.1 = BREAKEVEN PAYMENT = VALUE OF INFORMATION FOR FRED = $100,000

9.4 VALUE OF INFORMATION FOR CONSTANT RISK ATTITUDE (Δ PROPERTY)

VoI for constant risk attitude

= CE (lottery with information) - CE (lottery without information)

because subtracting X from outcomes is equivalent to subtracting X from certain equivalent:

let X = VoI

Eu (lottery without information = Eu (lottery with information - X from outcomes)

Take u^{-1} of both sides

for constant risk attitude only, value of information (VoI)

VoI = CE (lottery with information) - CE (lottery without information) and no payment (X=0)

Here: it turns out that Fred has a constant risk aversion

⇒ CE without information = 1M

CE with information = $u^{-1}(0.91)$

= 1.1M

⇒VoI = 1.1M - 1M

= 0.1M = $100,000

The solution is simple because Fred has an exponential utility.

⇒ Could have found the solution directly:

$$\begin{cases} CE(L) = 1 \text{ million} \quad [= U^{-1}(U(L))] \\ CE(L'(x=0)) = 1.1 \text{ million} \end{cases}$$

⇒VoI = 0.1M

EXAMPLE

Value of information of tests 1 and 2 for a risk-averse decision maker (see problem sect. 9.2)

Disutility for Lynn (fig. 9.9)

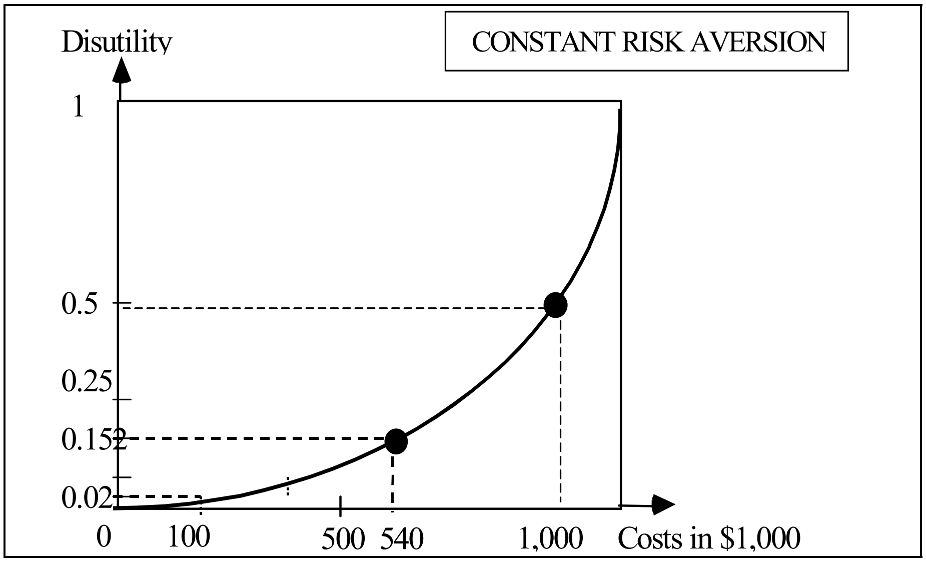
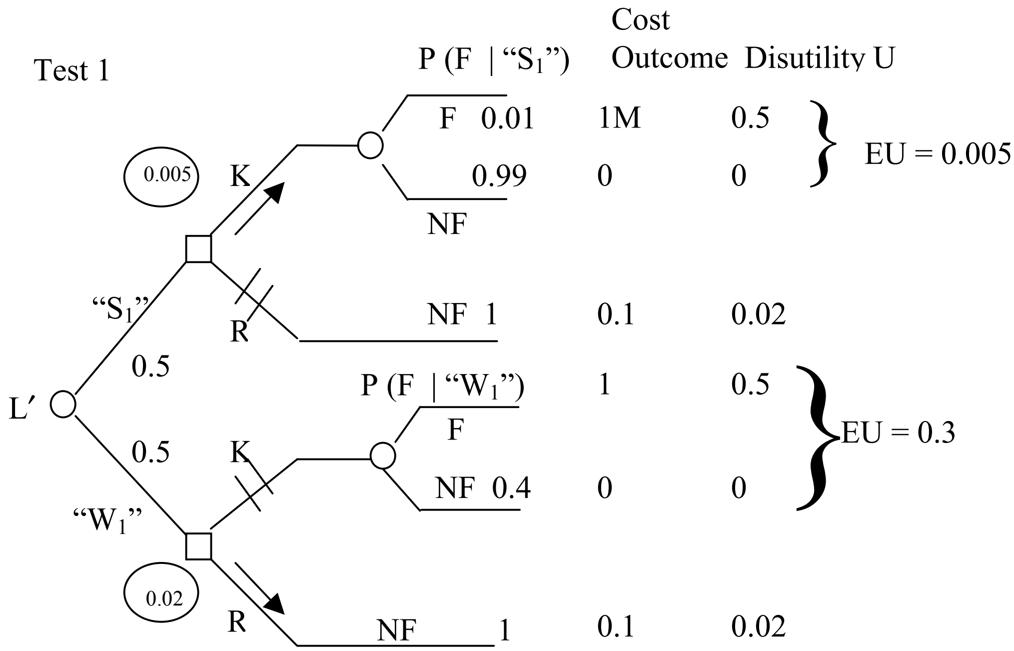


FIG. 9.9 DISUTILITY CURVE FOR LYNN (CONSTANT RISK AVERSION)

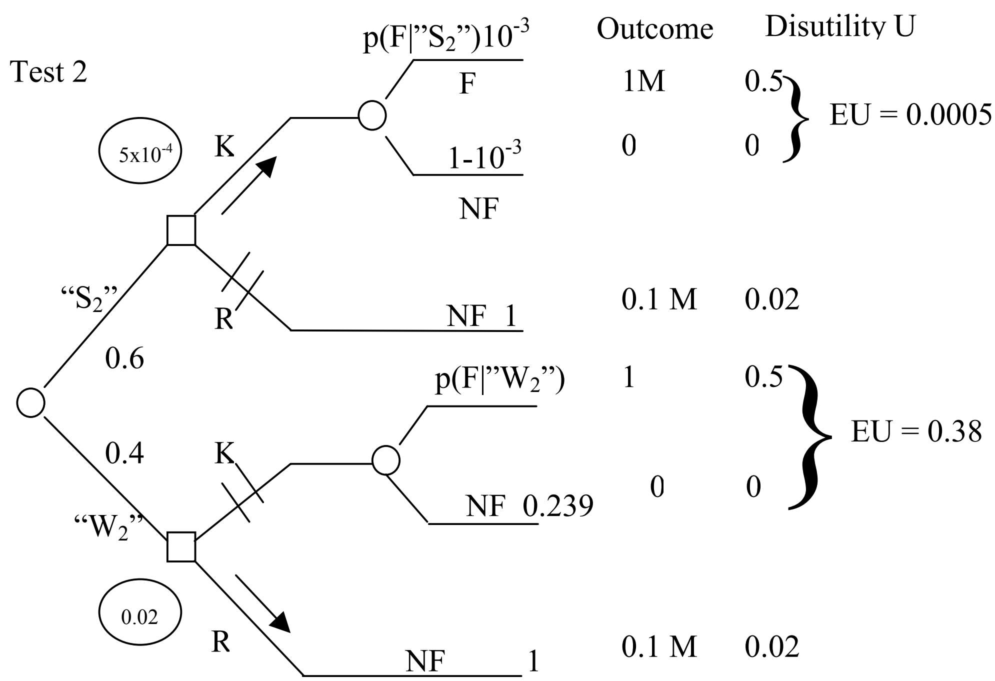
$\underline{U}$ ($1,000,000) = 0.5 $\underline{U}$ ($40,000) = 0.0125 $\Rightarrow \underline{U}^{-1}(0.0125)$ = $40,000

$\underline{U}$ ($100,000) = 0.02 $\underline{U}$ ($10,000) = 0.0083 $\Rightarrow \underline{U}^{-1}(0.0083)$ = $10,000

CHOICE OF A TEST: VALUE OF TESTS 1 AND 2 (SEC. 9.2) FOR THE RISK-AVERSE DM (LYNN)



EU (L′ 1) = 0.5 x 0.005 + 0.5 x 0.02 = 0.0025 + 0.01 = 0.0125 ⇒ CE (L′ 1)
= $^{-1}$ (0.0125) = ~\$40,000 costs ⇒ VoI_1 = \$540,000 - \$40,000 = \$500,000 (benefit)



EU (L'_2) = 0.6 x 5 x 10^{-4} + 0.4 x 2 x 10^{-2} = 3 x 10^{-4} + 8 x 10^{-3} = 0.0083
⇒ CE (L'_2) = U^{-1} (0.0083) ~ \$10,000 ⇒ VoI_2 = \$540,000 -\$10,000 = \$530,000 (benefits)

Decision given test costs: (Δ property)

VoI of Test 2 greater than for Test 1
- Cost of Test 1: \$50,000 ⇒ net result = \$450,000 benefit
- Cost of Test 2: \$70,000 ⇒ net result = \$460,000 benefit
- ⇒ For the risk averse (Lynn) the decision is Test 2 then K R

