Qualified-Bound-Pricing Method for Combinatorial Contract Auctions

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ABSTRACT

Prediction of some financial indices, such as unemployment rate, is an interesting problem in financial markets. One possible approach is to make use of the market force to do the prediction. An auction mechanism is commonly used as a platform to achieve this aim. Usually, the concerned index is divided and digitalized to several states and the value of each state is determined by the bids on it. Under the principle of Dutch Auction. combinatorial contracts auctions are considered to enhance liquidity, transparency and efficiency in the trading platform.

The objective of the study is to find the qualified orders. The linear programming technique based on the concept of aggressiveness is employed to compute the qualified and unqualified orders. The results are presented to demonstrate the effectiveness and the efficiency of the proposed method.

KEYWORDS QBP. order aggressiveness. order qualification. CCA

1 Introduction

Recently, some financial institutes or investment banks, for instance, Goldman Sachs and Deutsche Bank, can offer a platform for trading certain options for transferring such risks directly. These financial institutes or investment banks have adopted a pari-mutuel mechanism to price the various economic derivatives on certain indices [1,2], such as US non-farm payrolls [3]. The Pari-mutuel Digital Call Auction (PDCA) principle patented by Longitude is the first mechanism commercially available for trading such derivatives [4].

The main aim of this paper is to develop a new trading algorithm as an alternative to PDCA. The new method is called Qualified-Bound-Pricing (QBP) method and it is is based on the concept of aggressiveness. We will show that the QBP method is promising in the design of auction platform.

1.1 Problem Specification

The underlying index/figure is divided into several states. For example, an auction on global GDP growth this year can be divided into 3 states.

states	1	2	3
range(%)	$(-\infty, -3]$	$(-3, 3)$	$[3, \infty)$

Table 1. GDP example: dividing states

Now define a contract on a state as a paper agreement so that on maturity, the contract worth a notional \$N (\$100 in this paper) if it is on the winning state and worth \$0 if it is not on the winning state.

Then the customers enter orders, buying and selling contracts betting on one or a combination of states ,with a price limit and a quantity limit. An order to buy states 1 and 2 in the above example with price limit \$70 and quantity limit 1000 means:

1. The customer will be willing to pay at most \$70 for one state 1 contract and one state 2 contract.
2. The quantity accepted by the auction organizer x, should not exceed 1000, i.e., $x \leq 1000$.
3. The customer will pay $x*$(price of state 1 contract and state 2 contract) to the auction organizer. He/she will receive \$100x if the GDP growth is actually lower than 3%. If the GDP growth is not less than 3%, he/she loses the amount invested.

The sell order can be implemented into the system similarly.

Define a complete set to be the combination of one contract in each outcome(state 1, state 2 and state 3 contracts together). Obviously one complete set worths a notional no matter which state is the winning state. If there exists orders forming complete sets and with their price limits greater than their notional value, then the auction organizer takes no risk to accept these trades. The commission charge is ignored in this paper. The key issue is how to determine the qualified orders and find auction settlement prices (ASP) for the states from a given set of customer orders.

The auction organizer collects all bids, determines the qualified orders, and gives the ASP for each state. The ASP $\mathbf{p}$ is determined based on the principle of Dutch Auction, i.e., given by the price limit of the least qualified orders. Notice that customers pay the trading price which is computed by

$$\sum_{j=1}^{S} c_j p_j \tag{01QP}$$

instead of the price limit.

In Section 2, we will introduce our method in two stages. Numerical examples are given in Section 3 to illustrate the effectiveness of our method. Some concluding remarks are given in Section 4.

1.2 Definitions and notations

We explain format, notations and special terms used in the following parts.

An order is in the format

$$[\pi, q, \mathbf{c}]$$

where π is the price limit, q is the quantity limit, $\mathbf{c}$ is a 0-1 vector with length S, which stands for the number of states. For example, buy order $[70, 100, (1\ 1\ 0)]$ is to buy at most 100 shares of state 1 & 2 with the price limit \$70.

The Fill x is the amount traded in the end. $x > 0$ means the order is qualified and $x = 0$ means the order is unqualified.

