The basic variables in a basic solution are not necessarily all nonzero. This is noted by the following definition.

**Definition.** If one or more of the basic variables in a basic solution has value zero, that solution is said to be a *degenerate basic solution*.

We note that in a nondegenerate basic solution the basic variables, and hence the basis $B$, can be immediately identified from the positive components of the solution. There is ambiguity associated with a degenerate basic solution, however, since the zero-valued basic and nonbasic variables can be interchanged.

So far in the discussion of basic solutions we have treated only the equality constraint (8) and have made no reference to positivity constraints on the variables. Similar definitions apply when these constraints are also considered. Thus, consider now the system of constraints

$$Ax = b$$
$$x \geq 0,$$

which represent the constraints of a linear program in standard form.

**Definition.** A vector $x$ satisfying (10) is said to be *feasible* for these constraints. A feasible solution to the constraints (10) that is also basic is said to be a *basic feasible solution*; if this solution is also a degenerate basic solution, it is called a *degenerate basic feasible solution*.

### 2.4 THE FUNDAMENTAL THEOREM OF LINEAR PROGRAMMING

In this section, through the fundamental theorem of linear programming, we establish the primary importance of basic feasible solutions in solving linear programs. The method of proof of the theorem is in many respects as important as the result itself, since it represents the beginning of the development of the simplex method. The theorem itself shows that it is necessary only to consider basic feasible solutions when seeking an optimal solution to a linear program because the optimal value is always achieved at such a solution.

Corresponding to a linear program in standard form

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax = b, \\
& \quad x \geq 0,
\end{align*}
\]

a feasible solution to the constraints that achieves the minimum value of the objective function subject to those constraints is said to be an *optimal feasible solution*. If this solution is basic, it is an *optimal basic feasible solution*.

**Fundamental theorem of linear programming.** Given a linear program in standard form (11) where $A$ is an $m \times n$ matrix of rank $m$,