9.5 SUMMARY

STRUCTURE OF A DECISION TREE

- GENERAL DESCRIPTION

ACTIONS STATES OF NATURE ACTIONS.. OUTCOMES VALUES

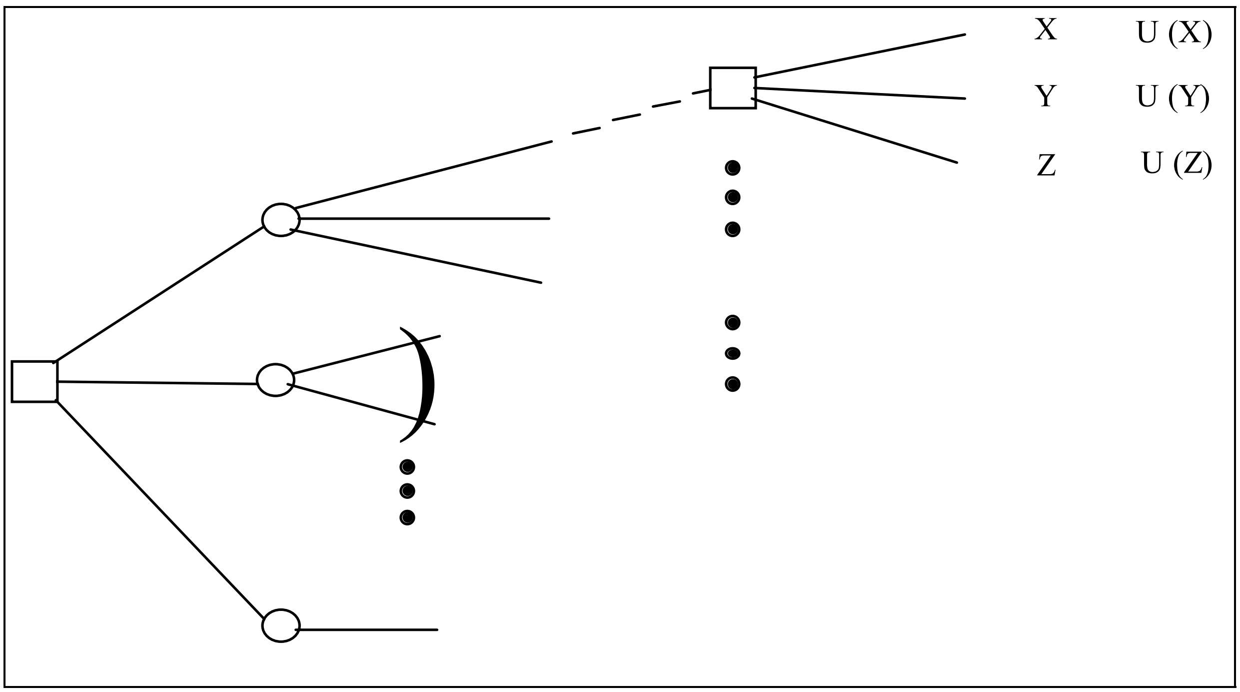


FIG. 9.12: GENERAL STRUCTURE OF DECISION TREES
(DISCRETE OR CONTINUOUS SETS OF ACTIONS AND STATES OF NATURE)

PROBABILITIES ARE ASSOCIATED TO THE DIFFERENT STATES OF NATURE. JOINT PROBABILITIES ARE ASSOCIATED TO SEQUENCES ("PATHS") OF EVENTS **(SCENARIOS)**.

- <u>EVALUATION OF THE PROBABILITIES ASSOCIATED WITH EACH SEQUENCE</u>
  DESCRIPTION OF THE UNCERTAINTIES THROUGH CONDITIONAL PROBABILITIES
  DEFINITION OF CONDITIONAL PROBABILITY (OR BAYES THEOREM)

$$p(A \mid B) = \frac{p(A, B)}{p(B)}$$

TOTAL PROBABILITY THEOREM:

$$\begin{aligned} p(B) &= \sum_i p(B \mid A_i) \times p(A_i) \\ &= \sum_i p(B, A_i) \end{aligned}$$

- <u>VALUATION OF THE OUTCOMES</u> (THEORY IN NEXT SECTION):

  FOR EACH POSSIBLE OUTCOME, ASSESS A "UTILITY" VALUE (UTILITY MEASURES INDIVIDUAL'S SATISFACTION FOR EACH FINAL STATE)

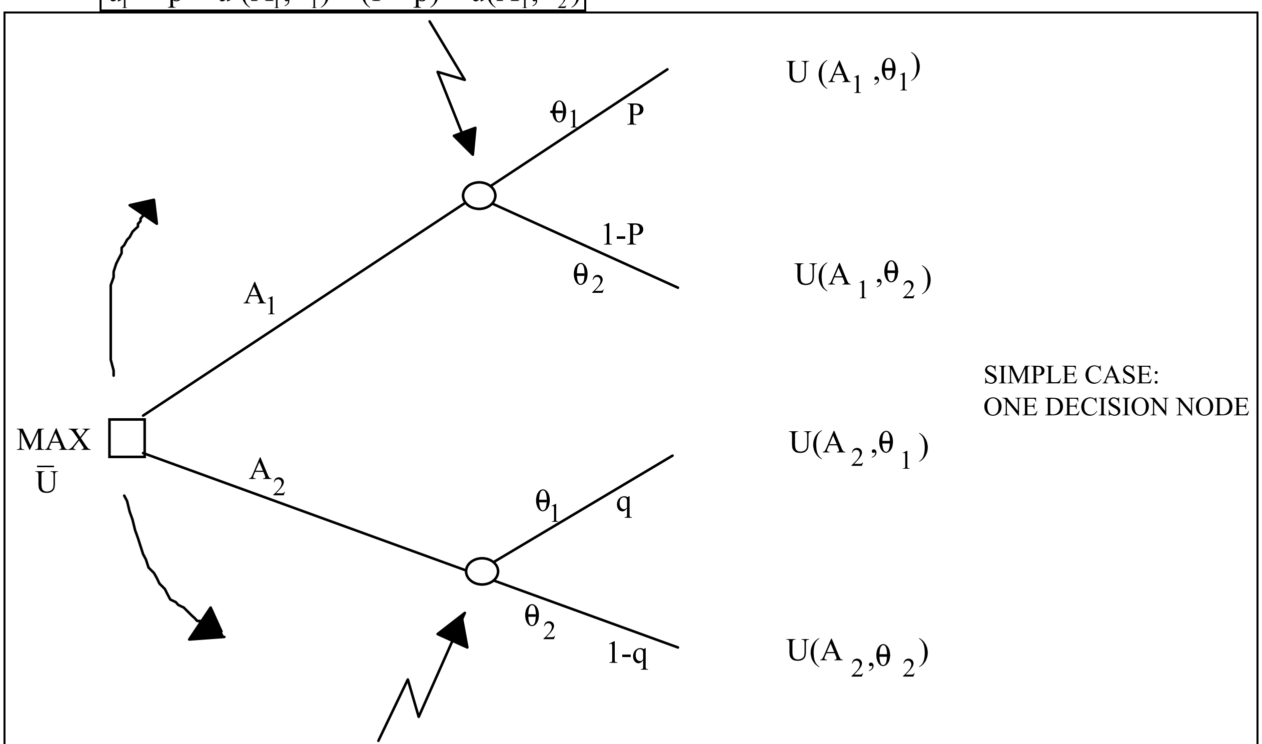
  GOAL OF THE RATIONAL DECISION MAKER:

<u>MAXIMIZE HIS OR HER EXPECTED UTILITY</u>

GOAL OF THE ANALYST: COMPUTE THE EXPECTED UTILITY AS REVEALED

EVALUATION OF A DECISION TREE

$$\bar{u}_1 = p \times u(A_1, \theta_1) + (1-p) \times u(A_1, \theta_2)$$



$$\bar{u}_2 = p \times u(A_2, \theta_1) + (1-q) \times u(A_2, \theta_2)$$

FIG. 9.13: EVALUATION OF A DECISION TREE

θ_1: STATES OF NATURE (STATE VARIABLES)

A_I: ACTIONS OR STRATEGIES (DECISION VARIABLES)

NOTE ON DECISION TREES:

- STATES OF NATURE CAN BE A SEQUENCE OF EVENTS: ANALYSIS OF THEIR PROBABILITIES IS DONE THROUGH EVENT TREES.
- THE RELEVANT STATES OF NATURE CAN BE A FAILURE MODE (FOR EXAMPLE); THE ANALYSIS OF THEIR PROBABILITIES IS DONE THROUGH FAULT TREES (THEORY IN PART 3, SECTION 13).

ALGORITHM FOR RESOLUTION:

- COMPUTE EXPECTED UTILITY FOR EACH DECISION ALTERNATIVE (RIGHT OF THE TREE).
- SELECT ALTERNATIVE, WHICH MAXIMIZES EXPECTED UTILITY FOR EACH DECISION VARIABLE; REPLACE VARIABLE BY CHOSEN ALTERNATIVE.
- REPEAT UP TO FIRST DECISION NODE.

VALUE OF INFORMATION

= BREAKEVEN POINT SUCH THAT EU (LOTTERY WITHOUT INFORMATION)

= EU (LOTTERY WITH INFORMATION AND INFORMATION PRICE SUBTRACTED FROM OUTCOME GAINS)

WHEN THE U FUNCTION IS LINEAR OR EXPONENTIAL (CONSTANT RISK ATTITUDE):

VOI = CE (WITH INFORMATION) - CE (WITHOUT INFORMATION)

SECTION 10

MULTI-ATTRIBUTE DECISIONS

10.1 ATTRIBUTES, OBJECTIVES AND PREFERENCES

IN THE GENERAL CASE THERE ARE SEVERAL ATTRIBUTES TO A DECISION. SOME MAY NOT BE DIRECTLY MARKETABLE.

ATTRIBUTES X_1 (EX.: DOLLARS)

X_2 (EX.: HUMAN LIVES)

X_3 (EX.: NUMBER OF TREES ALONG A ROAD)

THE UTILITY OF EACH OUTCOME IS A FUNCTION OF THE FORM:

$u\,(X_1, X_2, X_3)$.