The Contract Aggressiveness is the difference between the price limit and the trading price of the contract. If the order is $[\pi, q, \mathbf{c}]$, and the ASP is $\mathbf{p}$, then the contract aggressiveness is given by:

$$\pi - \mathbf{c}^T \mathbf{p} \tag{02QP}$$

If interest and commission are ignored, selling some contracts at a price π is equivalent to buy the complementary contracts at a price of $100 - \pi$. Hence we transform all sell orders to buy orders for simplicity in later parts, if not specified.

We exam a simple example of 2 buy orders:

$$[30, 1, (1\ 0\ 0)] \quad [70, 1, (0\ 1\ 0)]$$

The two orders cannot form a complete set unless the price of state 3 is 0. We introduce the concept of zero fill z in the QBP method to deal with this problem. The zero fills are single state buy orders with price limit 0 and unlimited quantity. For instance, the zero fill in state 3 is $(0, \infty, [0\ 0\ 1])$.

π_i	price limit
q_i	order quantity
$\mathbf{c}_i$	shape of order i
x_i	fill of order i
z_j	zero fill in state j
$\mathbf{p}$ or p_j	auction settlement price (ASP)
M	number of complete sets

Table 2. Table of notations

2 The QBP Method

The backbone of the QBP method is:

(i) Qualification-Disqualification: Determine qualified and unqualified orders by linear programming (LP) models.

(ii) Bounding-Pricing: Make use of the price limits of qualified orders as the bound constraints of ASP $\mathbf{p}$.

2.1 Qualification-Disqualification

The first stage of our method is to classify the qualified and unqualified orders, i.e., find the fill for each order.

2.1.1 Maximization of Total Aggressiveness

We model the problem of qualification-disqualification into an linear programming problem. The objective is to maximize the total aggressiveness because more aggressive orders must have higher priority to be chosen.

The ASP is not included in this stage because the orders are qualified by have the possibility of finding a match to form complete set(s) with high enough sum of price limits. It is not restricted to be expressed by the ASP. Therefore, an linear programming model is sufficient for this stage as we only need to determine the fill of each order.

Moreover, excluding $\mathbf{p}$ in qualification-disqualification dramatically improve the computational performance of the QBP method. We can deal with an auction with a huge number of orders and states in a few seconds.

Formulation (1):

$$\text{Max} \quad J \equiv \sum_{i=1}^{n} \pi_i x_i - 100M$$

maximize profit

$$\text{s.t.} \quad \begin{cases} x_i \le q_i & i = 1, \cdots, n \\ \sum_{i=1}^{n} c_{ij} x_i + z_j = M & j = 1, \cdots, S \end{cases}$$

p_j

Decision variables in formulation (1) are fill $\mathbf{x}$, zero fill $\mathbf{z}$ and the number of complete sets M. J is the total aggressiveness.

The total aggressiveness J is computed from:

$$\sum_{i=1}^{n} \left(\pi_i - \sum_{j=1}^{S} c_{ij} p_j \right) x_i + \sum_{j=1}^{S} (0 - p_j) z_j$$

There are only two sets of constraints in formulation (1). The first set of constraints $x_i \le q_i$ is the natural condition for the fill x_i and the second set of constraints $\sum_{i=1}^{n} c_{ij} x_i + z_j = M$ refers that the orders can form a complete set. It counts the number of qualified orders (including zero fills) in each state and set them to a fixed number M, which is exactly the number of complete sets of customer orders.

2.1.2 Priority Ranking System

The output of the LP problem is fill $\mathbf{x}$, the number of complete sets M and the maximum aggressiveness J.

We introduce a priority ranking system because it is possible that there is no sufficient information to determine the unique fills. We rank the orders and maximize the higher priority one first. The entering time of the orders may be one choice for ranking.

We maximize the fill of order 1 while keeping all the constraints of formulation (1) and adding one constraint

$$\sum_{i=1}^{n} \pi_i x_i - 100M = J$$

Then we fix the fill found (x_1^F), maximize the fill of order 2 and do the same process. The algorithm is:

Weight ...

for k = 1 to n

$$
\begin{aligned}
&\text{Max} \quad x_k^F \equiv x_k \\
&\text{s.t.} \quad \begin{cases} x_i \le q_i & i = 1, \cdots, n \\ \sum_{i=1}^{n} c_{ij} x_i + z_j = M & j = 1, \cdots, S \\ \sum_{i=1}^{n} \pi_i x_i - 100M = J & \\ x_t = x_t^F & t = 1, \cdots, k-1 \end{cases}
\end{aligned}
$$

where x_t^F is given by the previous optimizations.