TWO POSSIBLE APPROACHES

SINGLE ATTRIBUTE OR MULTI-ATTRIBUTE DECISION ANALYSIS

<u>SINGLE ATTRIBUTE</u>

- ASSUME CONSTANT MARGINAL RATE OF SUBSTITUTION AMONG ATTRIBUTES. PLACE A DOLLAR VALUE ON EACH ATTRIBUTE
- FORM SINGLE OBJECTIVE FUNCTION
- MAXIMIZE EXPECTED UTILITY

IMPLIES: SAME RISK ATTITUDE FOR ALL THE ATTRIBUTES

<u>MULTI-ATTRIBUTE</u>

- DEFINE TRADEOFFS AMONG ATTRIBUTES BY INDIFFERENCE CURVES
- ESTABLISH UTILITY AS A FUNCTION OF SEVERAL VARIABLES
- MAXIMIZE EXPECTED UTILITY

IMPLIES: MARGINAL RATE OF SUBSTITUTION CAN VARY AT DIFFERENT LEVELS OF UTILITY. RISK ATTITUDE CAN VARY FOR THE DIFFERENT ATTRIBUTES.

IN THE MULTI-ATTRIBUTE APPROACH, NOT ONLY CAN THE “PRICES” VARY, BUT THE UTILITIES CAN VARY ACROSS ATTRIBUTES (EVEN IF THE PRICES WERE THE SAME)

NOTE: IF THE “PRICES” ARE NOT CONSTANT, THE INDIFFERENCE CURVES ARE NOT LINEAR.

EXAMPLE:

DECISION INVOLVING DOLLARS (X_1), HUMAN LIVES (X_2) AND TREES (X_3)

(E.G., CUTTING TREES ALONG A ROAD TO IMPROVE THE SAFETY OF AUTOMOBILE DRIVERS)

10.2 SINGLE-OBJECTIVE APPROACH

REDUCE ALL ATTRIBUTES TO A SINGLE-OBJECTIVE FUNCTION

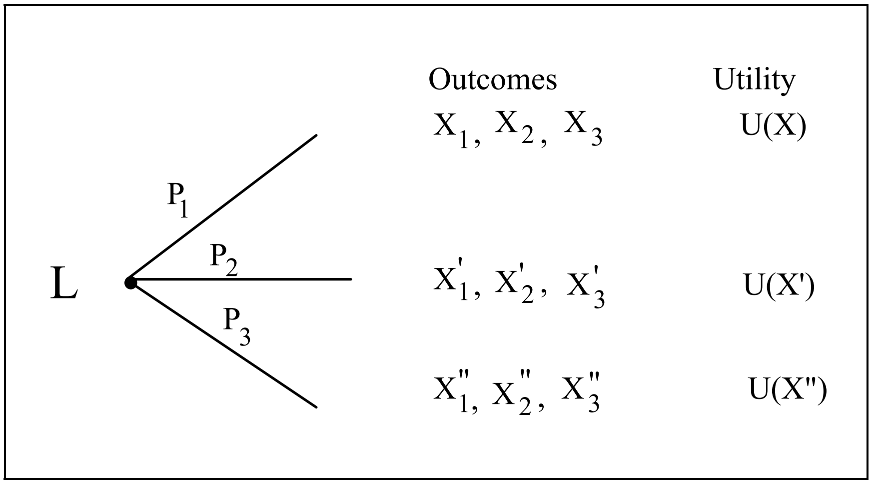


FIG. 10.1: LOTTERY WITH SEVERAL ATTRIBUTES

- PLACE A VALUE a ON HUMAN LIVES
- PLACE A VALUE b ON TREES
- FORM THE DOLLAR EQUIVALENT OF EACH OUTCOME

$$X = X_1 + a X_2 + b X_3$$

ASSUMPTION: THE MARGINAL RATE OF SUBSTITUTION BETWEEN TWO ATTRIBUTES IS HELD CONSTANT.

COMPUTE THE UTILITY OF (X_1, X_2, X_3) AS THE UTILITY OF A DOLLAR VALUE

$u(X_1, X_2, X_3) = u(X) = u(X_1 + a X_2 + b X_3)$

U, A, AND B ARE REVEALED BY THE DECISION MAKER

- U BY THE CHOICES AMONG LOTTERIES
- a AND b AS VALUES TO HER ("WILLINGNESS TO PAY") OF LIVES AND TREES

ADVANTAGE: SIMPLICITY

DISADVANTAGE: THE UTILITY OF ALL THE TREES MAY NOT BE THE UTILITY CORRESPONDING TO X_3 xb (THE LAST TREE MAY HAVE A VERY HIGH VALUE)

10.3 MULTI-ATTRIBUTE APPROACH: A GENERAL DEFINITION

(REFERENCE: KEENEY AND RAIFFA.)

PROBLEM: CONSTRUCT A UTILITY FUNCTION OF THE MEASURE OF ALL ATTRIBUTES (CONTINUOUS, MONOTONIC) FOR ALL VARIABLES.

SEVERAL APPROACHES:

- [WEIGHTED SCORING PROCEDURES: ASSIGN PRICES TO DIFFERENT ATTRIBUTES AND CALCULATE EV OF WEIGHTED SUM OF SCORES; PROBLEM: ASSUMES RISK NEUTRALITY AND DOES NOT ALLOW FOR CASES IN WHICH ATTRIBUTES ARE DEPENDENT (BACK TO SINGLE ATTRIBUTE WITH LINEAR UTILITY FUNCTION).]
- MULTI-ATTRIBUTE-UTILITY THEORY (MAUT): MORE COMPLEX THAN WEIGHTED SCORING TECHNIQUE, BUT NONE OF THE PROBLEMS.

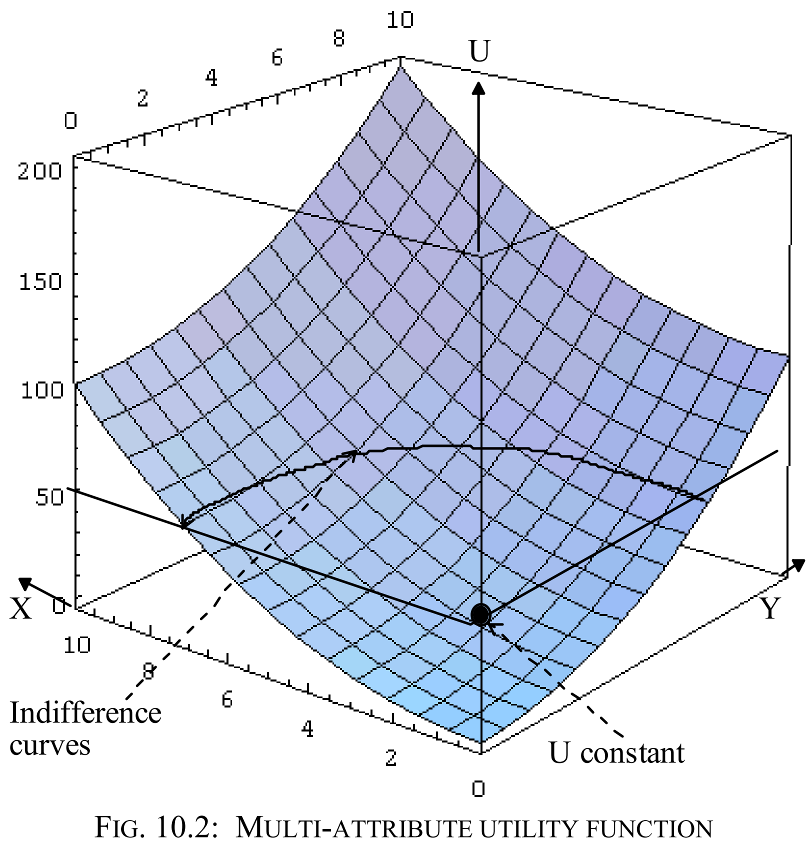


FIG. 10.2: MULTI-ATTRIBUTE UTILITY FUNCTION

- THE MULTI-ATTRIBUTE UTILITY SURFACE IS <u>CONTINUOUS</u> AND <u>MONOTONIC</u> WITH RESPECT TO ALL ATTRIBUTES. (SAME AXIOMS)
- WITH MORE THAN ONE ATTRIBUTE ONE MOVES FROM ONE LINE TO ONE (HYPER) SURFACE $U(X_1, X_2, ..., X_N)$.
- DEPENDING ON THE NATURE OF THE UNDERLYING UTILITY FUNCTION, THE ASSESSMENT OF THIS SURFACE CAN BE EASY OR VERY COMPLEX.
- HOW WOULD YOU ELICIT THE SHAPE OF THIS SURFACE FROM THE DECISION MAKER? INDIFFERENCE CURVES FOR U = CT; USE OF LOTTERIES.

POSSIBLE SOLUTION:

CONSTRUCT UTILITY AS A FUNCTION OF THE MEASURES OF ALL ATTRIBUTES

GENERAL CASE:

$U(X_1, X_2, ..., X_N)$; SET $U(\text{MIN}) = 0$; $U(\text{MAX}) = 1$

VARY GROUPS OF PARAMETERS ONE AT A TIME USING INDIFFERENCE CURVES

➔ GENERATE SURFACES BY CUTS ALONG PLANES: U = CONSTANT

COMPLICATED — ALMOST UNMANAGEABLE UNLESS ONE CAN MAKE SOME INDEPENDENCE ASSUMPTIONS REGARDING THE PREFERENCES FOR THE DIFFERENT ATTRIBUTES.

10.4 BACKGROUND: STRUCTURE OF PREFERENCES

FROM PREFERENCE TO UTILITY

PREFERENCES:

- EVERY DAY DECISIONS
- DEAL WITH CERTAIN OUTCOMES
- CAN EXHIBIT INDEPENDENCE – OR HIGH DEPENDENCE (BUY PC SOFTWARE IF YOU OWN A MAC?)

INDIFFERENCE CURVE. MARGINAL RATE OF SUBSTITUTION.

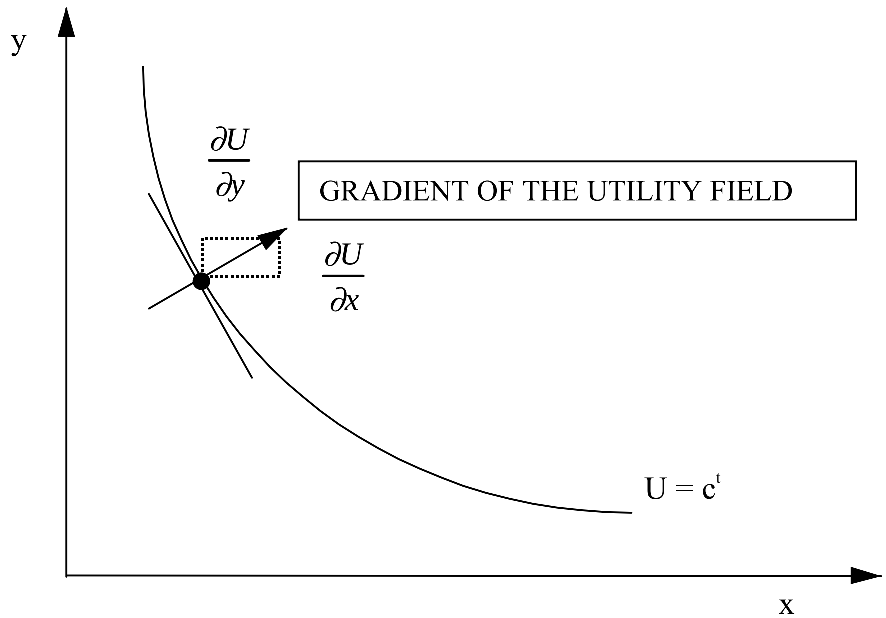


FIG. 10.3: INDIFFERENCE CURVE.

$$\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = 0 \text{ (U ct); } -\frac{dx}{dy} = \frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x}} = \text{MRS}$$

THE SLOPE OF THE GRADIENT AT ANY GIVEN POINT (X, Y) REPRESENTS THE "PRICE" OF ONE ATTRIBUTE WITH RESPECT TO THE OTHER AT THAT POINT.

THE GRADIENT OF THE UTILITY FIELD (∂ U/ ∂ X, ∂ U/ ∂ Y) REPRESENTS THE WILLINGNESS TO TRADE ONE ATTRIBUTE FOR THE OTHER FOR EACH STARTING POINT.

Gradient field representation of preferences

Example:

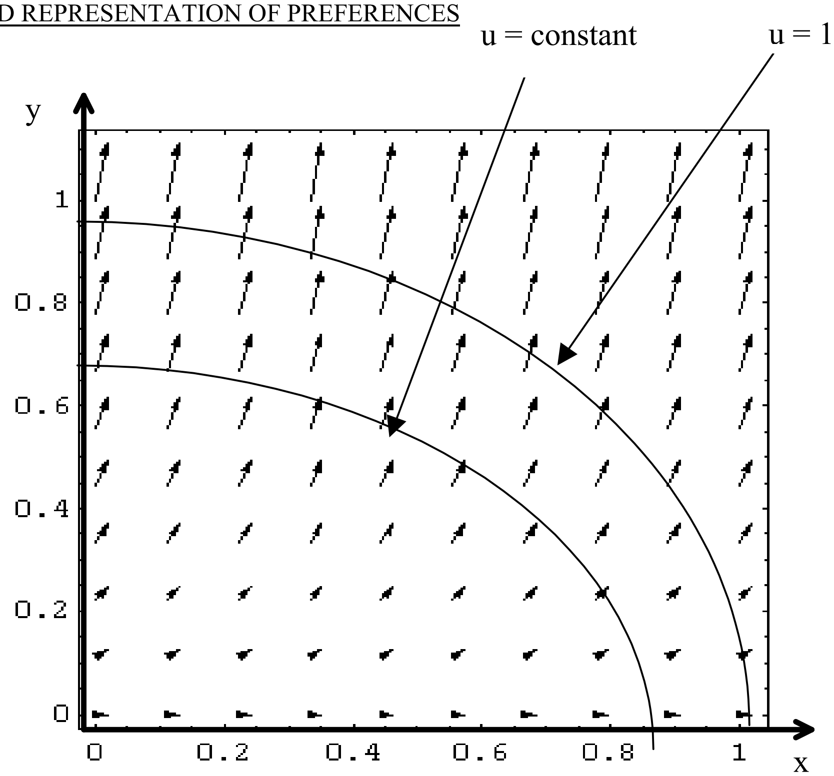


Fig. 10.4: Preference gradient field (x = √1-y)

- From economics we use the concepts of "marginal rate of substitution"("price") or "indifference curve"
- Preferences can be expressed in a gradient field, showing the direction of the attribute that is "most wanted" for a particular bundle of attributes
- Does this particular gradient field show preference dependence or independence?

Independence:

- preference for X (paid in Y's) does not depend on Y or on level of X
  $\Rightarrow (\partial U / \partial x) = a$
- preference for Y does not depend on X but depends on the level of Y
  ex: $(\partial U / \partial y) = by + c$ ex: $U = ax + by^2 + cy + d$; $U = ct \Rightarrow x = -\alpha y^2 - \beta y + \gamma$
  $\Rightarrow$Here, the MRS is constant for X and variable for Y but independent of X
  Here: U (x, y) = x + y2 U = 1 $\Rightarrow$ y = √1-x
- X has a constant price (in terms of ΔU)
- Y has a variable price in terms of ΔU (dependent on the Y level)
  (here: "addiction to y")
  These fields represent the answers of the DM to the questions: "How many apples are you willing to pay for an additional orange starting from x apples and y oranges?"

10.5 INDEPENDENCE ASSUMPTIONS FOR MULTI-ATTRIBUTE PROBLEMS

<u>TWO ATTRIBUTES:</u>

<u>SIMPLE UTILITY INDEPENDENCE</u>

ATTRIBUTES X_1, X_2

MEASURES x_1, x_2

X1 IS UTILITY-INDEPENDENT OF X2 WHEN CONDITIONAL PREFERENCES FOR LOTTERY ON X1 GIVEN X2 DO NOT DEPEND ON THE PARTICULAR LEVEL OF X2

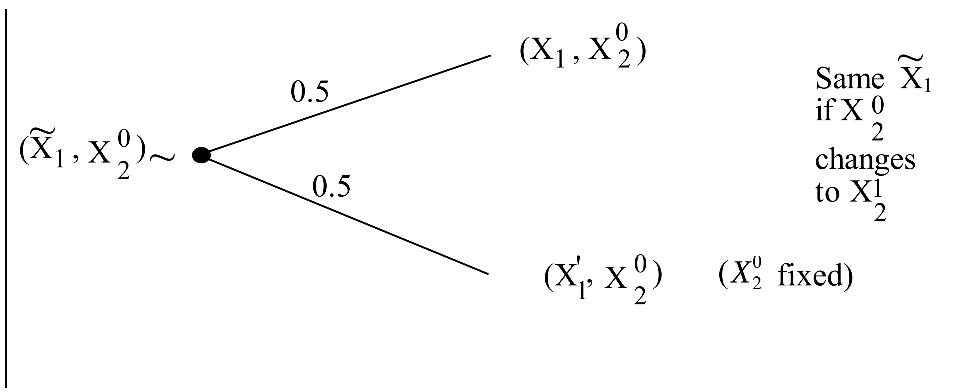


FIG. 10.5: UTILITY INDEPENDENCE

implies $\mathbf{u(x_1,x_2) = f(x_2) + g(x_2) \times u(x_1,x_2^0)}$

$(\tilde{X}_1, X_2^0$ CERTAIN EQUIVALENT FOR X_1, $\forall X_2^0)$

<u>MUTUAL UTILITY INDEPENDENCE</u> (IF UTILITY INDEPENDENCE IS TRUE BOTH WAYS)
THE CERTAIN EQUIVALENT FOR ONE ATTRIBUTE IS NOT INFLUENCED BY THE LEVEL OF THE OTHER => SIMPLIFIES ENCODING (FIX ONE ATTRIBUTE, ENCODE THE UTILITY CURVE FOR THE OTHER) $U(x_1, x_2) = U_1(x_1) + U_2(x_2)$

<u>THREE ATTRIBUTES:</u>

<u>PREFERENTIAL INDEPENDENCE</u>

THE PAIR OF ATTRIBUTES X_1 AND X_2 IS PREFERENTIALLY INDEPENDENT OF X_3 IF THE CONDITIONAL PREFERENCES IN THE (X_1, X_2) SPACE GIVEN X_3 DO NOT DEPEND ON X_3.

PREFERENCES AND TRADE-OFFS AMONG EACH PAIR OF ATTRIBUTES IS INDEPENDENT OF THE OTHER ATTRIBUTES $\Rightarrow$ ONE CAN HANDLE THE ATTRIBUTES 2 BY 2 (IT SIMPLIFIES ENCODING). (WEAK INDEPENDENCE). WE CAN FIX X_3^0 AND USE INDIFFERENCE CURVES BETWEEN x_1 AND x_2.

Additive Independence

X_1 and X_2 are additive-independent if the paired preference comparison of any two lotteries defined by two joint probability distributions on $X_1 \times X_2$ depends only on their marginal probability distributions => Any variation in attribute 1 can be compensated by a variation in attribute 2 and vice versa.