It is clear that the fill $\mathbf{x}$ is determined by the sequential optimization. obviously. there are at most $n-1$ such optimizations for optimizing the fill in priority. To further reduce the number of optimizations, the sensitivity analysis technique can be employed to speed up this process. It provides information about which decision variables are fixed and do not need to be re-calculated. For a practical example of 150 orders, we find that there are only 5 optimization steps to determine the fill of each order instead of 150 optimization steps.

2.2 Pricing

After stage 1, the qualified orders and their fills are determined. The orders being qualified and unqualified will impose some bounds on the prices of the state. i.e. the prices of the states must not violate the price limits of those qualified orders:

$$x_i = q_i \Rightarrow \sum_{j=1}^{S} c_{ij} p_j \le \pi_i \tag{03QP}$$

$$0 < x_i < q_i \Rightarrow \sum_{j=1}^{S} c_{ij} p_j = \pi_i \tag{04QP}$$

$$x_i = 0 \Rightarrow \sum_{j=1}^{S} c_{ij} p_j \ge \pi_i \tag{05QP}$$

where qualified orders are served as upper bound condition and unqualified orders are served as lower bound condition. Notice that there always exists at least one ASP satisfying these three types of constraints.

Alternatively, PDCA is a nonlinear programming method which find qualified orders and the ASP at one time. The formulation is transformed from a working paper of Longitude by using our notations. PDCA introduces the concept of opening orders in their formulation, where a certain amount of money is placed in each state. Then PDCA tries to maximize the total number of complete sets formed [1].

QBP and PDCA differs in three perspectives. Firstly, QBP is separated into two stages and PDCA solves the problem as a whole. Secondly, QBP is linear while PDCA is nonlinear and binary variables are needed. At last, the objective of QBP is fair for the customer in a sense by maximizing the aggressiveness while PDCA maximizes the number of complete sets is good for the auction organizer.

3 Examples and Performance

Example 1: This example is a comparison of the PDCA method by Longitude and the QBP method [1]. Orders are:

$$[30, 300, (1\ 0\ 0)] \quad [40, 200, (0\ 1\ 0)] \quad [50, 100, (0\ 0\ 1)]$$

The solution of PDCA: fill $\mathbf{x} = [100, 100.8333, 100]$. On the other hand, the QBP method gives: fill $\mathbf{x} = [100, 100, 100]$.

PDCA	30	40	30
upper bound	30	40	50
lower bound	10	20	30

Table 3. Feasible price boundaries revealed by QBP method

Example 2: We simulate a 1000 orders 10 states problem. A "fair" price is set at first. The order quantity and shape are random and the price limit follows normal distribution, with mean computed by the "fair" price and variance HK$5 [1].

492 orders out of 1000 are qualified and there is no zero fill.

fair price	5	5	5	5	10	15	15	15	15	10
upper bound	5.00	5.32	4.87	4.29	10.55	15.46	14.62	14.66	15.50	9.79
lower bound	5.00	5.30	4.86	4.28	10.55	15.46	14.61	14.64	15.47	9.77

Table 4. Feasible price boundaries revealed by QBP method

We generated some data and tested the performance of the QBP method. The test was performed using a Pentium 4 2.4GHz desktop, and the software environment was Microsoft Windows XP Professional SP1 with Java 1.4.1_02 and Lindo API 2.0 installed. The testing data are randomly generated and the table was based on 500 test scenarios each (including CPU time used in stage 2).

[1] The data can be retrieved from http://hkusua.hku.hk/~h0392020/mco-eg-data1.htm

	Number of orders			
States	500	1000	1500	2000
18	0.6s	2.7s	4.0s	6.4s
28	1.4s	3.4s	5.8s	7.6s
38	2s	3.8s	6.0s	8.3s

Table 5. Computational performance

4 Conclusions

The QBP method is a linear programming model which contains LP methods to find the market driven price for various financial indices. It is easy to program and fast to solve.

Moreover, the QBP method is efficient, i.e., no possible trading after the auction is possible. The QBP method is risk-controllable. In most practical cases, it is risk-neutral. The QBP method implement fairness by introducing the Priority Ranking System and gives unique solution by sequential LP optimization. These results demonstrate the effectiveness and efficiency of the QBP method.

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