No "synergies"

X_1, X_2: "heaven" for both attributes (most preferred)

X'_1, X'_2: "the pits" for both attributes (least preferred)

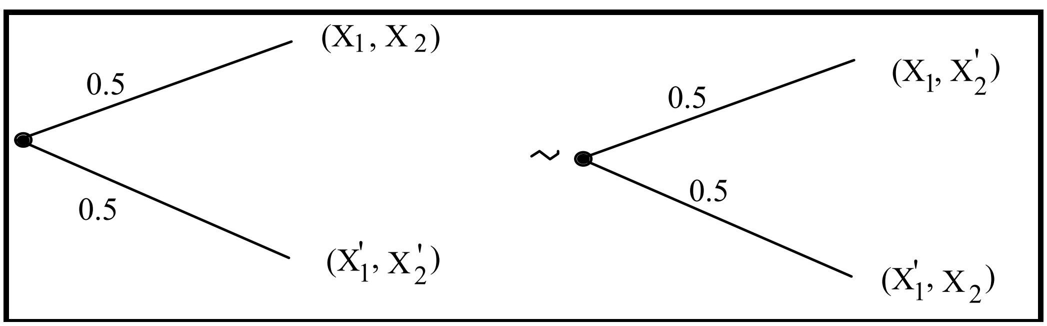


Fig. 10.6: additive independence

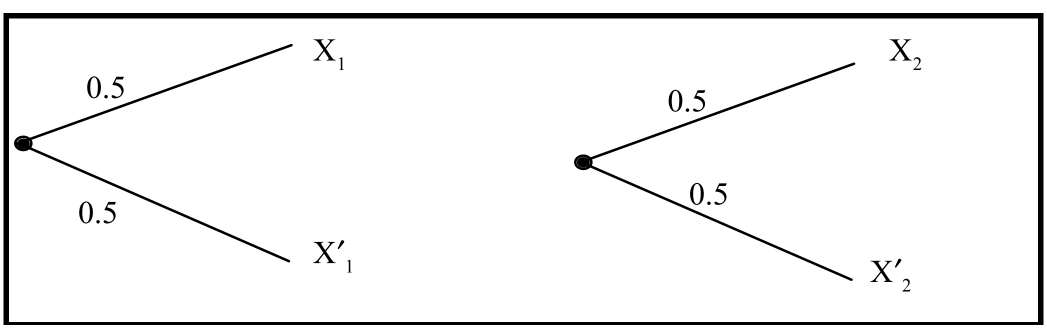


Fig. 10.7: marginal distributions for Fig. 10.6

Marginal distribution of second attribute X_2 in L_1 and in L_2 $(0.5{:}\ x_2;\ 0.5{:}\ x'_2)$

Implies: $u(x_1, x_2) = k_1 u_1(x_1) + k_2 u_2(x_2)$

FORM ALMOST ALWAYS USED BECAUSE ASSUMPTION OFTEN VERIFIED IN INDUSTRIAL SETTINGS. IN THAT CASE, THE PROCEDURE FOR THE MULTI-ATTRIBUTE APPROACH IS:

- DETERMINE SCALING CONSTANTS K_1 AND K_2 (FOR EXAMPLE, BY SETTING THE VARIABLES AT EXTREME VALUES).
- DETERMINE UTILITY FUNCTIONS U_1 AND U_2 AS IN THE CASE OF THE SINGLE ATTRIBUTE ASSESSMENT (FIXING THE OTHER VARIABLE AT CONVENIENT LEVEL).

ADDITIVE INDEPENDENCE CAN OCCUR WITH N ATTRIBUTES.

ADVANTAGE:
ALLOWS FOR DIFFERENT RISK ATTITUDES FOR THE DIFFERENT ATTRIBUTES.

DISADVANTAGE:
MORE DIFFICULT TO CONDUCT THAN SINGLE-ATTRIBUTE ANALYSIS. UNNECESSARY FOR DIRECTLY (AND REVERSIBLY) MARKETABLE COMMODITIES (RISK ATTITUDE IS THAT OF MONEY).

EX: THE DECISION TO CUT ALL THE TREES ALONG A SCENIC ROAD TO DECREASE THE NUMBER OF VICTIMS IN CAR ACCIDENTS. (UNCLEAR WHETHER THERE WAS ADDITIVE INDEPENDENCE BETWEEN VICTIMS AND TREES.)

10.6 EXAMPLES

<u>EXAMPLE 1: CHOICE OF A PIPELINE SITE</u>

THREE ALTERNATIVE ROUTES FOR A PIPELINE THAT AFFECT AN ENDANGERED SPECIES. TOTAL POPULATION 100,000 CREATURES. UNCERTAINTY ABOUT EFFECT ON CREATURES.

ALTERNATIVE 1	COSTS 70 MILLION		
	LOSS OF CREATURES IN 1,000	$\begin{cases} .1 & 95 \\ .9 & 75 \end{cases}$	(PROBABIL.AND OUTCOMES)
ALTERNATIVE 2	COSTS 100 MILLION		
	LOSS OF CREATURES IN 1,000	$\begin{cases} .5 & 60 \\ .5 & 45 \end{cases}$	(PROBABIL. AND OUTCOMES)
ALTERNATIVE 3	COSTS 120 MILLION		
	LOSS OF CREATURES IN 1,000	$\begin{cases} .4 & 20 \\ .6 & 5 \end{cases}$	(PROBABIL. AND OUTCOMES)

$$\underline{U} = X_1 + \tfrac{1}{100} X_2^2 \qquad \text{(DISUTILITY)}$$

$\uparrow$ X_1: \$ COSTS IN M; $\uparrow$ X_2: NUMBER OF CREATURES LOST (IN THOUSANDS)

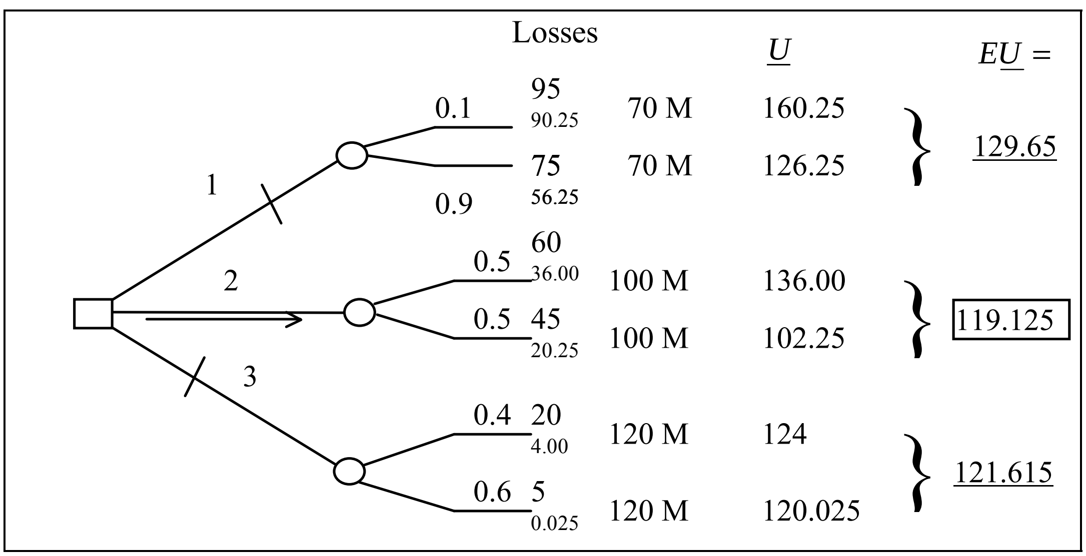


FIG. 10.8: CHOICE OF A PIPELINE SITE

BUT IF I CAN DECREASE THE COSTS OF ALTERNATIVE 3 BY 5M$\Rightarrow$ 115 M THEN $E\underline{U}_3 = 116.615$

PREFERRED ALTERNATIVE.

EXAMPLE 2: ROOT BEER AND PEANUTS

(ADAPTED FROM KEENEY AND RAIFFA, 1976)

PROBLEM:

YOU ARE IN CHARGE OF ORGANIZING THE FOOD AND DRINK FOR THIS YEAR'S FINAL IN THE FOOTBALL (AKA "SOCCER") WORLD CUP. YOU KNOW THAT MOST OF YOUR EUROPEAN FRIENDS WILL WANT TO ENJOY A GOOD BOTTLE OF ROOT BEER WITH THE GAME. AT THE SAME TIME, THEY WILL WANT SOME PIZZA TO EAT. SO YOU HAVE TO PROVIDE FOR TWO THINGS: ROOT BEER AND PIZZA. BUT HOW MUCH OF WHICH, GIVEN THAT YOU HAVE A LIMITED BUDGET?

FURTHERMORE, DEPENDING ON WHAT TEAM WILL WIN, YOU WILL EITHER NEED MORE ROOT BEER OR MORE PIZZA!

X WILL BE THE LEVEL OF "ROOT BEER SHORTAGE;" SO X = 2 MEANS YOU SHOULD HAVE PROVIDED FOR TWO MORE SIX PACKS. Y WILL BE THE LEVEL OF "PIZZA SHORTAGE;" SO Y = 2 MEANS YOU SHOULD HAVE BOUGHT TWO MORE PIZZAS. IN THE BEST OF ALL WORLDS, YOU WOULD HAVE X = 0 AND Y = 0; BUT THIS IS NOT THE BEST WORLD AND YOUR BUDGET ONLY ALLOWS YOU FOR SO MUCH ROOT BEER OR PIZZA.

AFTER LONG DELIBERATION AND TALKS WITH YOUR FRIENDS, YOU HAVE COME UP WITH THE FOLLOWING MULTI-ATTRIBUTE UTILITY FUNCTION (UTILITY INDEPENDENCE!):

$$U(X, Y) = 0.72\, U_X(X) + 0.13\, U_Y(Y) + 0.15\, U_X(X)\, U_Y(Y)$$

$$U_X(X) = 1 + 0.375\,(1 - e^{X/7.692})$$

$$U_Y(Y) = 1 + 2.033\,(1 - e^{Y/25})$$

YOU HAVE TWO POSSIBLE ALTERNATIVES: GO HEAVY ON THE ROOT BEER OR GO STRONG ON THE PIZZA. THE PROBLEM IS THE OUTCOME OF THE WORLD CUP FINAL. IF CAMEROON WINS, YOUR EUROPEAN FRIENDS WILL WANT MORE PIZZA, YET IF GREAT BRITAIN WINS, THEY WILL WANT MORE ROOT BEER. THE DECISION TREE DEPICTS YOUR SITUATION:

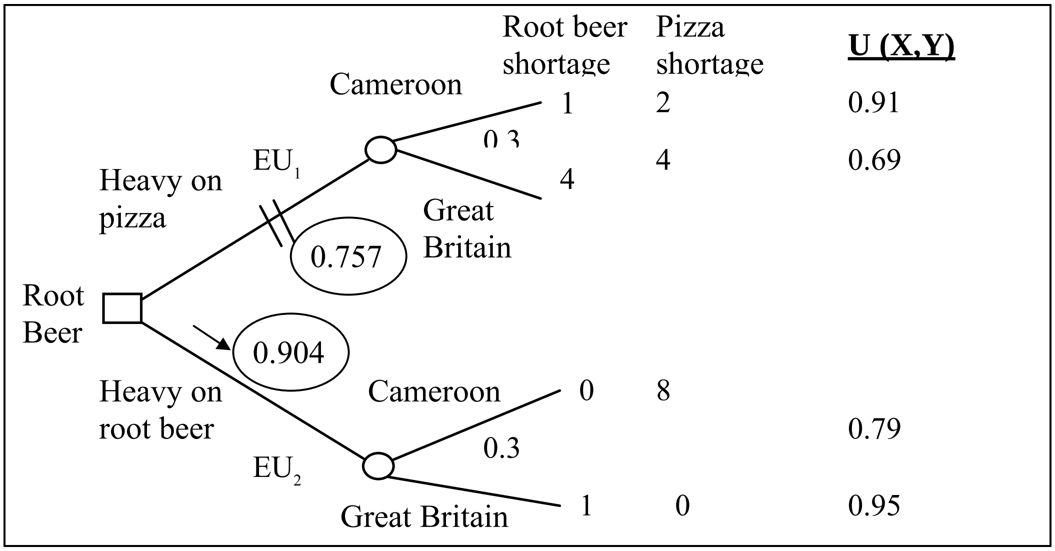


FIG. 10.9: ROOT BEER OR PIZZA?

10.7 ENCODING

ASSUME ADDITIVE INDEPENDENCE

⇒ ENCODING OF SINGLE ATTRIBUTE UTILITIES

EXERCISE: CONSTRUCTION OF JACK'S UTILITY CURVE. AN ALTERNATIVE TO ASKING FOR A CERTAIN EQUIVALENT (CE). FIX CE AND OUTCOMES AND ASK FOR PROBABILITY.

QUESTION 1:

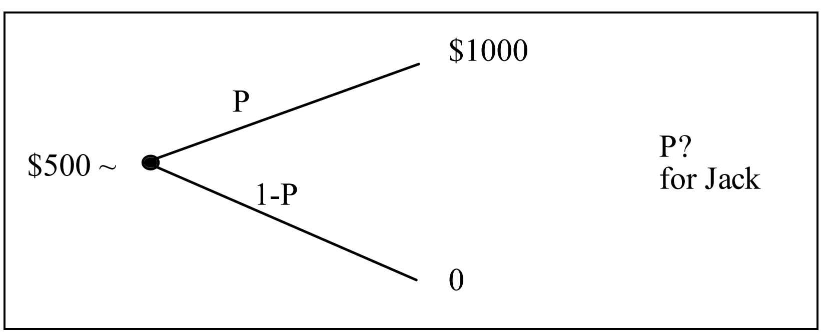


FIG. 10.10: QUESTION 1

JACK'S ANSWER: P = 0.7

⇒ U (500) = EU = 0.7

QUESTION 2:

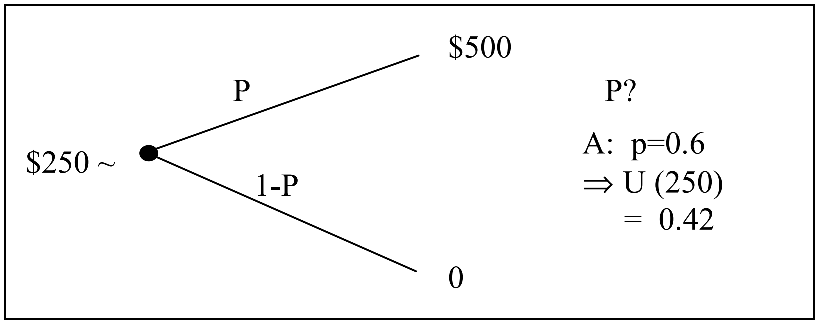


FIG. 10.11: QUESTION 2

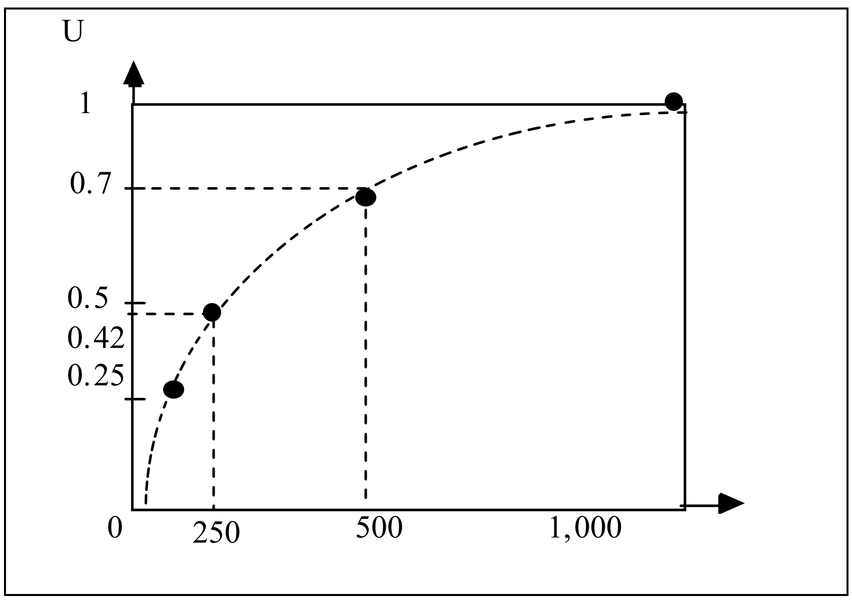


FIG. 10.12 JACK'S UTILITY CURVE

SECTION 11

COLLECTIVE DECISION MAKING AND DECISIONS IN THE PUBLIC SECTOR

READINGS: PATÉ-CORNELL, "ACCEPTABLE DECISION PROCESSES AND ACCEPTABLE RISKS IN PUBLIC SECTOR REGULATION."
TRIBUS, "DECISION ANALYSIS APPROACH TO SATISFYING THE REQUIREMENTS OF THE FLAMMABLE FABRICS ACT." (OPTIONAL READING)
FISHBURN, "FOUNDATIONS OF DECISION ANALYSIS: ALONG THE WAY" (OPTIONAL READING).

11.1 CASE OF SEVERAL DECISION MAKERS

DESCRIPTIVE VS. NORMATIVE ANALYSIS

DESCRIPTIVE (PROCEDURAL) APPROACH

- BARGAINING PROCESS. CAN BE DESCRIBED IN SOME CASES THROUGH GAME THEORY. RESULT DEPENDS ON BARGAINING POWER, GOOD WILL, AND INDIVIDUAL RESOURCES (OR RESOURCES OF EACH COALITION).
- VOTE. INFLUENCE OF INFORMATION (E.G., COLLECTIVE MEMORY). RESULTS IN THE PUBLIC SECTOR (GOVERNMENT): DECISIONS ARE OFTEN RISK-AVERSE AND CYCLICAL.

NORMATIVE (ANALYTICAL) APPROACH

PROBLEM: HOW SHOULD THE COLLECTIVITY MAKE DECISIONS FOR A GROUP OF INDIVIDUALS, GIVEN THEIR PARTICULAR UTILITY FUNCTIONS?
CASE OF GOVERNMENT DECISIONS:

- WHAT SHOULD BE THE GOVERNMENT'S RISK ATTITUDE?
- WHAT SHOULD BE ITS WILLINGNESS TO PAY FOR THE VARIOUS ATTRIBUTES OF A MULTI-ATTRIBUTE DECISION PROBLEM? (CALIBRATION)

NOTE: RISK ATTITUDE VS. WILLINGNESS TO PAY:
A VOCABULARY PROBLEM, IN PARTICULAR, IN THE PUBLIC SECTOR.

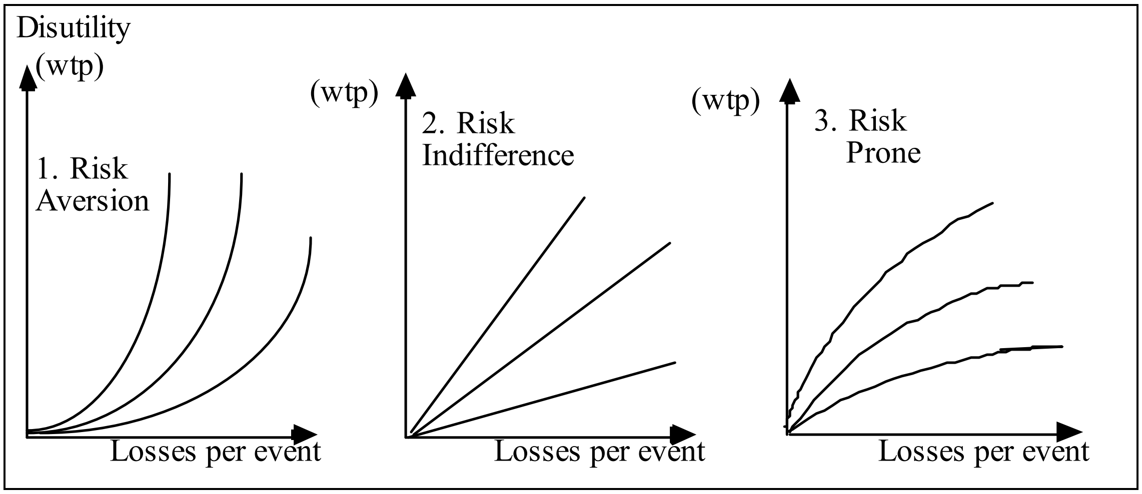


FIG. 11.1: PROBLEM WITH CLASSICAL DEFINITION OF RISK ATTITUDES

ADVANTAGES OF EACH

1. RISK AVERSION

   FITS THE POLITICAL PROCESS BASED ON VISIBILITY.

2. RISK INDIFFERENCE

   EQUALLY PROTECTS POTENTIAL VICTIMS REGARDLESS OF THE TOTAL NUMBER OF CASUALTIES (EQUITY ARGUMENT).

3. RISK PRONE

   MAY FIT THE POLITICAL PROCESS FOR LARGE DISASTERS WHERE ADDITIONAL LOSSES BECOME LESS OVERWHELMING.

11.2 ARROW'S IMPOSSIBILITY THEOREM

INDIVIDUALS HAVE "UTILITIES". GROUPS DON'T.

PROBLEMS WHEN TRYING TO CONSTRUCT A COLLECTIVE UTILITY (WITHOUT "CROSS-UTILITIES" OR A DICTATOR).

THE AXIOMS OF RATIONALITY OF CHOICES CANNOT APPLY TO GROUPS

ARROW HAS SHOWN THAT THE FOLLOWING 5 PROPERTIES ARE INCOMPATIBLE.

GIVEN THE RANKINGS OF A SET OF ALTERNATIVES BY EACH INDIVIDUAL IN A DECISION MAKING GROUP, THERE IS IN THE GENERAL CASE NO COLLECTIVE RANKING OF THOSE ALTERNATIVES THAT DOES NOT CONTRADICT ONE OF THE (VERY REASONABLE) FOLLOWING ASSUMPTIONS:

(A) **COMPLETE DOMAIN**

AT LEAST TWO INDIVIDUALS, THREE ALTERNATIVES, ALL PREFERENCES DEFINED. (NON TRIVIAL PROBLEM)

(B) **Positive association of social and individual ordering**

If the group originally preferred A to B and if any change in the individuals' tastes is (if anything) in favor of A, the group still prefers A to B. (reinforcement of preference)

(C) **Independence of irrelevant alternatives**

If an alternative is eliminated from the possible set, the group ordering is unchanged for the other alternatives.

(D) **Individual's sovereignty**

For each pair of alternatives A and B, there can be some set of individual orderings such that the group prefers A to B. (all options can be preferred; preferences are not dictated by the process)

(E) **Nondictatorship**

There is no individual with the property that whenever he (or she) prefers A to B the group will also prefer A to B, regardless of others' preferences. (preferences are not dictated by an individual)

A, B, C, D, and E are incompatible

There is an unavoidable issue: the group's utility for individual utilities.

Classical decision processes (e.g., majority vote) violate the axioms of rationality.

Illustration:

Nontransitivity (circularity) of votes

Example:

Three Decision makers: Peter, John, and Paul

Three Outcomes A, B, and C

Peter's preference	John's preference	Paul's preference
A > B > C	C > A > B	B > C > A

Vote (majority):

A>B (Peter & John)

B>C (Peter & Paul)

C>A (John & Paul)

circularity: A>B>C>A

By a majority-vote mechanism, the group can become "a money pump."

• INDIVIDUALS CAN HAVE PREFERENCES IN THE VON NEUMANN SENSE. GROUPS DO NOT. BUT GROUPS CAN FIND COMPROMISES AND HAVE SOCIAL VALUATION FUNCTIONS.

• THESE FUNCTIONS INCLUDE INTERPERSONAL COMPARISON OF PREFERENCES (INDIVIDUALS' STRENGTHS OF PREFERENCES).

• THEY CAN ALSO AGREE ON DECISION PROCESSES.

11.3 GROUP VALUATION FUNCTIONS

WITH THE FOLLOWING ASSUMPTIONS:

(1) **PREFERENTIAL INDEPENDENCE** OF EACH PAIR OF OUTCOMES WITH RESPECT TO THE OTHERS FOR THE WHOLE GROUP (PREFERENCES BETWEEN A AND B INDEPENDENT OF C).

(2) **ORDINAL POSITIVE ASSOCIATION** (IF A AND B ARE ORIGINALLY EQUALLY PREFERRED, AND IF A IS REPLACED BY A' WHICH SOME OF THE MEMBERS OF THE GROUP PREFER TO A WHILE OTHERS ARE INDIFFERENT BETWEEN A AND A', THEN THE GROUP PREFERS A' TO B)

THEN, THE VALUE FUNCTION OF THE GROUP HAS THE FOLLOWING FORM:

$$v(X) = \sum_i v_i^*(u_i(X))$$

WHERE $u_i(X)$ REPRESENTS THE UTILITY FUNCTION OF EACH OF THE INDIVIDUALS, $\mathbf{v_i^*}$ REPRESENTS A MONOTONIC CONTINUOUS TRANSFORMATION, CHARACTERISTIC OF THE FUNCTION THAT THE GROUP APPLIES TO THE PREFERENCES OF INDIVIDUAL i AT DIFFERENT LEVELS OF SATISFACTION.

FOLLOWING ANOTHER SET OF "REASONABLE" ASSUMPTIONS: k_i REPRESENTS THE "WEIGHT" OF EACH INDIVIDUAL IN THE GROUP. MARGINAL Δ OF UTILITY FOR EACH INDIVIDUAL AND FOR ALL RANGES OF OUTCOMES EQUALLY AFFECT v(X).

⇒ COMMONLY USED FORM:

$$v(X) = \sum_i k_i u_i(X) \qquad \text{(linear form)}$$

⇒ THREE STEPS IN FORMULATION OF A GROUP VALUATION FUNCTION:

- SPECIFY ALL OUTCOMES AND OBJECTIVES: X_I'S
- ASSESS INDIVIDUALS' UTILITY FUNCTIONS OVER $X = \{X_I\text{'s}\}$
- AGGREGATE UTILITY FUNCTIONS (E.G., BY ASSESSING WEIGHTS FOR INDIVIDUALS' UTILITIES)

EXAMPLE OF A COLLECTIVE CHOICE

CHOICE OF A RESTAURANT

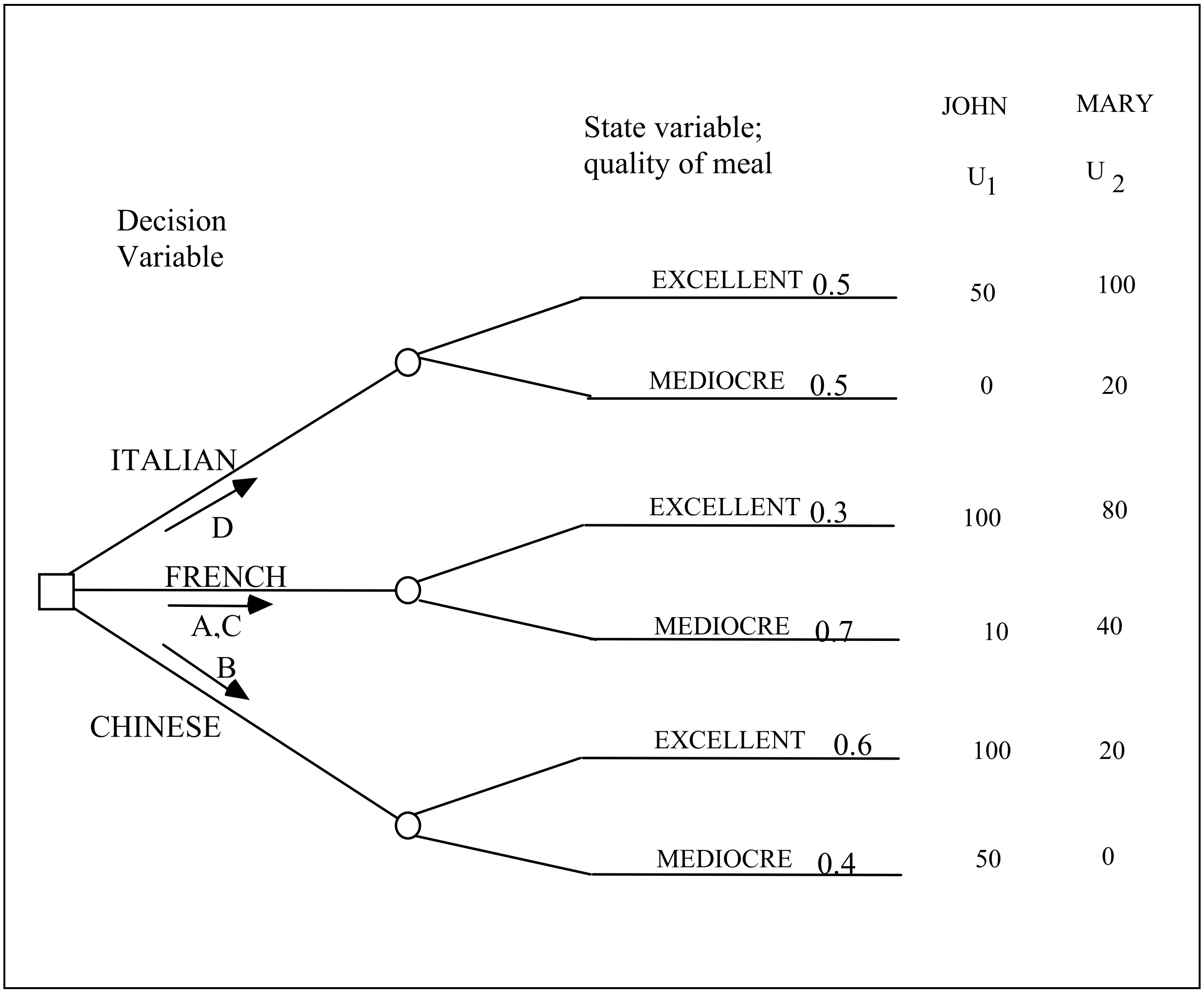


FIG. 11.2: COLLECTIVE CHOICE (TWO DECISION MAKERS)

VALUES OF DINNER ALTERNATIVES FOR THE GROUP

	ITALIAN	CHINESE	FRENCH
(A) $V(x) = V^*(U_1) + V^*(U_2)$	0.6032	0.6454	0.6521
(B) $V(x) = U_1 + U_2$	85	92	89
(C) $V(x) = 3V^*(U_1) + 7V^*(U_2)$	3.44	2.37	3.54
(D) $V(x) = 3U1 + 7U2$	495	324	475

How the group values the utility of each member:

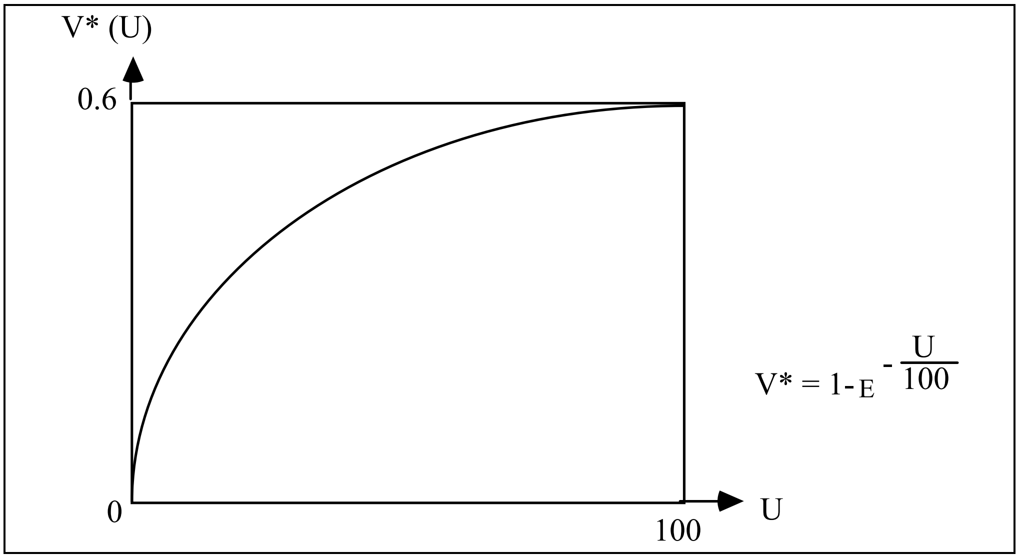


Fig. 11.3: Collective valuation of two utility functions for a specified group

NOTE: The group represented here is "risk averse" with respect to the utilities of its members, i.e., they care more about securing a certain level of happiness.

(A) $v(x) = v^*(u_1) + v^*(u_2)$ → French

(B) $v(x) = u_1 + u_2$ → Chinese

(C) $v(x) = 3\,v^*(u_1) + 7\,v^*(u_2)$ → French

(D) $v(x) = 3\,u_1 + 7\,u_2$ → Italian

Implication: The group agrees to behave as a single decision maker following a process that leads to V(x) and the maximization of EV(x).

11.4 DECISIONS IN THE PUBLIC SECTOR

Problem of scale in linear aggregation

- Social valuation function reflects interpersonal comparisons. aggregation mechanism reflects philosophical standpoint (relative weight of individuals).
- Values at stake in the aggregation process: individual freedom (to accept or not accept collective decision) vs. social justice (equal weight to each person or more weight to the least well off, and the transfers that it implies).
- Issues:
  Risk attitude vs. total income (resources).
  Trade-offs among the different attributes.
  Redistribution effects (among individuals).

- <u>Problem</u>: individual risk attitude depends on resource constraints and on available information (and perception) about the issues and alternatives.
- <u>Ethical issue</u>: Importance of informed consent.
- Alternative normative approach: <u>norm on the decision process</u> (with risk-benefit estimates as part of the information). Example: voting process with equal vote power to all individuals.

<u>The government as a "super decision maker"</u>

An analytical normative approach: what <u>should</u> be the government risk attitude and the government trade-offs among attributes, given individual preferences?

- Government risk attitude

  If the level of the outcomes is much lower than the level of the country resources, the government should be <u>risk indifferent</u> (even though individuals are often risk averse).

Arguments:

* Maximizes expected return on the whole set of public projects in the long run.

* Equity reasons: in the case of human safety, the potential victims should be equally protected independently from the perception biases introduced by the size of the possible outcomes.

- Government trade-offs among the different attributes could be based on the average <u>"willingness to pay"</u> among the different individuals.

Arguments:

* Uses globally for each attribute what the total population is willing to invest.

* Gives equal weight to all citizens.

<u>In practice:</u>

Elected officials make decisions according to their own preferences (delegation through votes). Time value and discounting may be influenced by election schedule. Make sure that "facts matter" - What and how much are we talking about? If perception is reality, make sure that perception is influenced by facts.

- DISTRIBUTION EFFECTS

  QUESTION OF REALLOCATION OF RESOURCES (WHO PAYS VS. WHO BENEFITS) IN THE DOMAIN OF RISKS. SAME ISSUES AS THOSE RAISED BY TAX POLICIES. SIMILAR POLITICAL AND PHILOSOPHICAL QUESTIONS.

- <u>ETHICAL ISSUE</u>: ENVIRONMENTAL EQUITY.

  IS COMPENSATION THE RIGHT MECHANISM? ARE ALL PARETO-OPTIMAL DEALS ETHICAL AND SOCIALLY ACCEPTABLE?

11.5 RISK ATTITUDE REVEALED BY PUBLIC REACTIONS

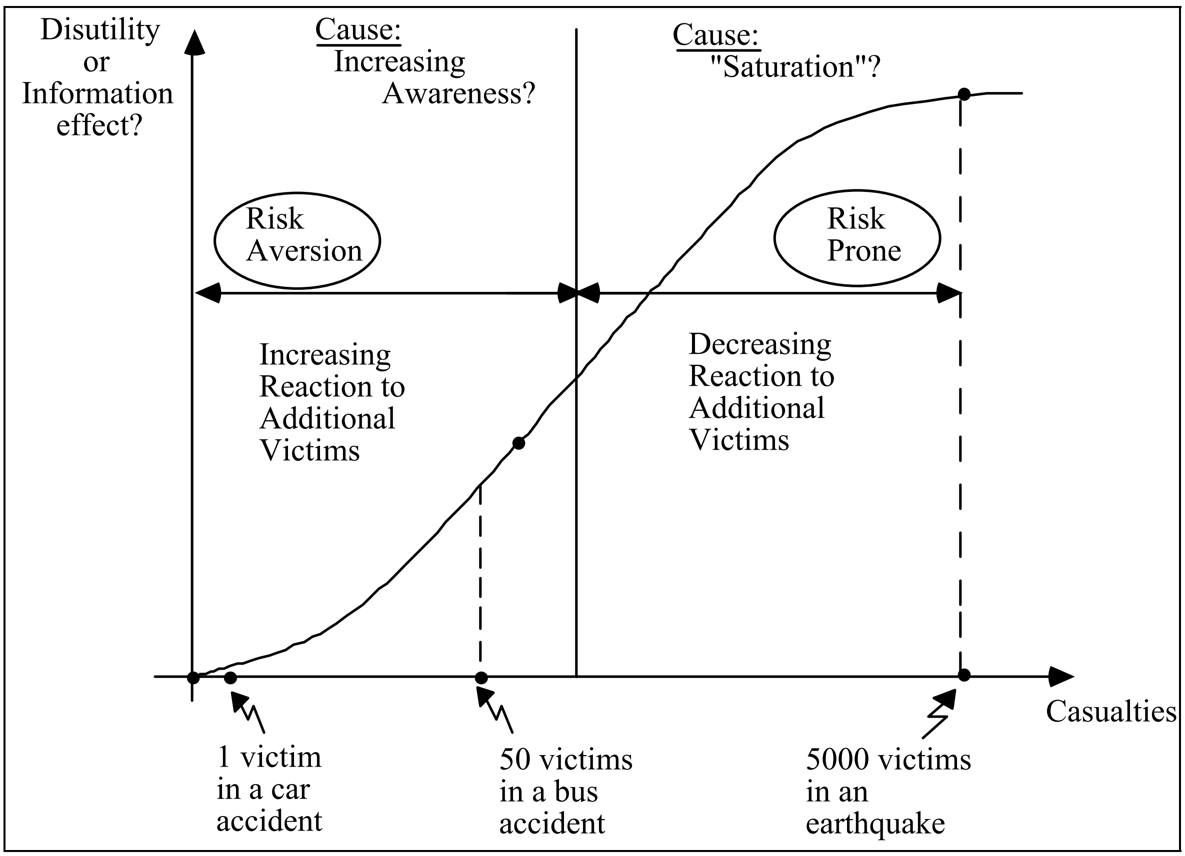


FIG. 11.4: PREFERENCES GENERALLY REVEALED BY THE PUBLIC